Screening in two-dimensional lattice gauge theories

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Project carried within the MPD programme "Physics of Complex Systems" of the Foundation for Polish Science and co-financed by the European Regional Development Fund in the framework of the Innovative Economy Programme.

INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
Foundation for Polish Science

EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND

Lattice 2012, Cairns, 28-06-2012

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Two-dimensional models of QCD – abelian case

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- Start with abelian $U(1)$ gauge group: QED_2:
  - Free electrodynamics in 1+1 dimensions – no true dynamics (at least in $\mathbb{R}^2$) but confining linear potential for probe charges
  - Schwinger, 1962: QED_2 with single massless flavour: solvable using bosonisation trick, exhibits charge screening $\forall Q_{ext}$.

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    - Schwinger, 1962: QED_2 with single massless flavour: solvable using bosonisation trick, exhibits charge screening $\forall Q_{ext}$.
    - Coleman, Jackiw, Susskind, 1975: perturbative addition of small mass. Only integer charges Q_{ext} are screened for non-zero mass (string breaking).
    - Plethora of numerical studies (most only for $Q_{ext} \in \mathbb{Z}$) both using bosonisation & lattice methods (and also DLCQ). Still field of active research (e.g. Dürr, 2012).

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- Limitation of large-N limit of fundamental matter – quenched fermion dynamics. Idea: use fermions in two-index representations of $SU(N)$, e.g. the adjoint.
- Adjoint fermions analysed theoretically (e.g. Kutasov, 1994) and by DLCQ (e.g. Bhanot, Demeterfi, Klebanov, 1993) but hardly any lattice calculation in 1+1 dimensions.

Wilson loops: confinement vs. charge screening

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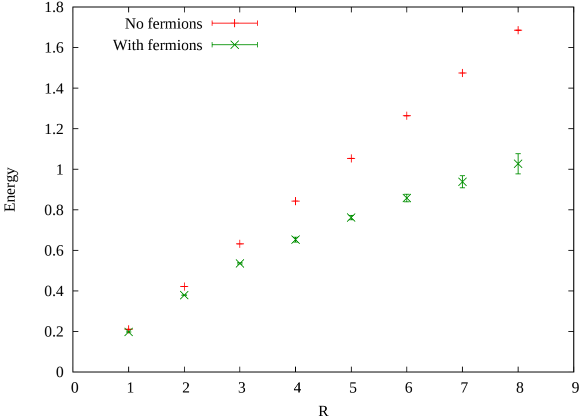
- Full QCD_4 – charge screening at large distances $\gtrsim 1fm$ ("string breaking") – hard to observe on the lattice.
- Conjecture: small overlap of Wilson loops onto broken-string ground state. Inclusion of other observables (static-light mesons) necessary (Bali et al., 2005).

Wilson loops and string breaking in QED_2

In terms of confinement/screening QED_2 resembles QCD_4.



Wilson loop energies, $g^{-2} = 1.5, \kappa = 0.245, V = 24 \times 24$.

Wilson loops and string breaking in QED$_2$

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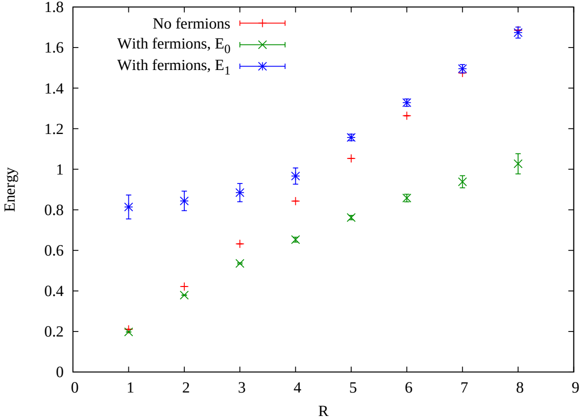
Extract 1st excited state: $W(R,T) \cong C_0 e^{-E_0(R)T} + C_1 e^{-E_1(R)T}$



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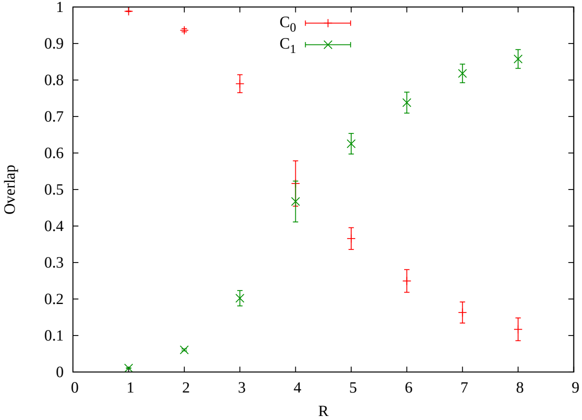

Overlap coefficients, $g^{-2} = 1.5, \kappa = 0.245, V = 24 \times 24$.

