Study of the conformal hyperscaling relation through the Schwinger-Dyson equation

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Reference:

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(arXiv: 1201.4157 [hep-lat])

Outline

1. Introduction
2. Hyperscaling relation
3. SD equation analysis
4. Summary

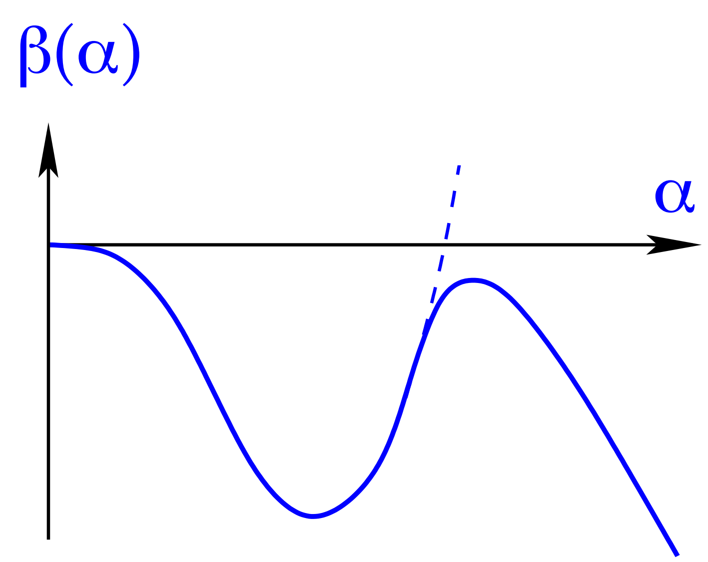
Introduction

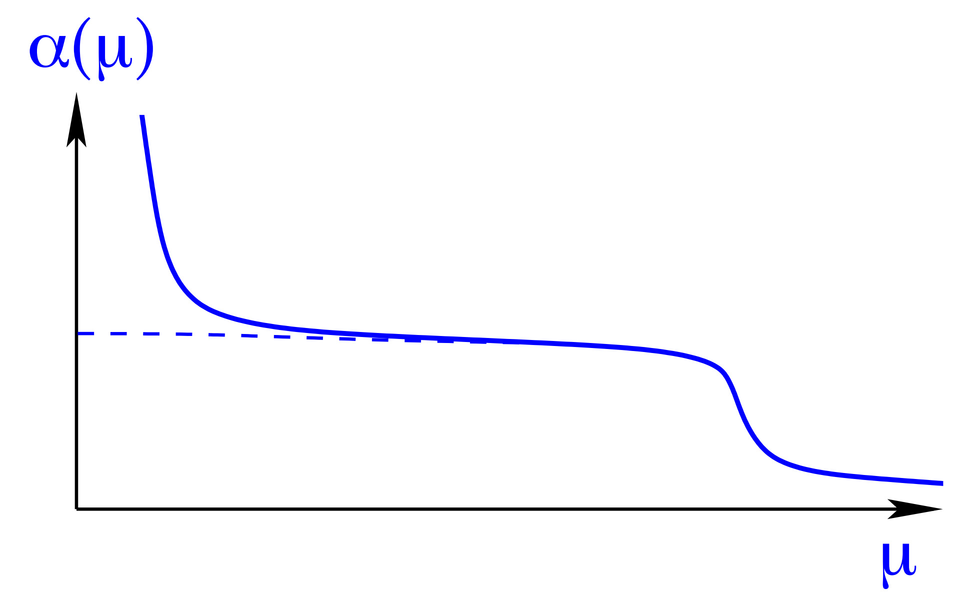
Technicolor model is an attractive candidate for the dynamical origin of the Electroweak symmetry breaking, though one based on naive scale up of QCD is phenomenologically disfavored...

A theory which has an (approximate) infrared fixed point with large mass anomalous dimension might be viable

Walking Technicolor

Yamawaki-Bando-Matumoto (1986)





Walking theory --- any example?

Large flavor QCD

$$\boxed{\text{RGE}} \quad \mu\frac{d}{d\mu}\alpha(\mu) = \beta(\alpha) = -\,b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$$

$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	$+$	$+$	$-$
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	$+$	$-$	$-$

Walking theory --- any example?

Large flavor QCD

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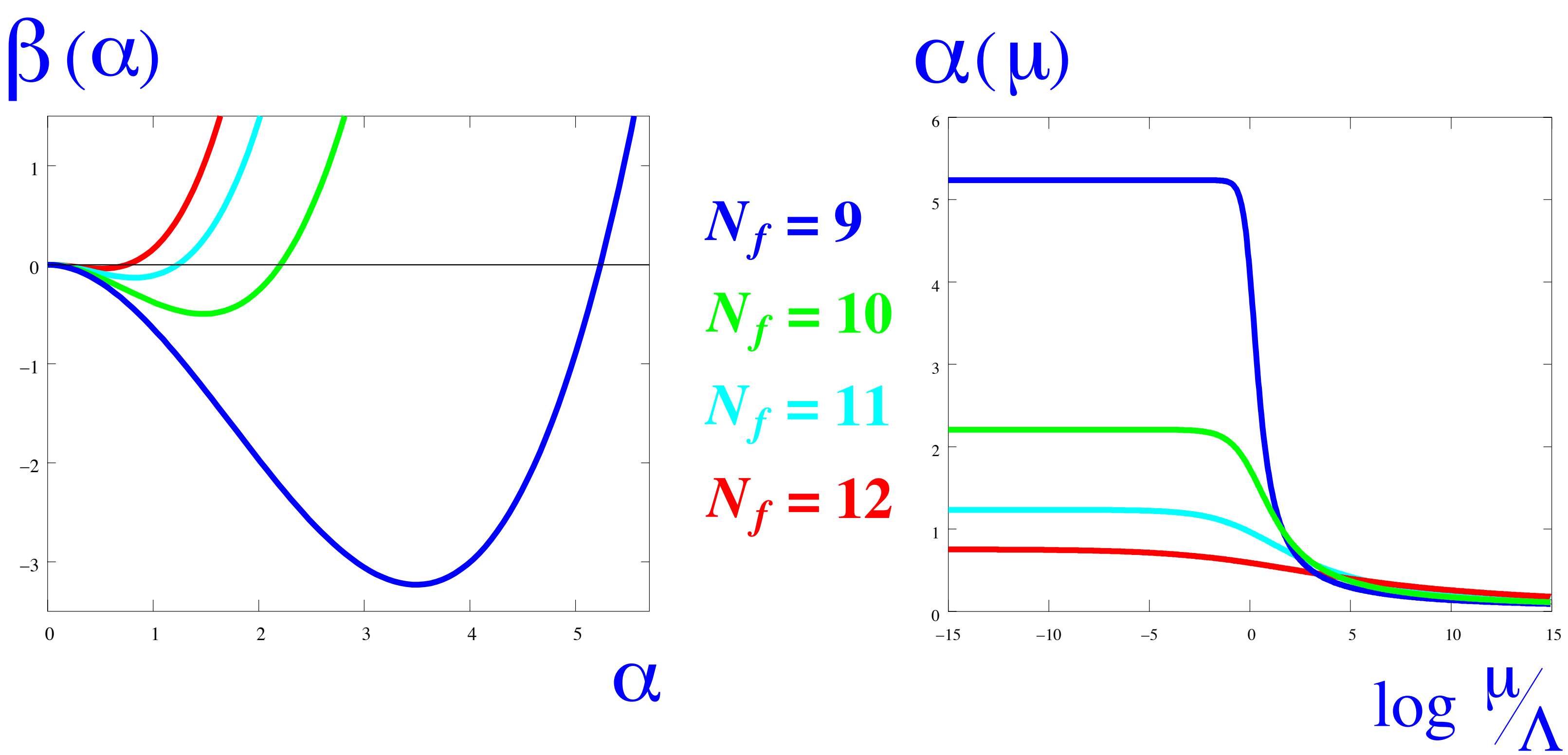
$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{6\pi}(33 - 2N_f)$	$+$	$+$	$-$
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	$+$	$-$	$-$

non-trivial infrared fixed point appears

$$\alpha_* = -b/c$$



Walking theory --- any example?

Large flavor QCD

Does this IRFP exist beyond perturbation?
What is the number of critical flavor?

Fully non-perturvative (lattice) study is desirable

Several ways of checking infrared conformality

- measure the running coupling
- study the spectrum
- ....

We discuss this here

2. Hyperscaling relation

Hyperscaling relation

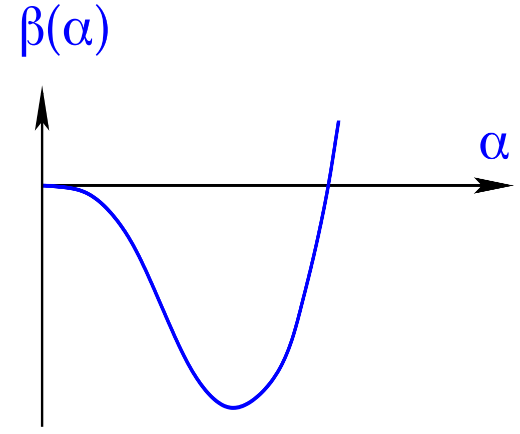
Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

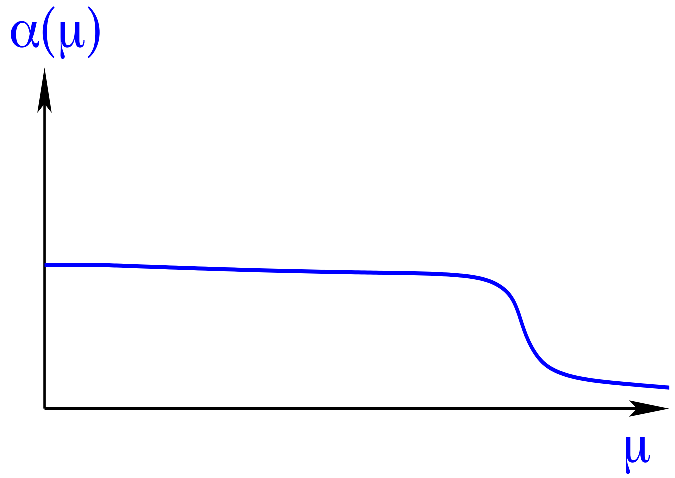
relations between physical quantities and the fermion mass in a mass-deformed infrared conformal theory

Hyperscaling relation

Ref : Miransky, PRD59 105003, 1999
Del Debbio, Zwicky, PRD82 014502, 2010

- Consider a theory which has an IRFP





Hyperscaling relation

Ref : Miransky, PRD59 105003, 1999
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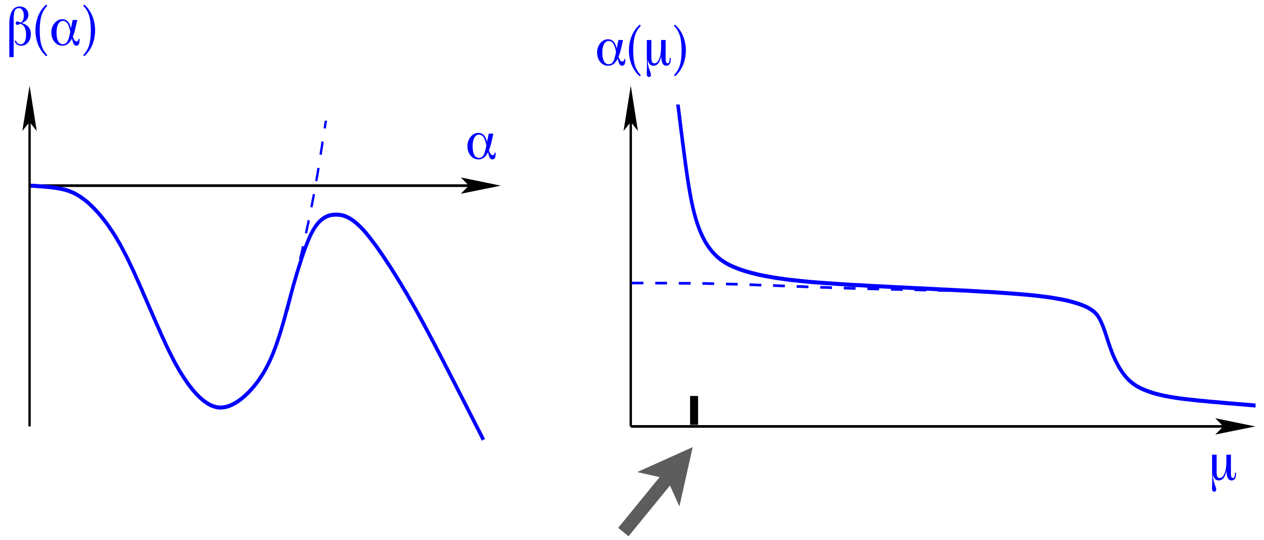
- Consider a theory which has an IRFP
- Deform it by introducing small fermion bare mass m



Hadrons emerge at this scale

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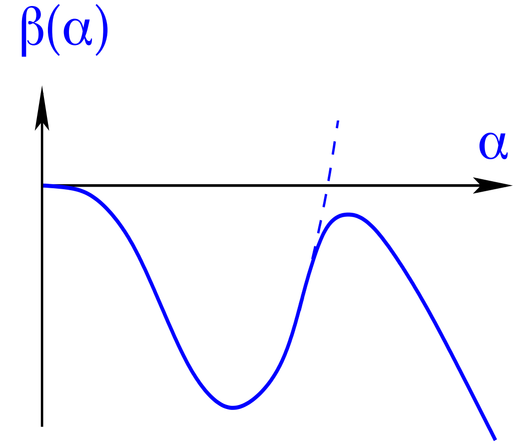