Fractional charges

- Despite numerous analyses for $Q_{ext} \in \mathbb{Z}$, equation (CJS, 1975)

$$\sigma = \# mg \left(1 - \cos(2\pi Q_{ext})\right)$$

  no lattice simulations for non-integer Q_{ext} (though note Hamer, Kogut, Crewther, Mazzolini, 1982).
- "Charged Wilson loop":

$$W_Q(R,T) \equiv \exp\{iQg \oint_{\Box} dl_\mu A_\mu(l)\} = \big(W(R,T)\big)^Q$$

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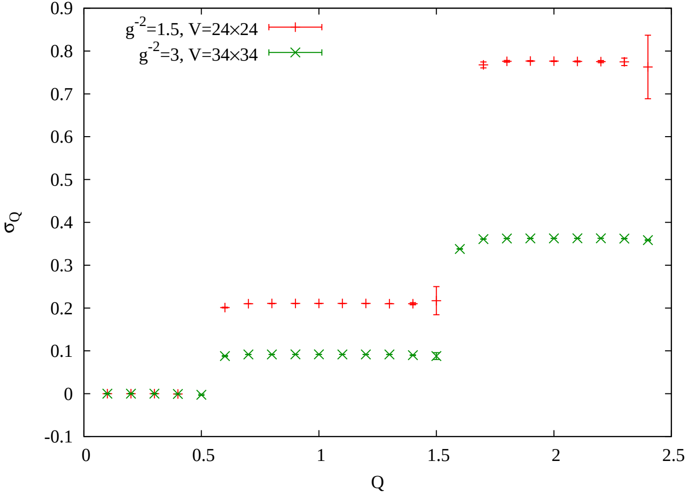

String tension vs. external charge in pure gauge QED$_2$

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- To understand this, we came back to pure gauge $U(1)$ theory.
- σ_Q with $Q \notin \mathbb{Z}$ is "projected" to the closest integer value, independent on lattice size up to $V \approx 200 \times 200$.

Fractional charges cntd.

- Manton, 1984: Continuum QED_2 on a (spatial) circle of radius 1
- Pure gauge theory: one can set the gauge so that $A_x(x,t)$ is x-independent and $A_x \in [0,1)$ – configuration space is periodic.

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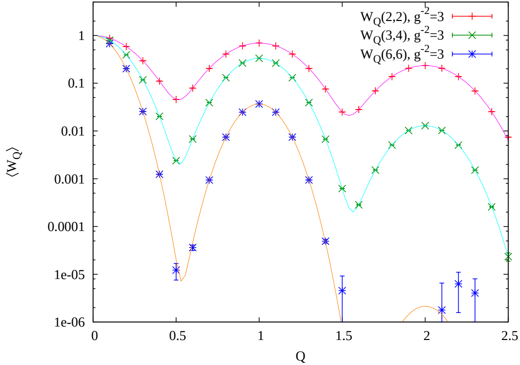
- Pure gauge theory: one can set the gauge so that $A_x(x,t)$ is x-independent and $A_x \in [0,1)$ – configuration space is periodic.
- One obtains a quantum-mechanical system with Hamiltonian $H = \frac{\pi L}{Q^2}\dot{A_x}^2$ and functions in Hilbert space satisfying $\psi(A_x = 0) = \psi(A_x = 1)$.
- Only integer-charged states satisfy periodicity. A state with arbitrary charge Q gets projected to integer charges:

$$\langle P_Q(A_x,\tau)P_Q(A_x,0)\rangle = \sum_{n=0} e^{-n^2 e^2 \pi\tau} \left(\frac{\sin(\pi(Q-n))}{\pi(Q-n)}\right)^2$$

Fractional charges cntd.

Ansatz for Monte Carlo: Wilson loops with $Q \in \mathbb{Z}$ as on infinite lattice, decomposition to integer-charged basis for $Q \notin \mathbb{Z}$:

$$W_Q(R,T) = \sum_{i=0}^{\infty} \left(\frac{I_i(\beta)}{I_0(\beta)}\right)^{RT} \left(\frac{\sin(\pi(Q-i))}{\pi(Q-i)}\right)^{2R+2T}$$



Monte Carlo Wilson loop data vs. the ansatz (not a fit).

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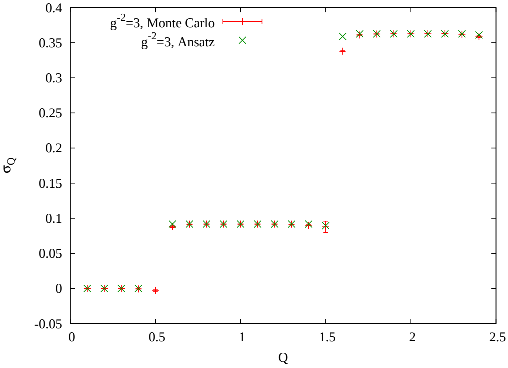

One-exponent fits to discrete data generated using the ansatz vs. Monte Carlo results

Conclusions & outlook

- Two-dimensional theories share many intrinsic features with those in 4 dimensions and can be used as a test bed for concepts relating to QCD$_4$.
- Wilson loops can be used as a probe of string breaking but very large statistics is required as overlap on the ground state becomes poor.
- "Topological finite-size effect" is an obstacle to implement fractional charges in a lattice simulation (different boundary conditions?).

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Plans for future:

- Move to non-abelian theories, in particular with adjoint matter, which are of great interest recently (technicolor, large-N equivalences).