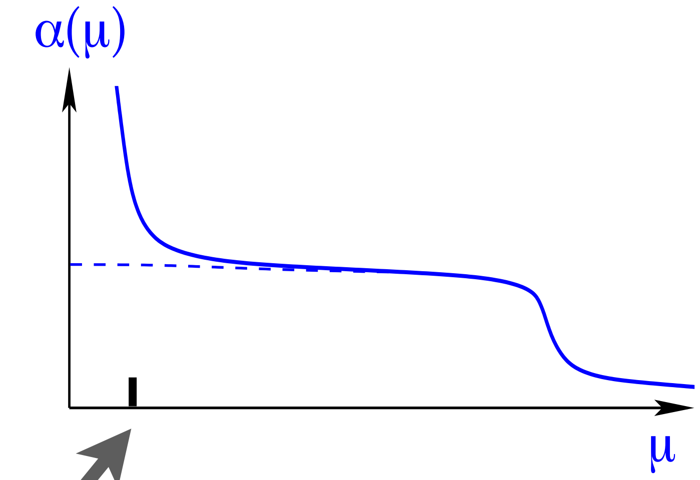

Hadrons emerge at this scale

Relations between low-energy physical quantities (for example, a hadron mass M_H) and the bare fermion mass m can be derived

$$\boxed{M_H \simeq (\text{const.})\ \mu\ \hat{m}^{1/1+\gamma_*}}$$

mass anomalous dimension at the fixed point

(normalized) bare quark mass: $\hat{m}(\mu) = m(\mu)/\mu$

a scale (in the case of lattice, one can consider $\mu = a^{-1}$)

Hyperscaling relation

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- Consider a theory which has an IRFP
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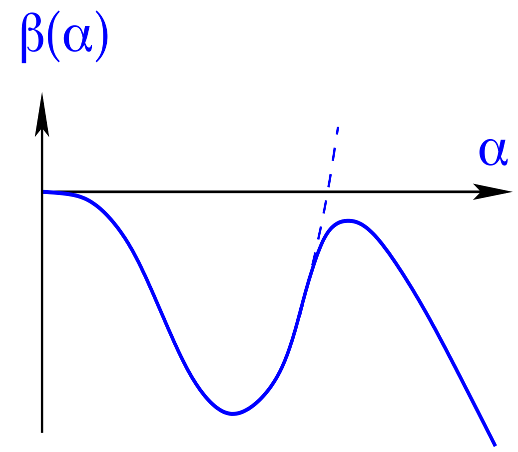
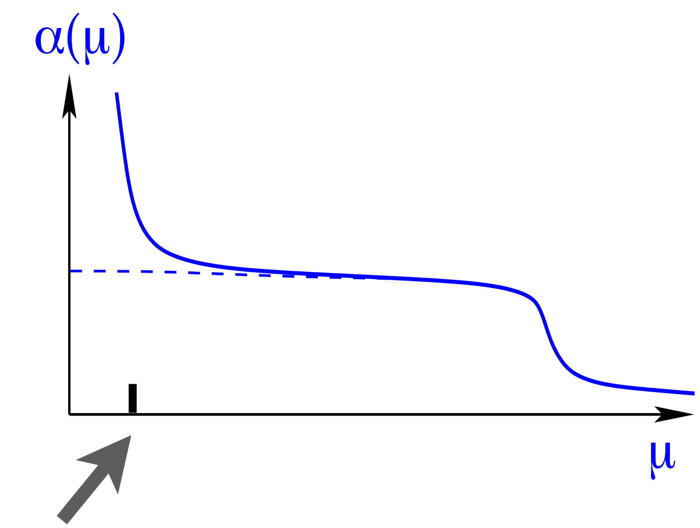




Hadrons emerge at this scale

Relations between low-energy physical quantities (for example, a hadron mass M_H) and the bare fermion mass m can be derived

When one considers a theory in a finite volume $\sim L^4$

$$\boxed{M_H = L^{-1} f\left(x\right) \quad \text{where} \quad x = \hat{L}\ \hat{m}^{1/(1+\gamma_*)}}$$

$$\hat{L} \equiv L\mu$$

Hyperscaling relation

Results of lattice simulation with various values of input $(\hat{L}, \hat{m})$
should satisfy the hyperscaling relation (with an appropriate value of γ_*)
if the theory has an IRFP

(Wouldn't it be interesting to see that **all the data** you have
with **different values of** $(\hat{L}, \hat{m})$ align in a single curve?)

Many lattice groups use this method to judge whether a theory
is conformal, and if it is, to estimate the value of γ_*

Hyperscaling relation

Results of lattice simulation with various values of input $(\hat{L}, \hat{m})$
should satisfy the hyperscaling relation (with an appropriate value of γ_*)
if the theory has an IRFP

Couple of questions arise here:

- How small m has be to observe the scaling?
  (What is the form of correction when it's not small enough?)
- When the original theory does not have an IRFP,
  how and how much the scaling relation is violated?

Schwinger-Dyson equation is a useful tool for such studies

- we know the phase structure, and a value of γ_* for a given theory
- analytic understanding can be obtained (to a certain extent)
- numerical calculations can be easily done in a wide range of parameter space on your PC

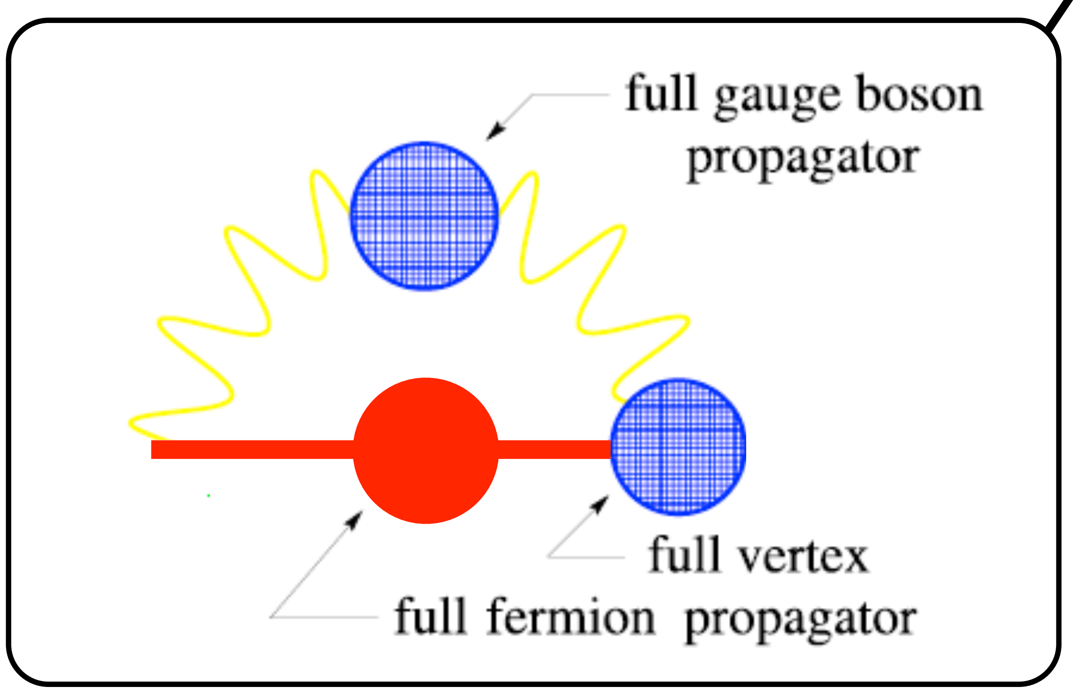
3. SD equation analysis

Schwinger-Dyson equation

Self-consistent equation for the
full fermion propagator $iS_F^{-1} \equiv A(p^2)\not{p} - B(p^2)$

bare fermion propagator

1 particle irreducible diagram



in equation...

Schwinger-Dyson equation

Self-consistent equation for the

full fermion propagator $iS_F^{-1} \equiv A(p^2)\not{p} - B(p^2)$

$$iS_F^{-1}(p) = \not{p} - m + C_2 \int \frac{d^4k}{i(2\pi)^4}\, \bar{g}^2(p,k)\, \frac{1}{(p-k)^2}\left(g_{\mu\nu} - \frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2}\right)\gamma^\mu\, iS_F(k)\, \gamma^\nu$$

We adopted...

Improved ladder approximation

$$C_2 = \frac{N_C^2 - 1}{2N_C}$$

$\bar{g}(p,k)$: running coupling

coupled equation for

$A(p^2),\ B(p^2)$

mass function

$\Sigma(p^2) \equiv B(p^2)/A(p^2)$

- advantage of using the SD equation

  we know whether a specific theory is in the chiral-symmetry-breaking phase or not, as well as a value of the mass anomalous dimension of that theory

Example: SU(3) gauge theory

- $\alpha_{\rm cr} = \pi/4$

  $\Leftrightarrow N_f^{\rm cr} \simeq 11.9$

$N_f < N_f^{\rm cr}$: confining
$N_f > N_f^{\rm cr}$: conformal

- $\gamma_* = 1 - \omega \quad \left(\omega \equiv \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}}\right)$

- strategy

We first generate "data" for various values of $(\hat{L}, \hat{m})$ through the SD equation, and do the hyperscaling analysis by using those data

- advantage of using the SD equation

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- strategy

By comparing the conclusion drawn by the hyperscaling analysis to the above **answer**, we can investigate properties of the hyperscaling analysis itself to generate data for $(\hat{L}, \hat{m})$, we formulate the SD eq. in a finite-volume spacetime

SD equation in the infinite volume:

$$
\begin{aligned}
A(p^2) &= 1 + C_2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{g}^2(p,k)}{k^2 A(k^2)^2 + B(k^2)^2} \\
&\quad \cdot \left[A(k^2) \frac{(p \cdot k)}{p^2 (p-k)^2} + 2A(k^2) \frac{\{p \cdot (p-k)\}\{k \cdot (p-k)\}}{p^2 (p-k)^4} \right], \\
B(p^2) &= m_0 + 3 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{g}^2(p,k)}{k^2 A(k^2)^2 + B(k^2)^2} \frac{B(k^2)}{(p-k)^2}.
\end{aligned}
$$

to put these in a finite-size box, all we have to do is replacing continuous momentum by the discrete one:

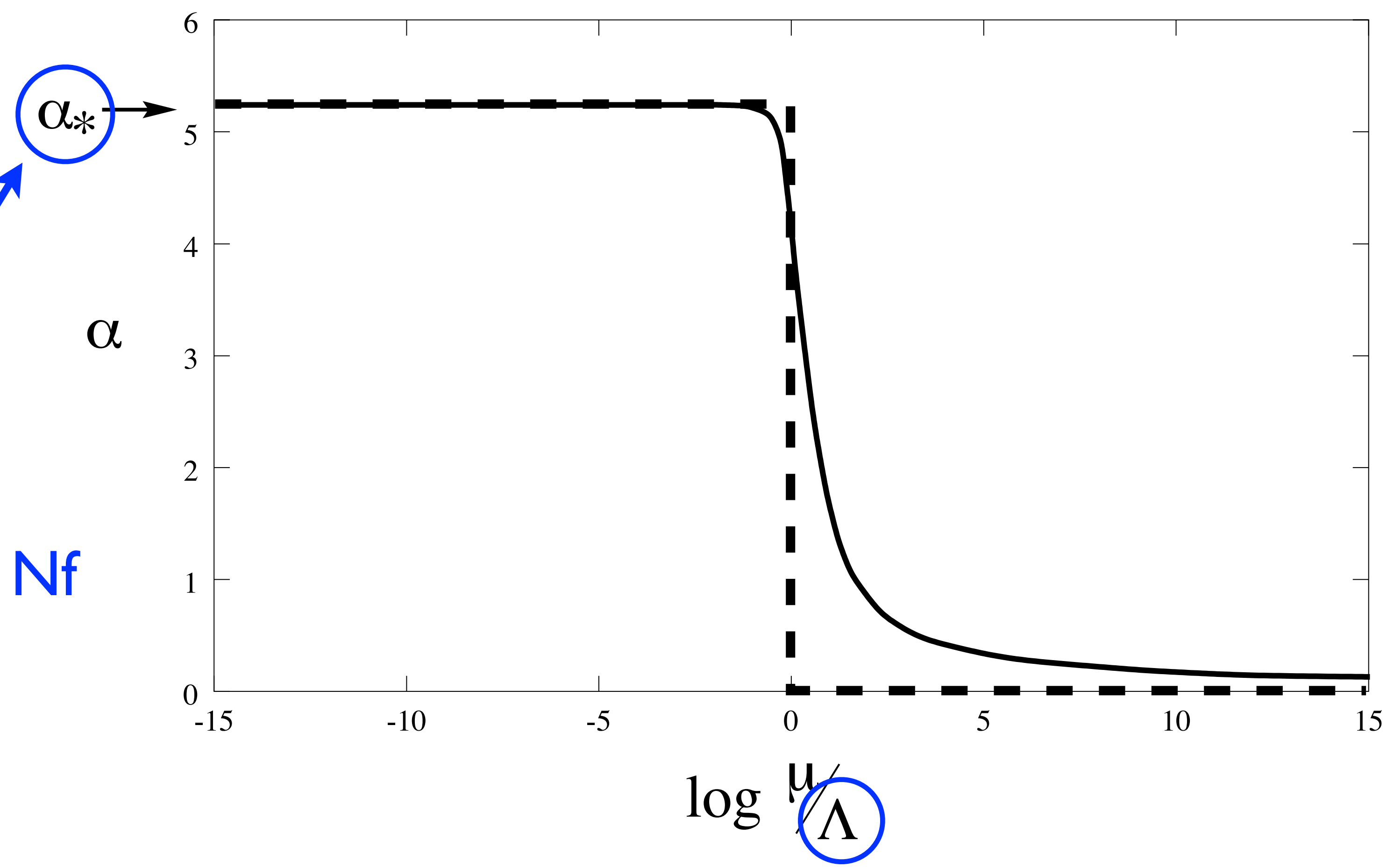
$$
p \longrightarrow p_n = \frac{2\pi n}{L}, \qquad \int_{-\infty}^{\infty} dp \; f(p) \longrightarrow \frac{2\pi}{L} \sum_{n=-\infty}^{\infty} f(p_n)
$$

Then, do the iteration to find a solution

setup for the current study

2-loop running coupling of the SU(Nc) gauge theory

Running coupling is approximated by the step function



fixed by Nc, Nf

Energy scale where the coupling begins to run

example of the solution

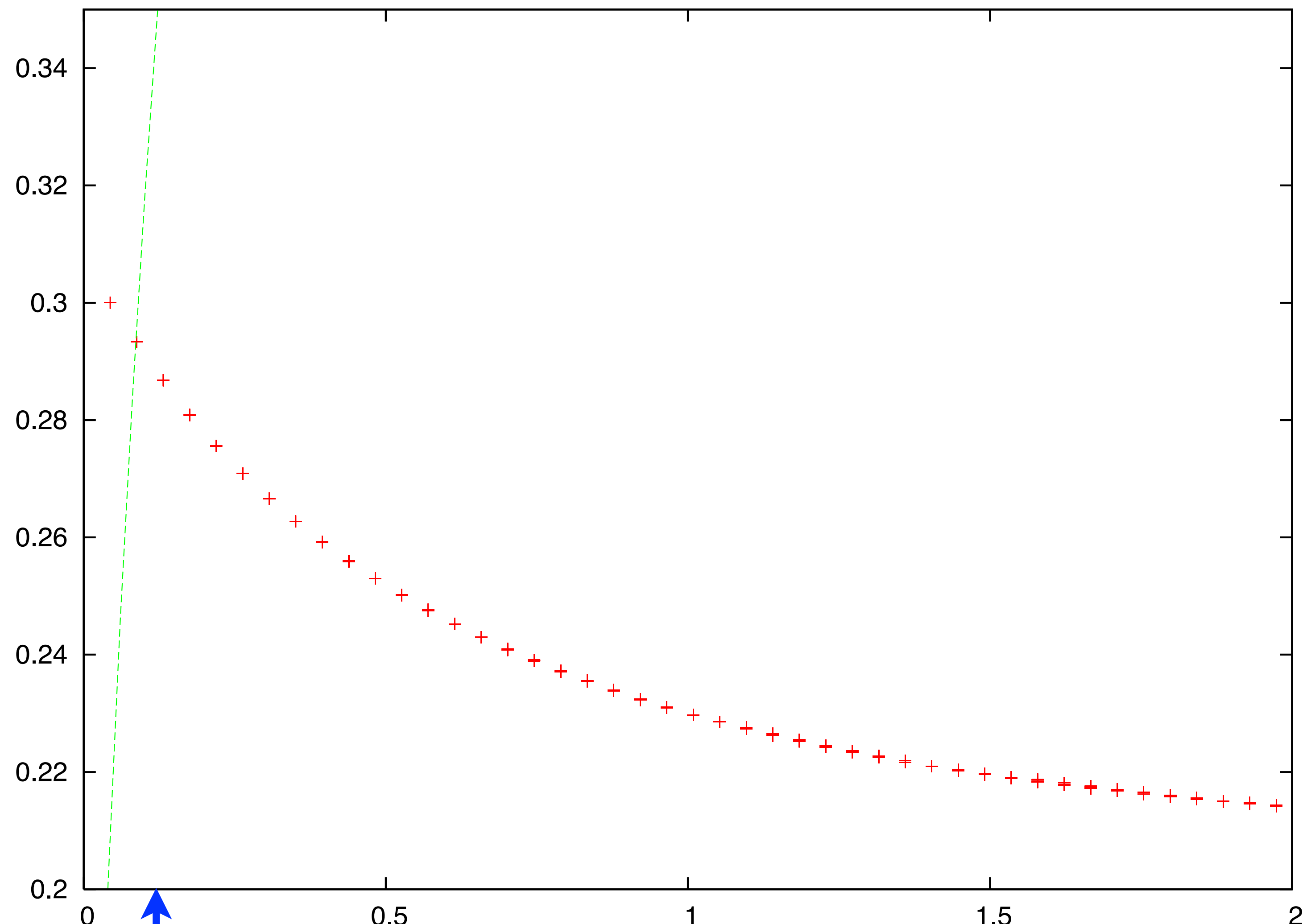
$N_C = 3$

$N_f = 12$

$L\Lambda = 30$

$m/\Lambda = 0.2$

$\Sigma(p^2)/\Lambda \equiv B(p^2)/A(p^2)/\Lambda$



p^2/Λ^2

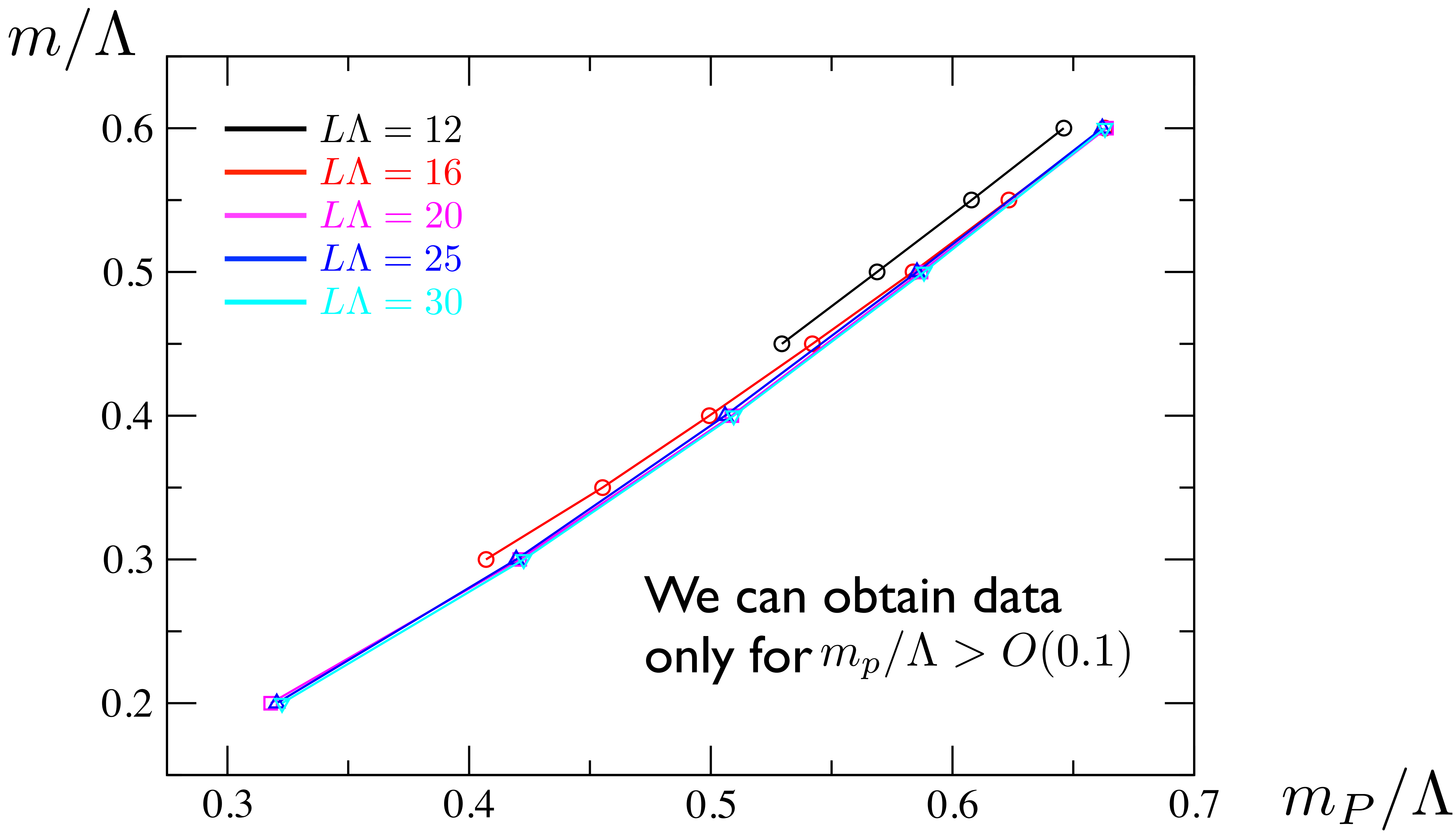
$$\boxed{\Sigma(p^2 = m_p^2) = m_p}$$

we take the "pole mass" m_p as a physical quantity

$$m_p \sim M_H$$

SU(3) gauge theory with 12 fundamental fermion

relation between m_P/Λ and m/Λ for various values of $L\Lambda$



SU(3) gauge theory with 12 fundamental fermion

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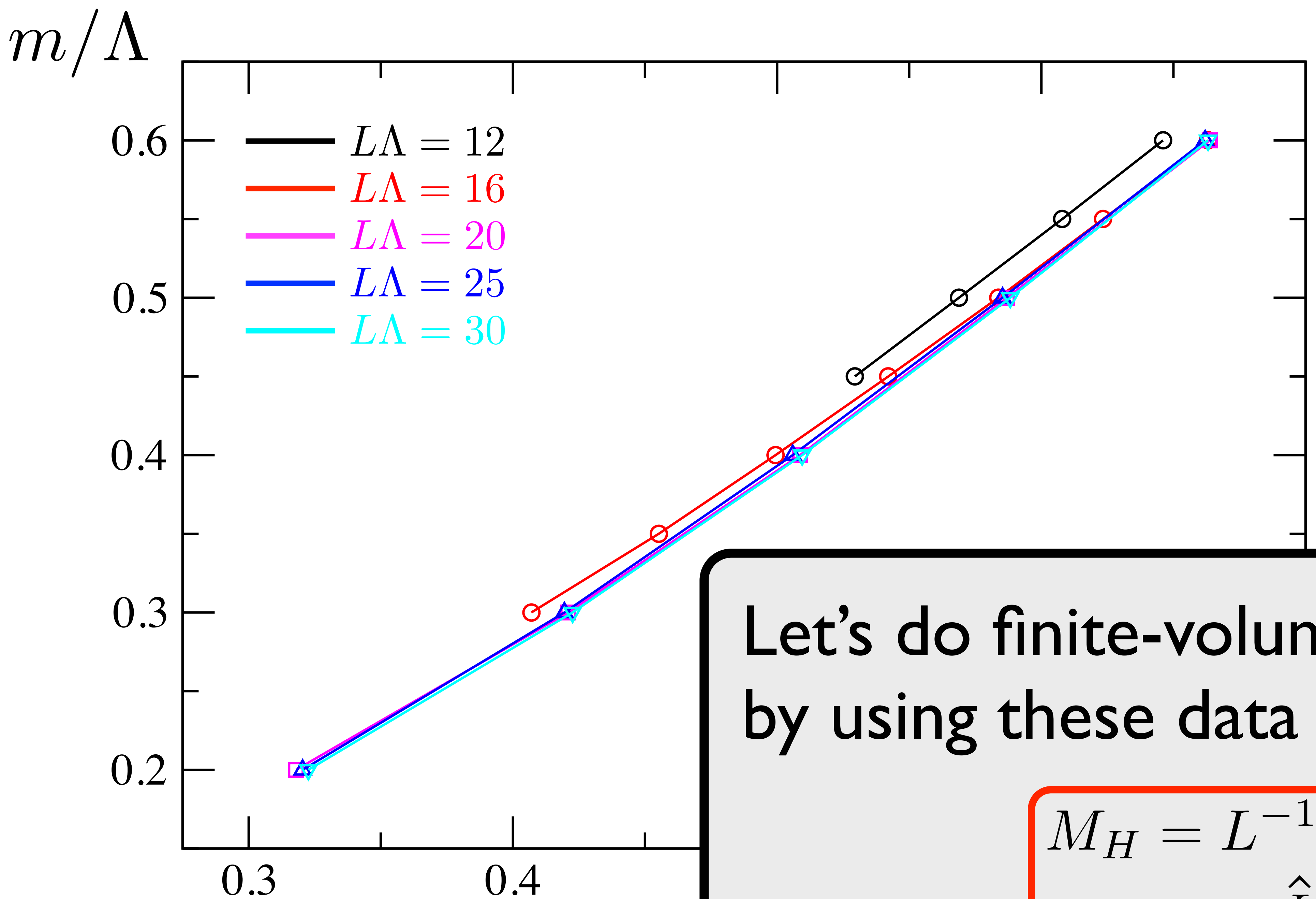
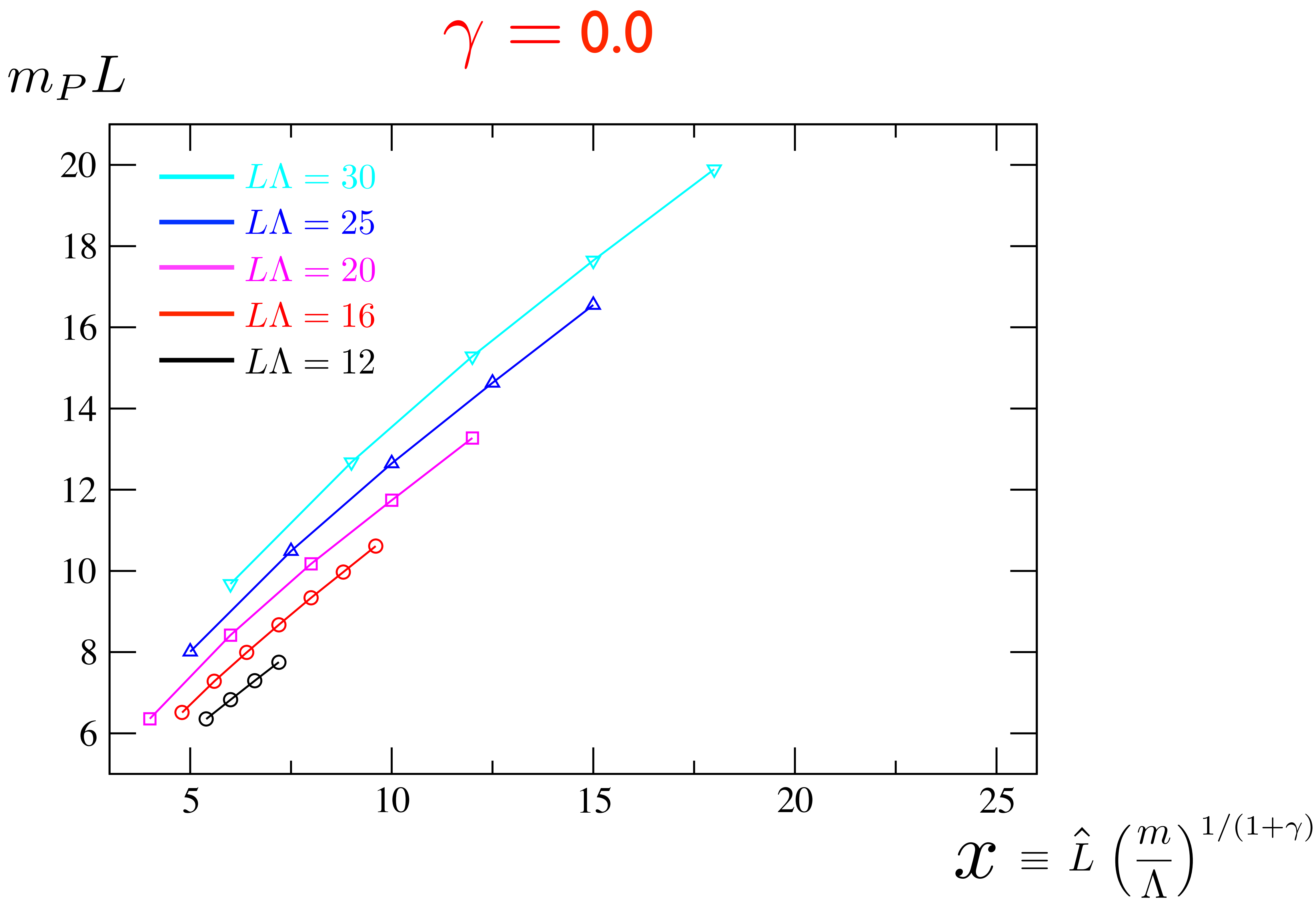


Let's do finite-volume scaling by using these data

$$\boxed{M_H = L^{-1} f(x) \quad \text{where } x = \hat{L}\,\hat{m}^{1/(1+\gamma_*)}}$$

SU(3) gauge theory with 12 fundamental fermion



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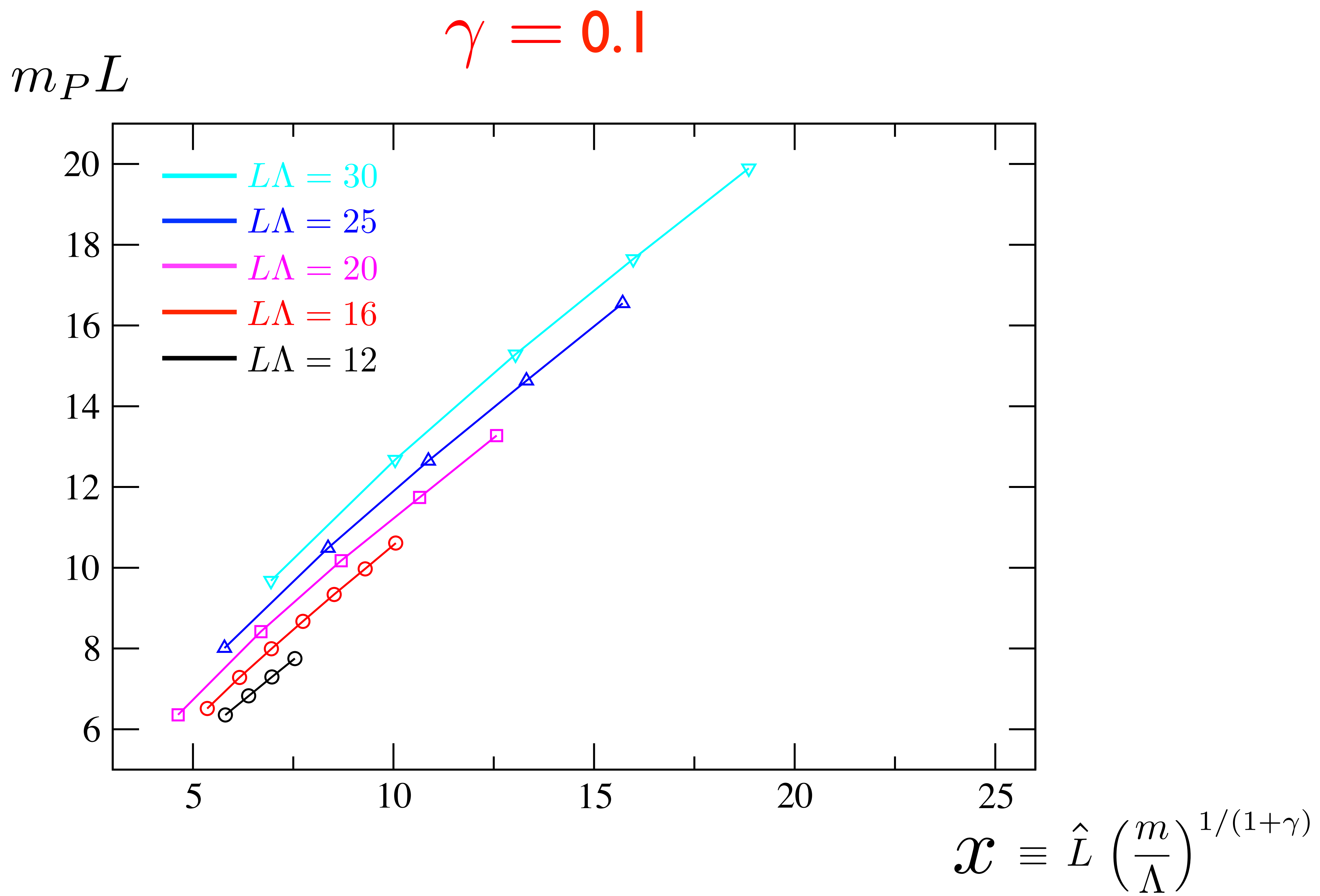
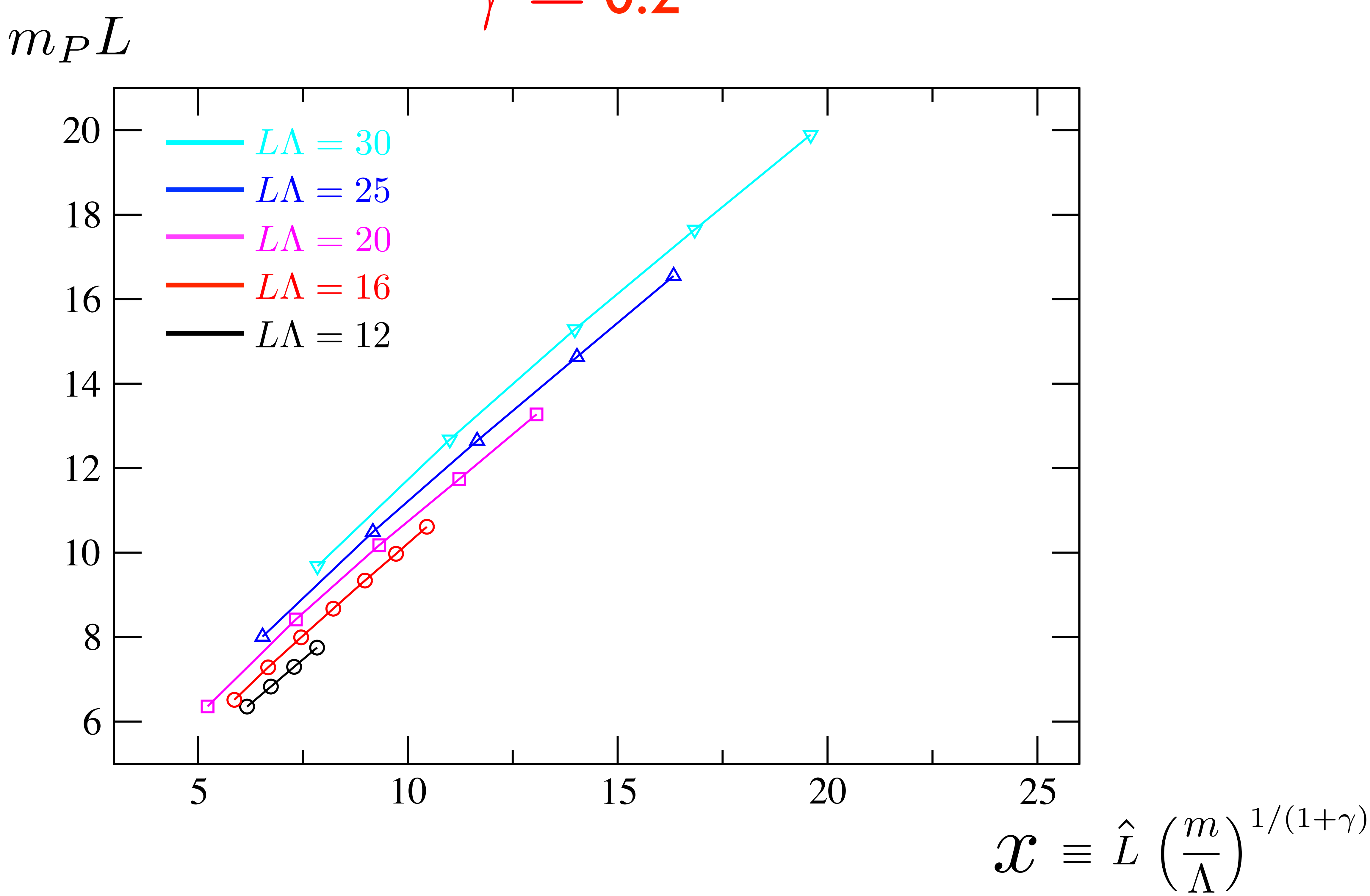


SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.2$



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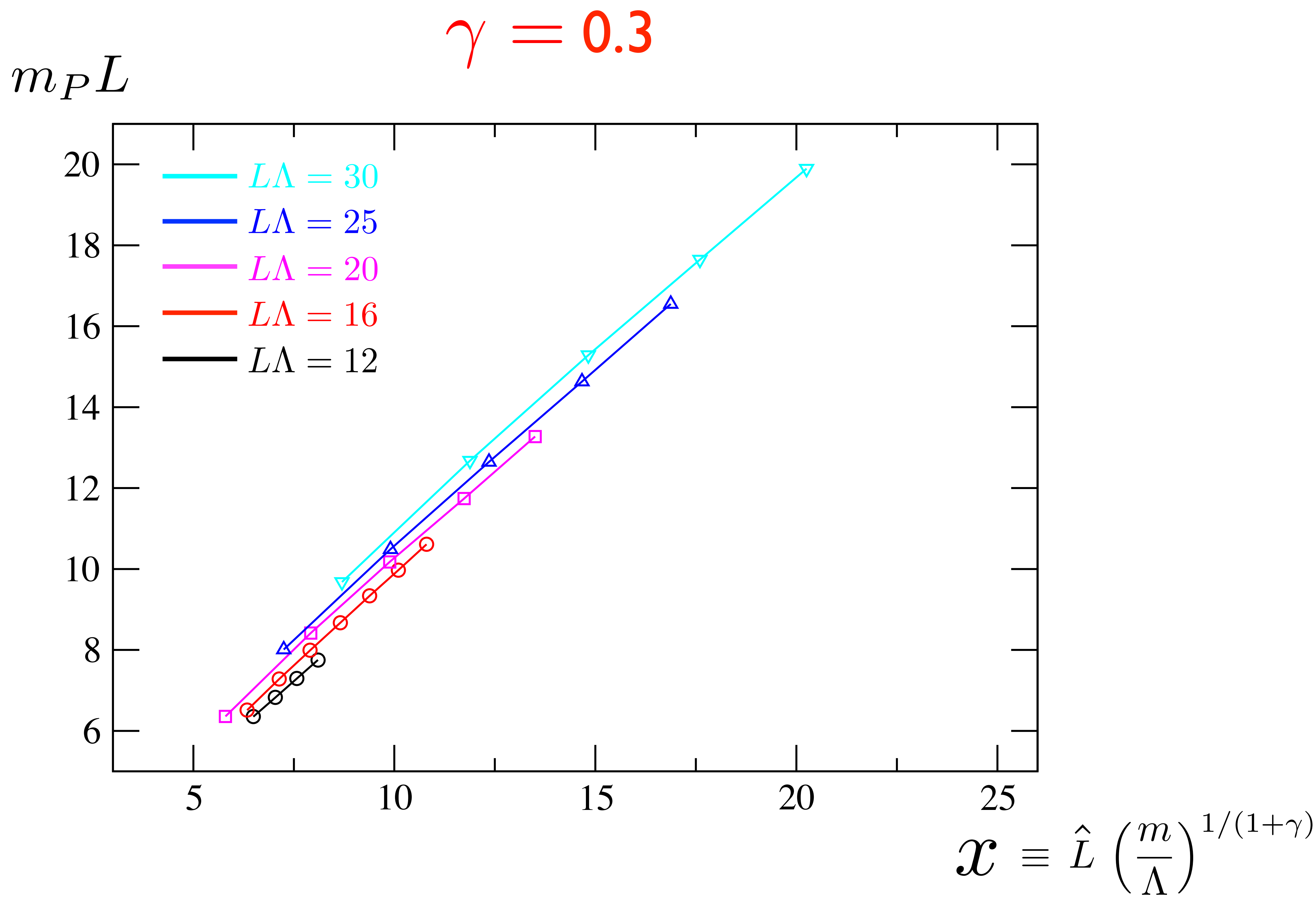
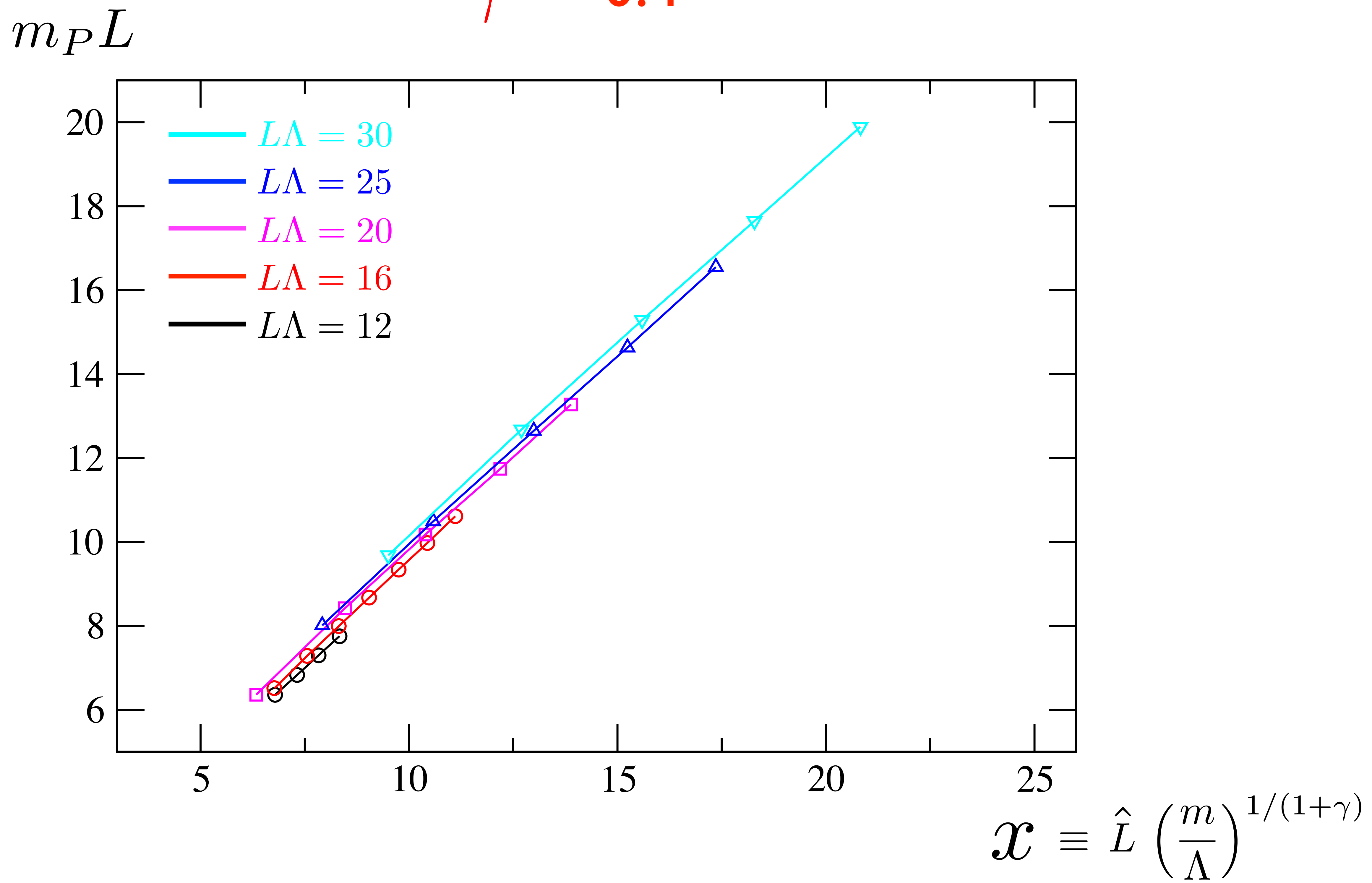


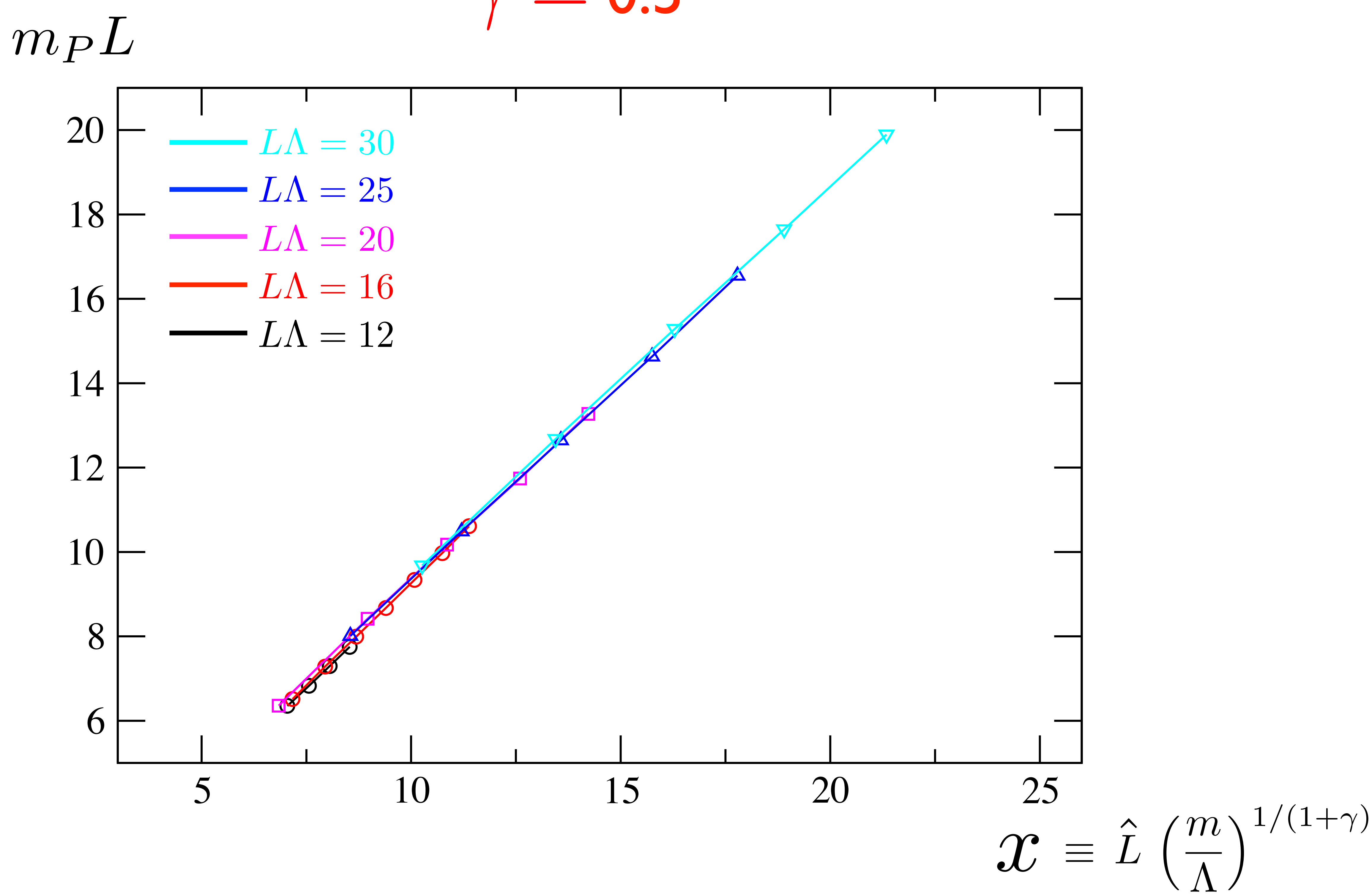
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.4$



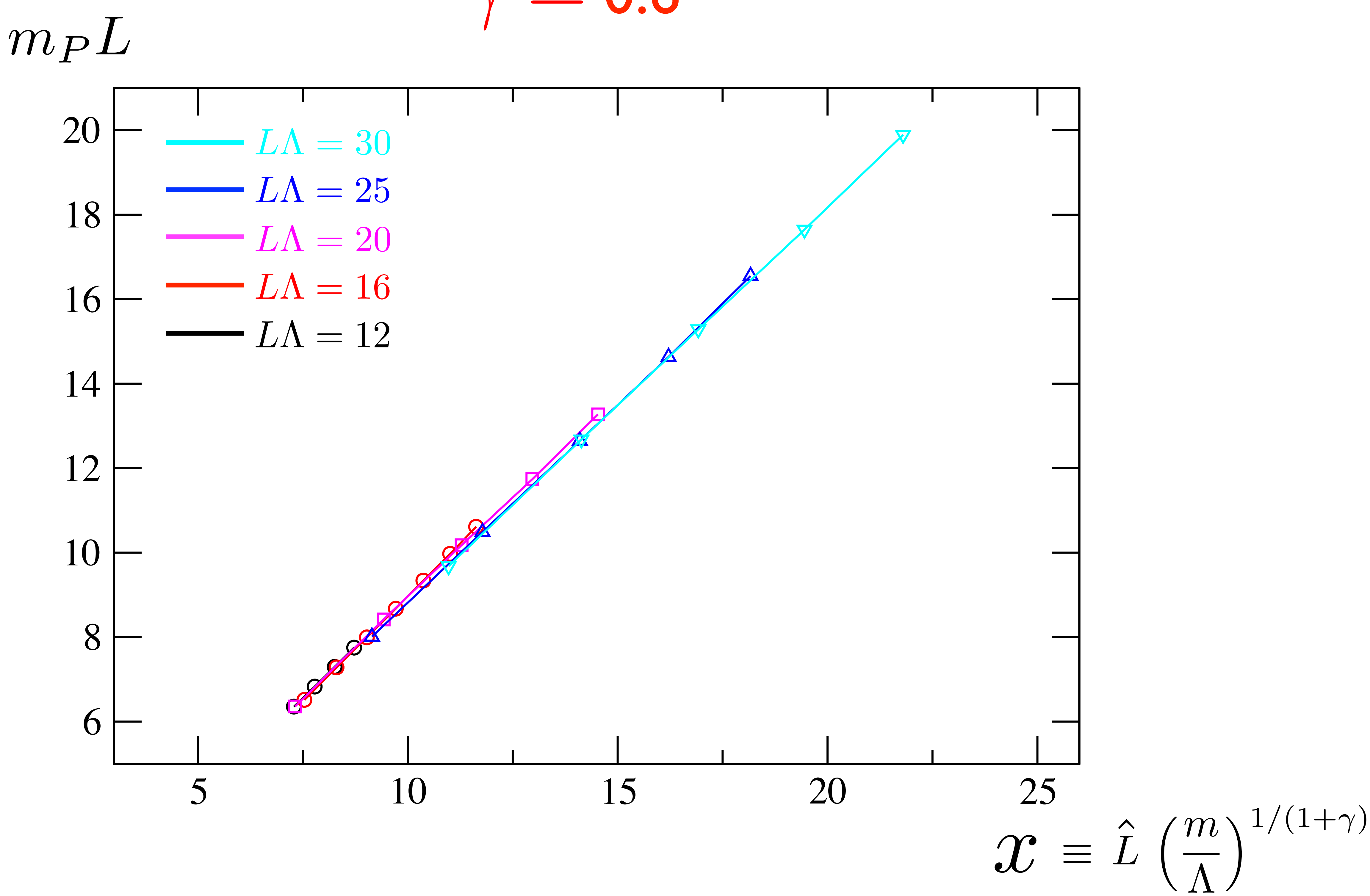
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.5$



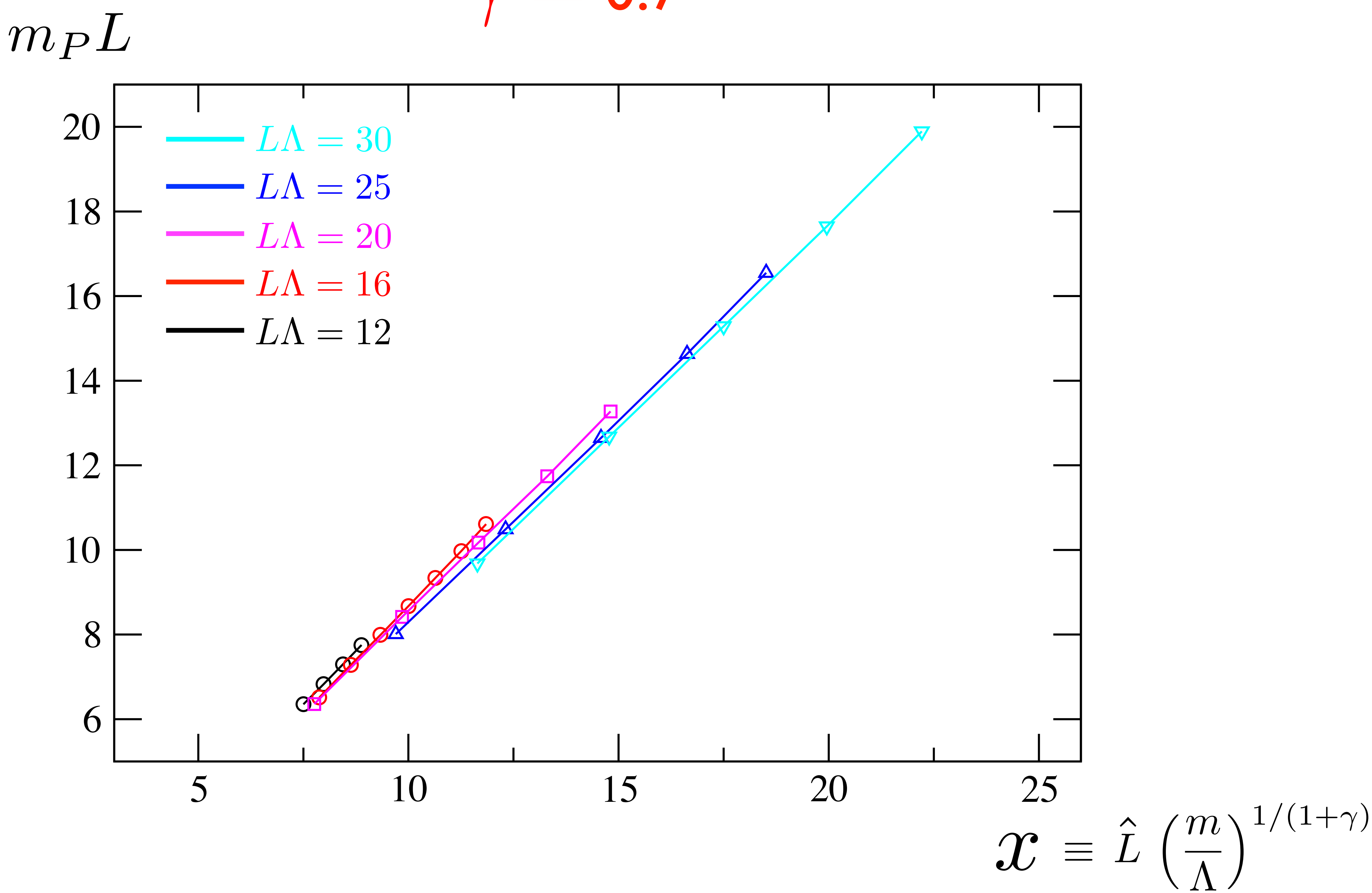
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.6$

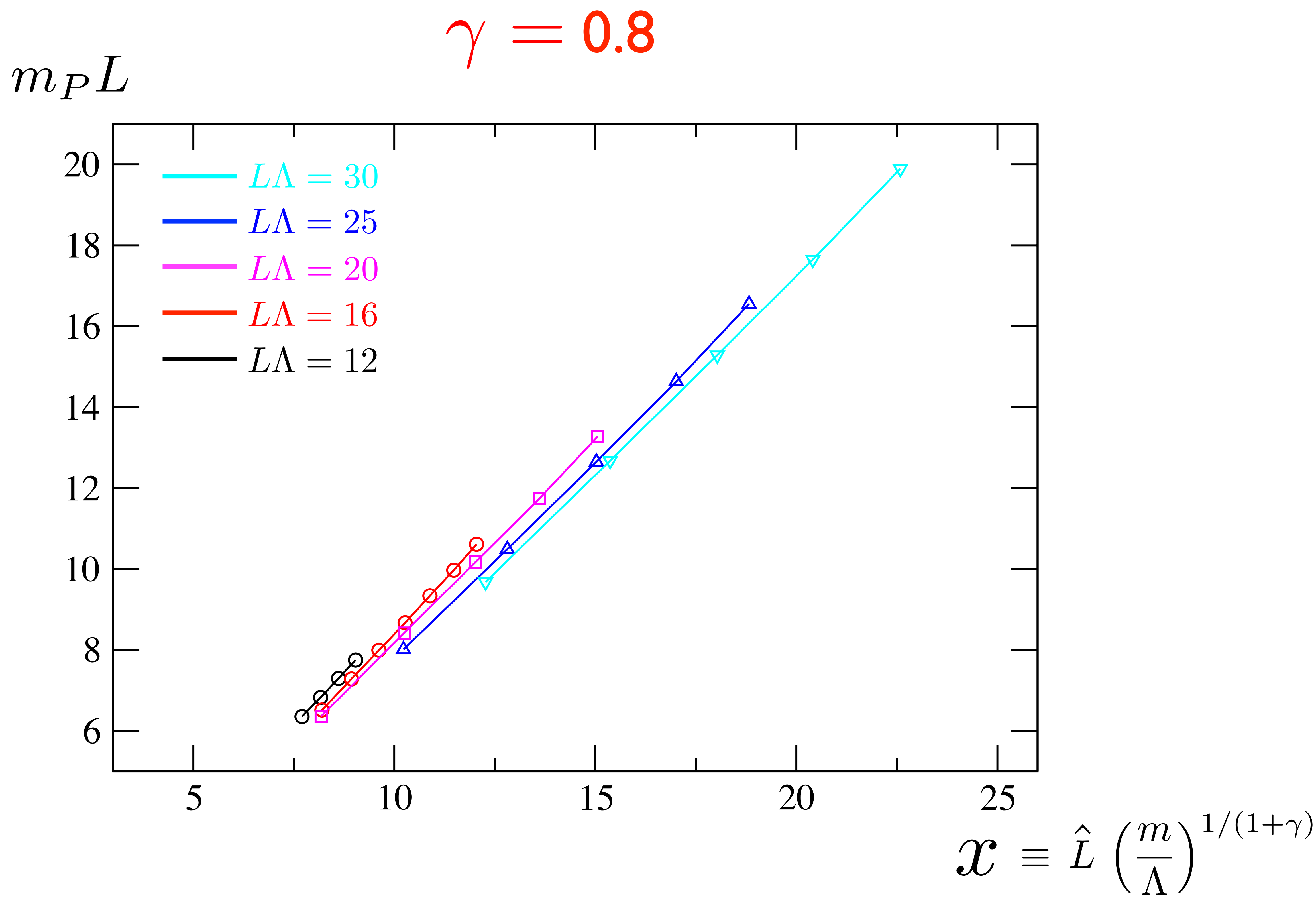


SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.7$

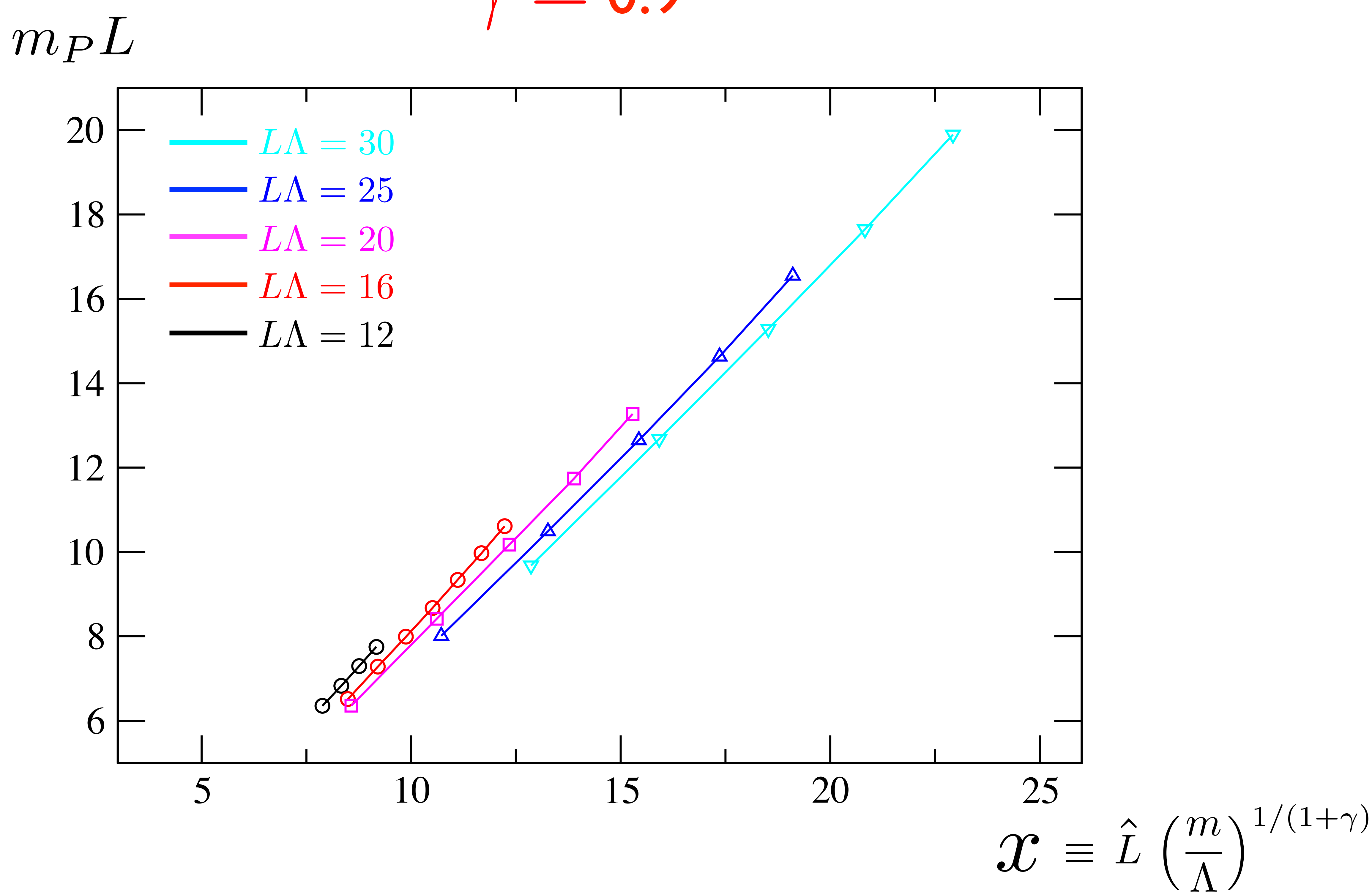


SU(3) gauge theory with 12 fundamental fermion



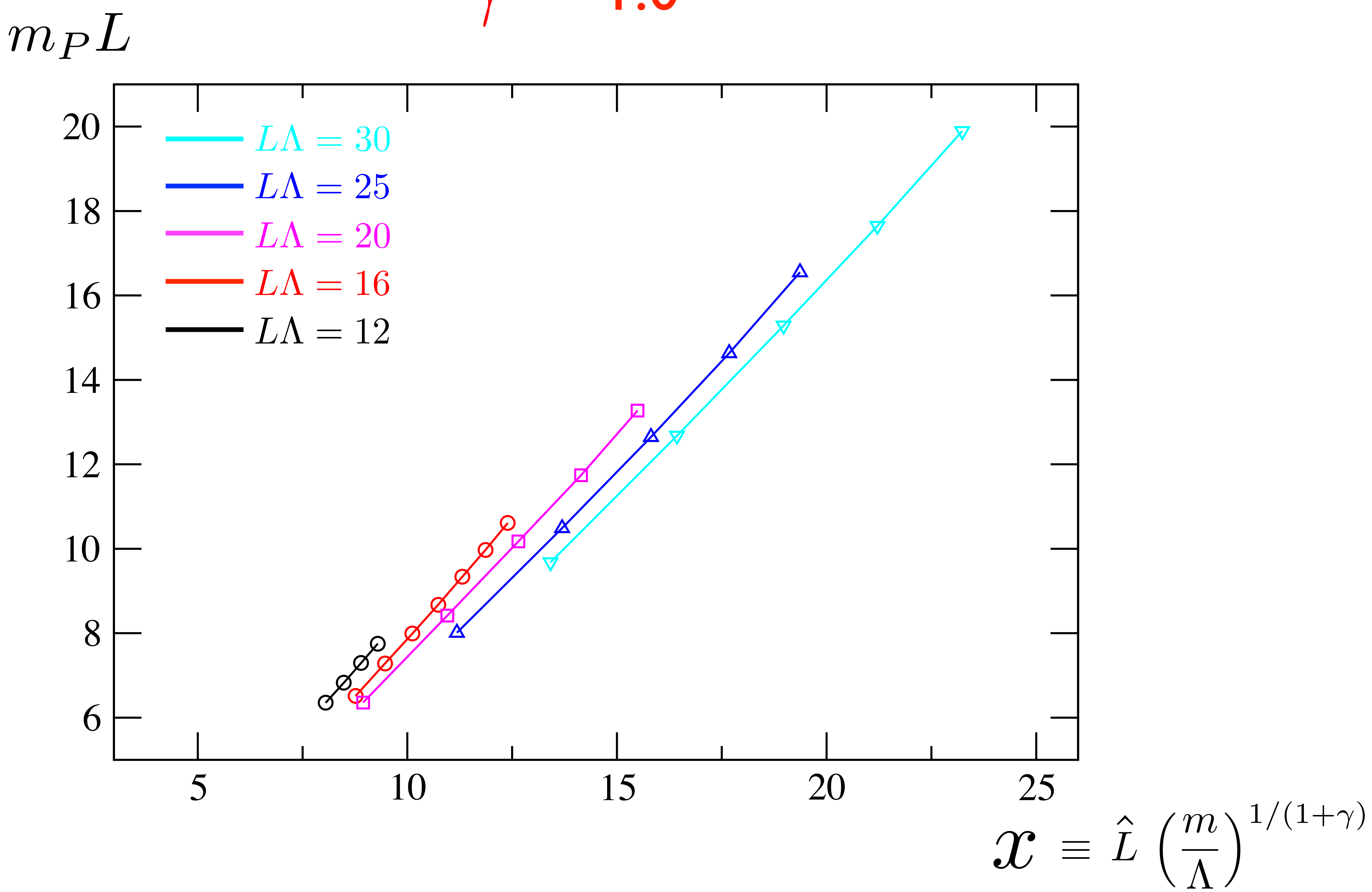
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 0.9$



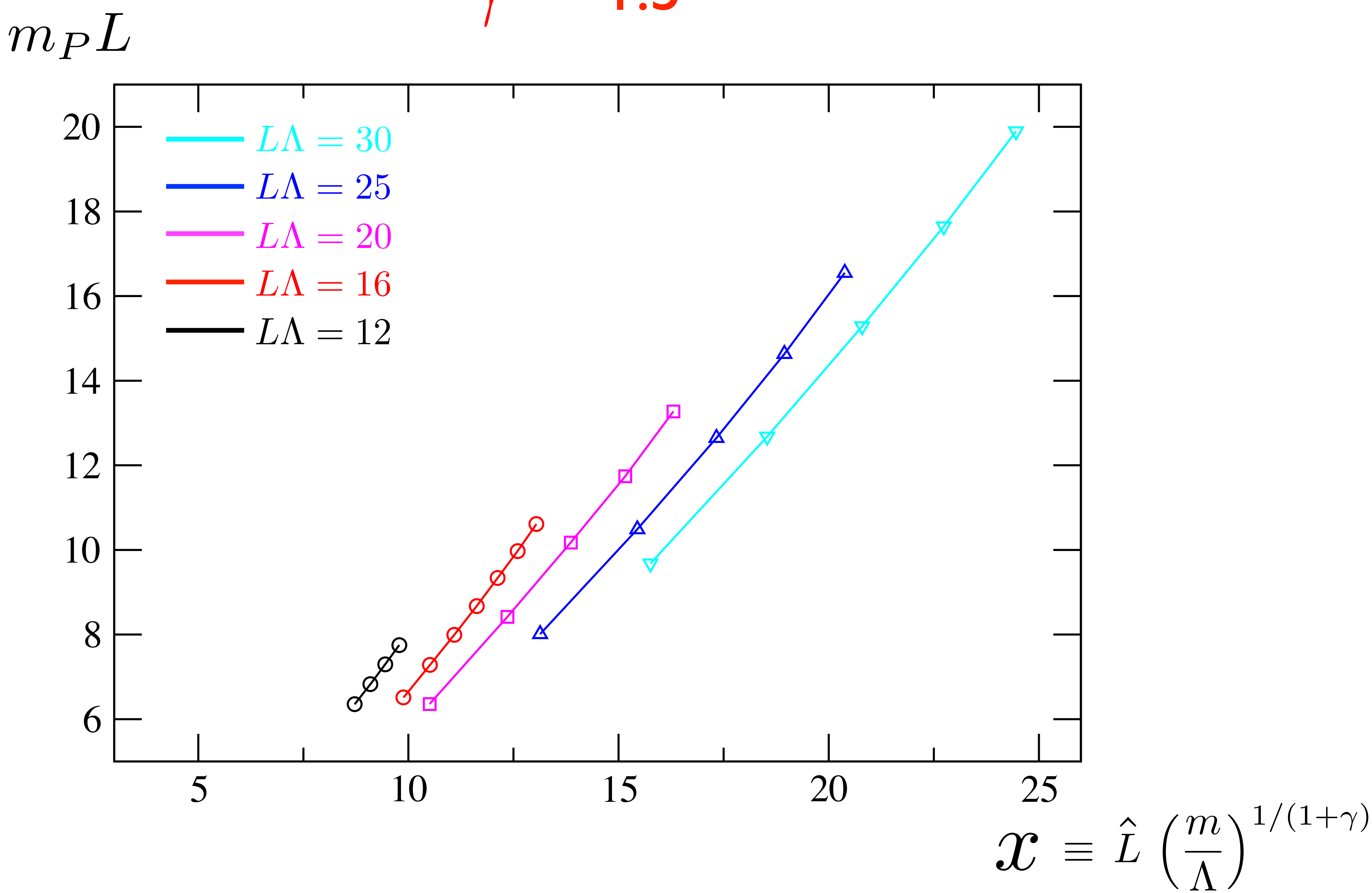
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 1.0$



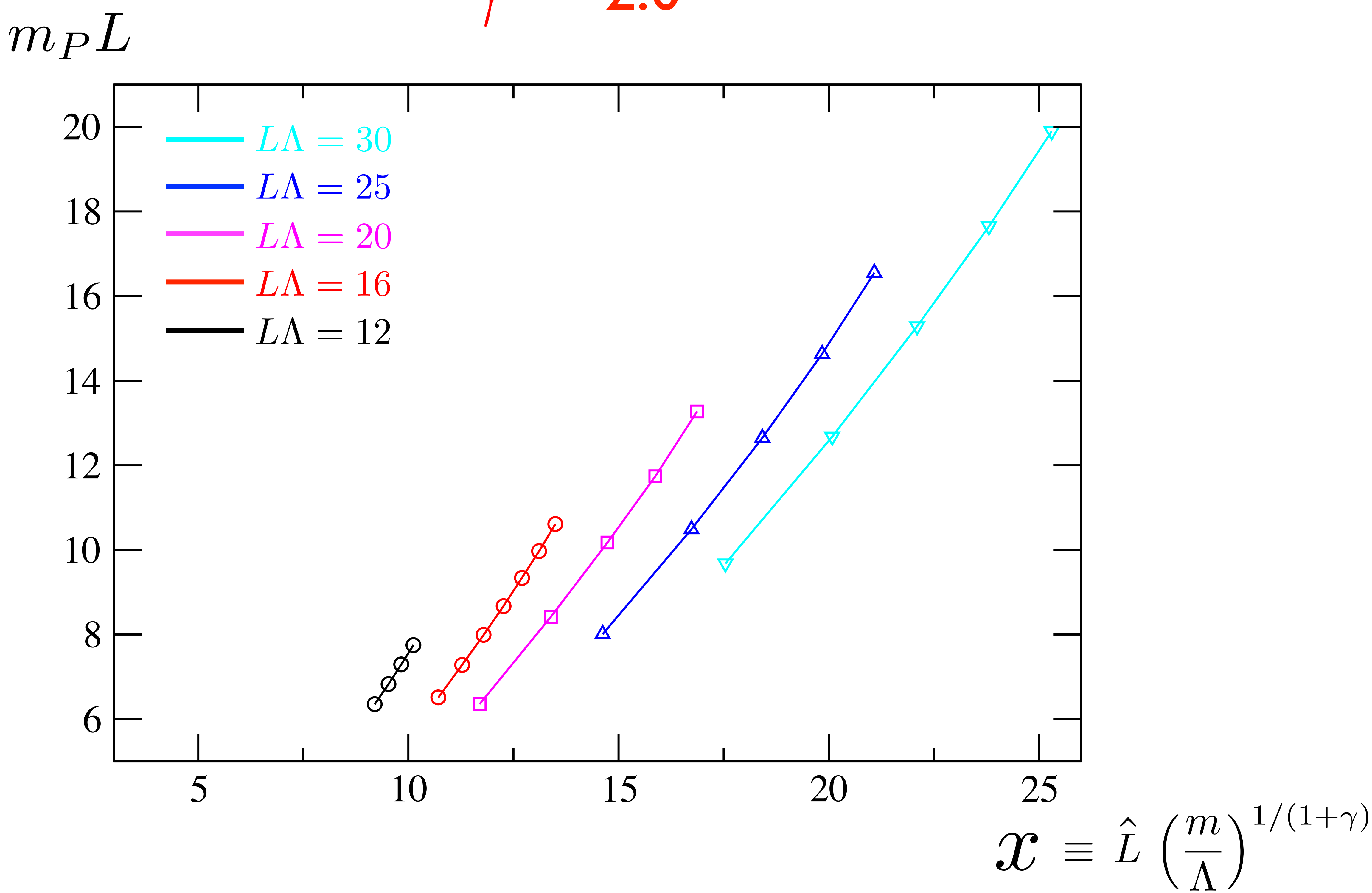
SU(3) gauge theory with 12 fundamental fermion

$\gamma = 1.5$



SU(3) gauge theory with 12 fundamental fermion

$\gamma = 2.0$



SU(3) gauge theory with 12 fundamental fermion

good scaling behavior for $\gamma = 0.5 \sim 0.6$

SU(3) gauge theory with 12 fundamental fermion

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but the actual value for $N_f = 12$ is...

$$\gamma_* = 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \simeq \underline{0.8}$$

SU(3) gauge theory with 12 fundamental fermion

good scaling behavior for $\gamma = 0.5 \sim 0.6$

but the actual value for $N_f = 12$ is...

$$\gamma_* = 1 - \sqrt{1 - \frac{\alpha_*}{\alpha_{\rm cr}}} \simeq \underline{0.8}$$

From the analytic solution of the SD equation, we can understand the above deviation is coming from the fact that the fermion mass, which was introduced to probe the IR conformality, itself is breaking the IR conformality

Relation between m and m_P which is obtained from the analytic solution of the SD equation

$$m/\Lambda \;=\; \xi \left[\; \frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*} + \; \frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*} \; \right]$$

Relation between m and m_P which is obtained from the analytic solution of the SD equation

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Hyperscaling

correction

Relation between m and m_P which is obtained from the analytic solution of the SD equation

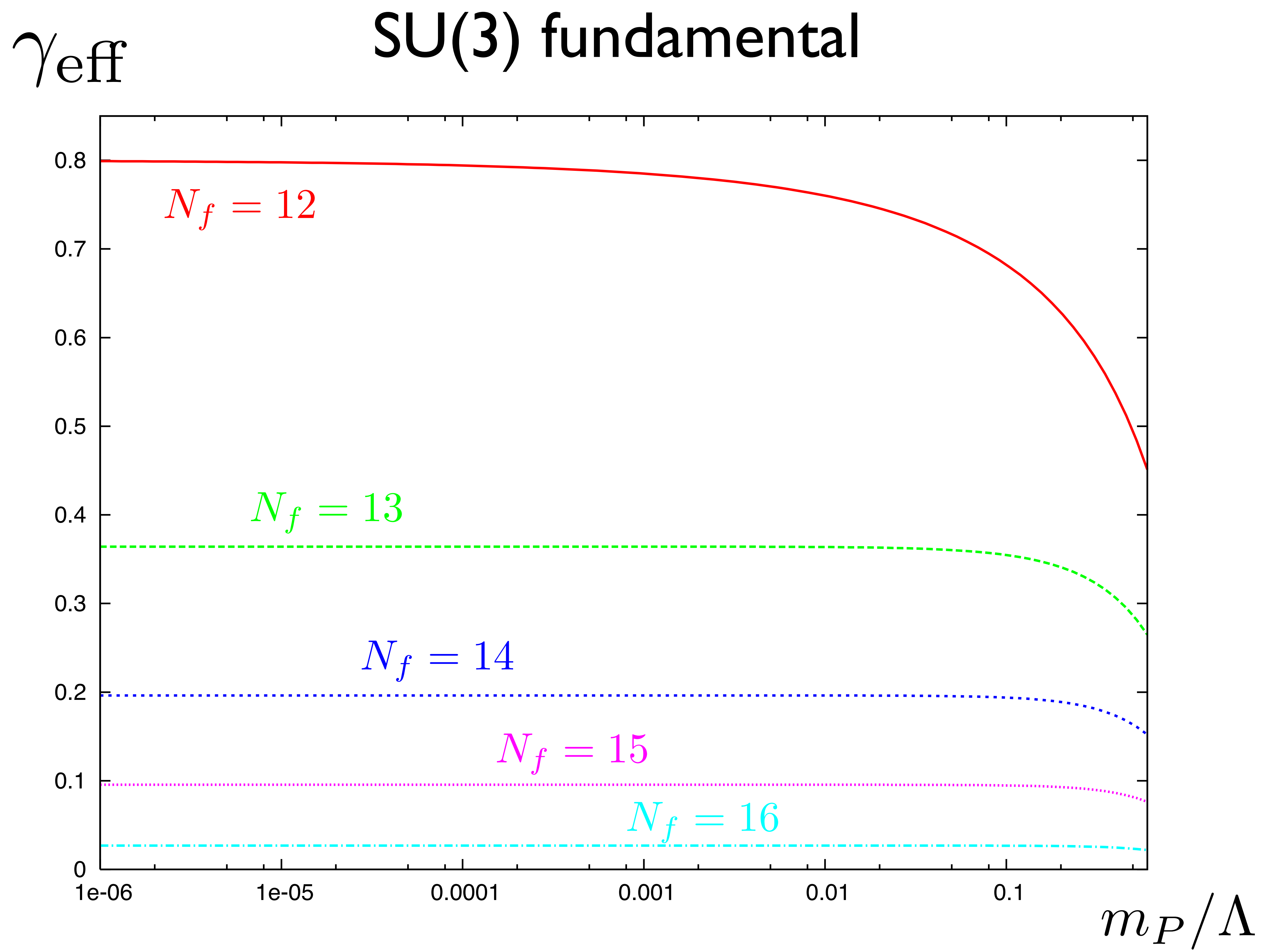
$$\boxed{m/\Lambda = \xi \left[\frac{\Gamma(1-\gamma_*)}{\Gamma(\frac{2-\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{1+\gamma_*}} + \underline{\frac{\Gamma(-1+\gamma_*)}{\Gamma(\frac{\gamma_*}{2})^2} \left(\frac{m_P}{\Lambda}\right)^{3-\gamma_*}} \right]$$

Hyperscaling

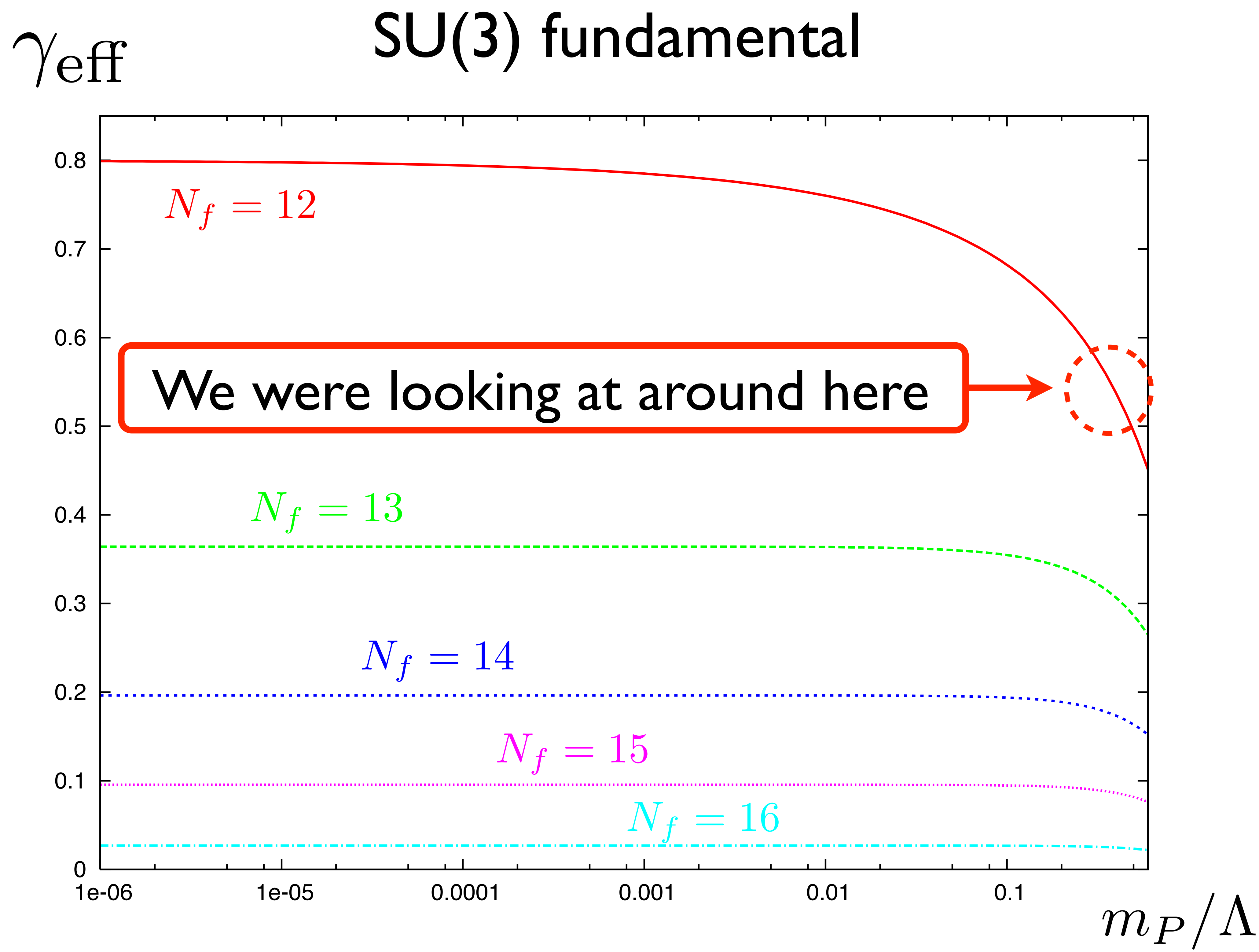
correction

The correction is large for a large value of m_P/Λ

Effective anomalous dimension



Effective anomalous dimension



Another question:

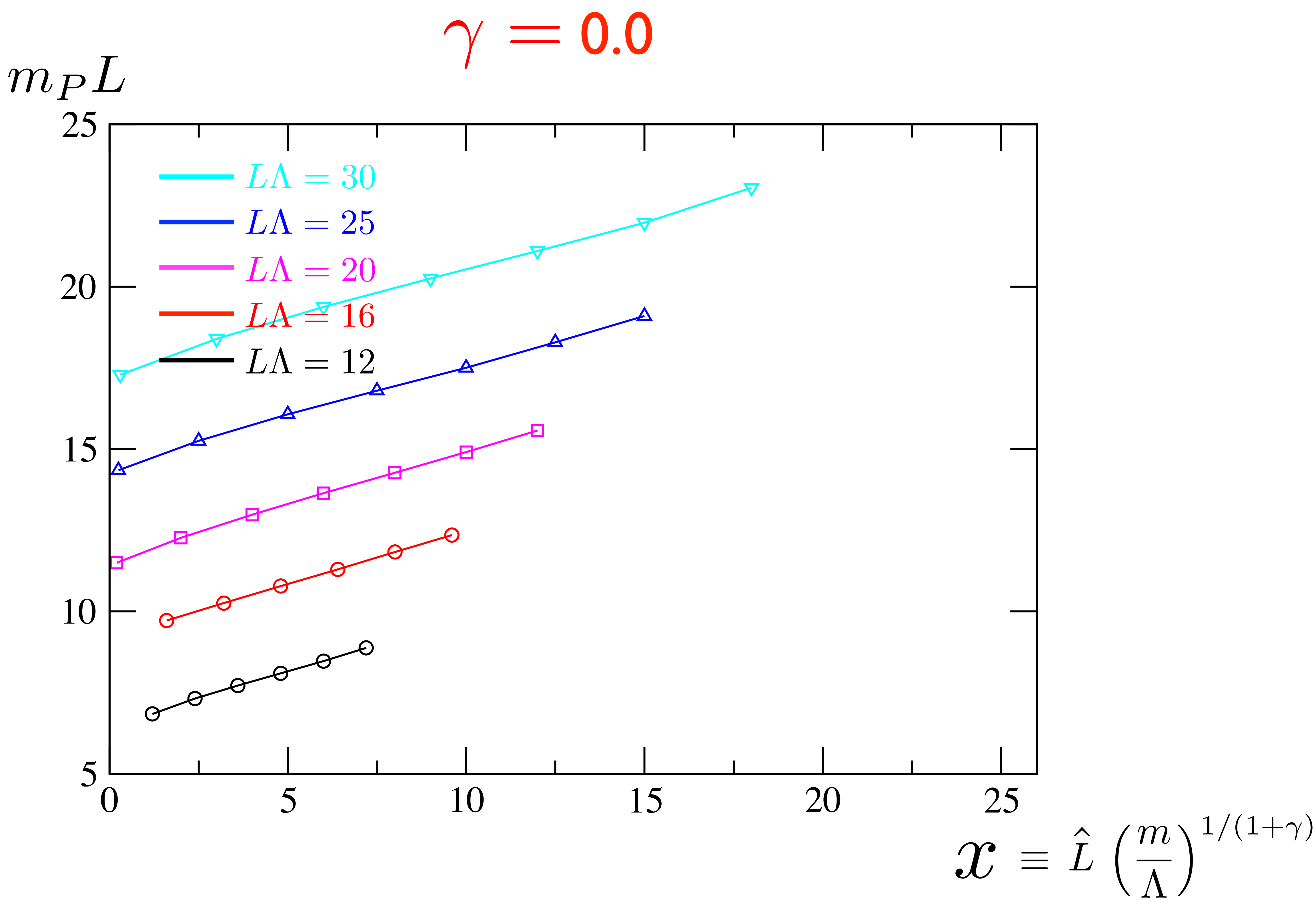
when the theory does not have an IRFP (namely, in the chiral symmetry breaking phase), how and how much the scaling relation is violated?

We show two examples:

SU(3), 9 flavor: deeply broken

SU(3), 11 flavor: close to the critical flavor

SU(3) gauge theory with 9 fundamental fermion



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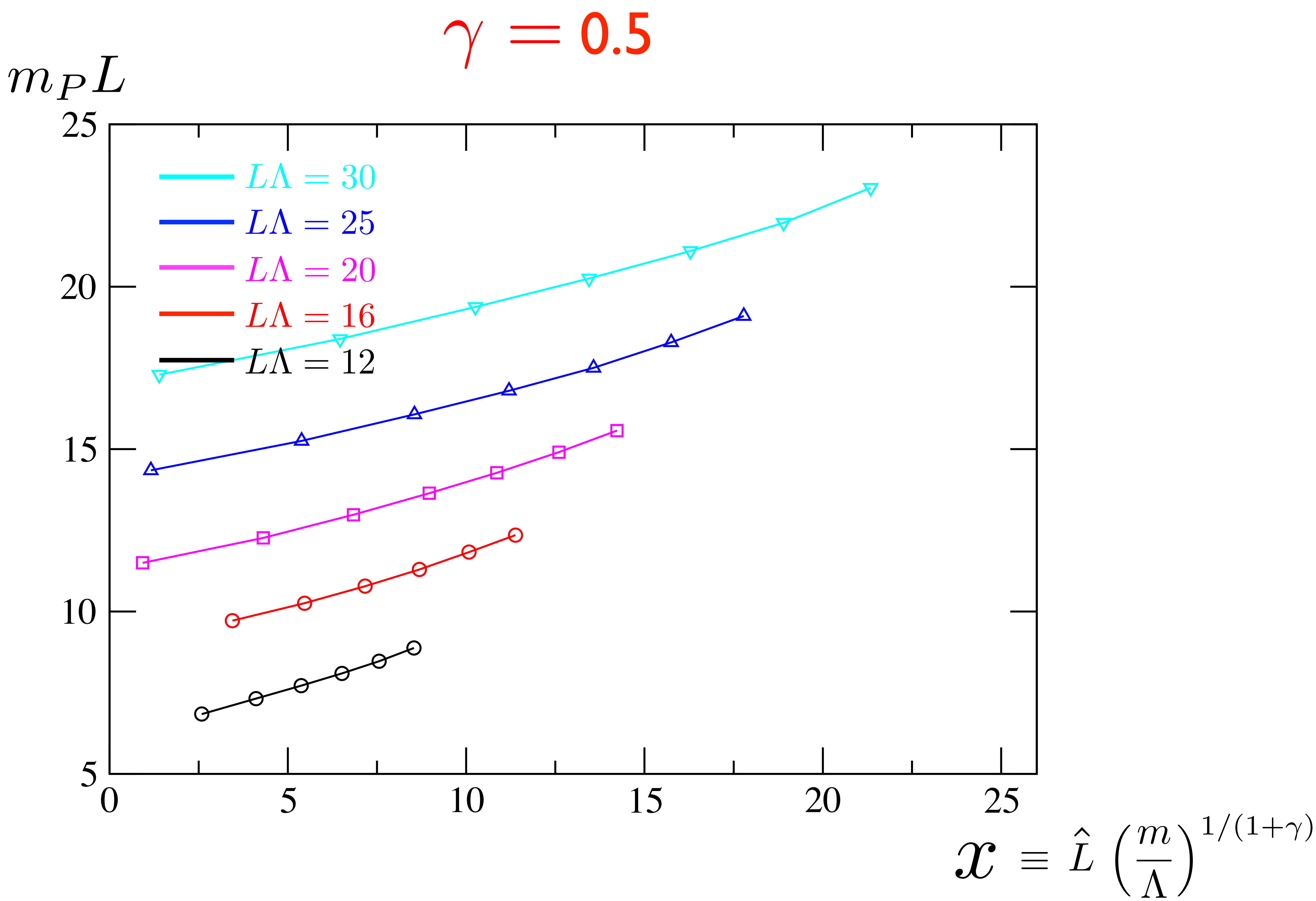

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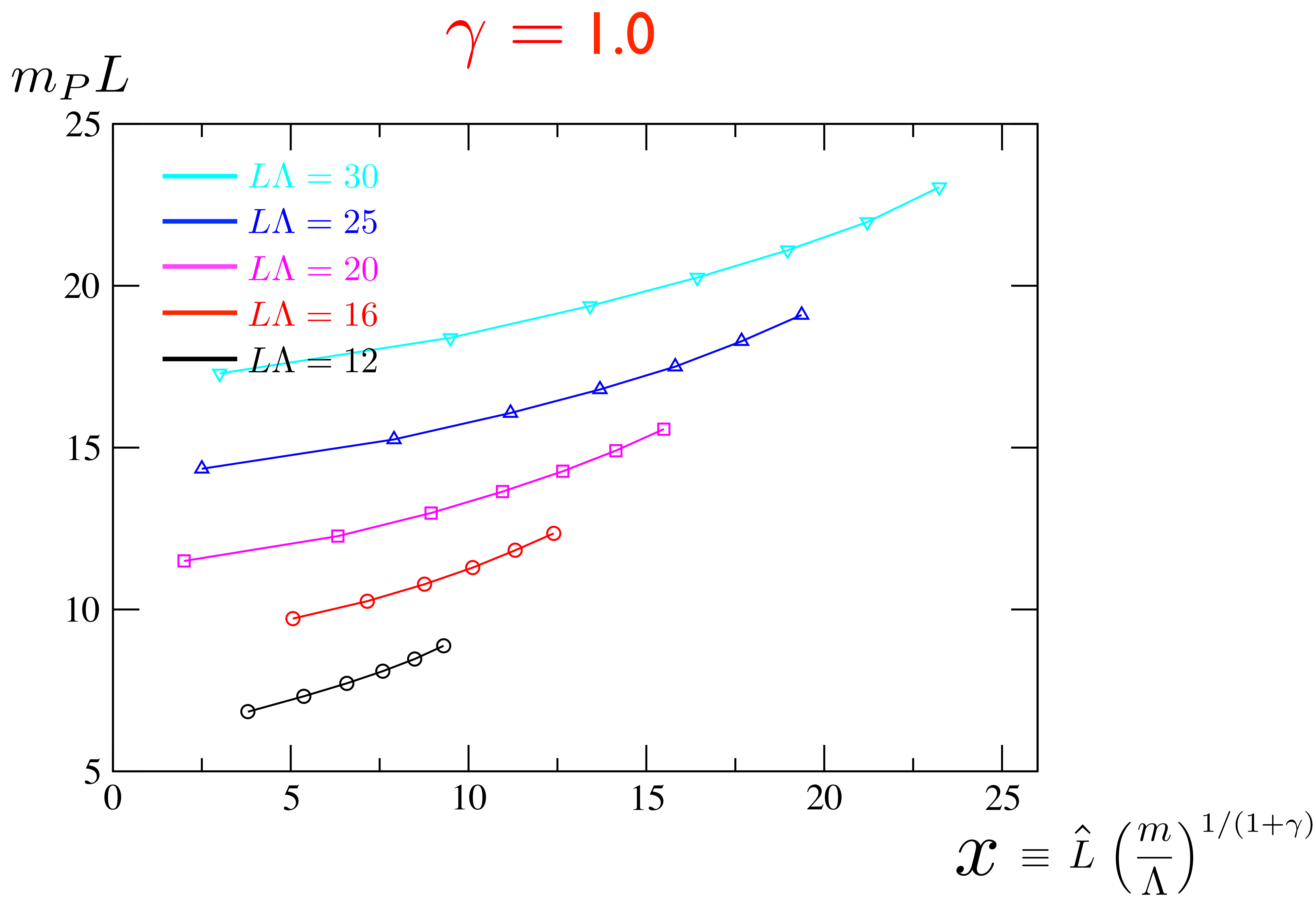

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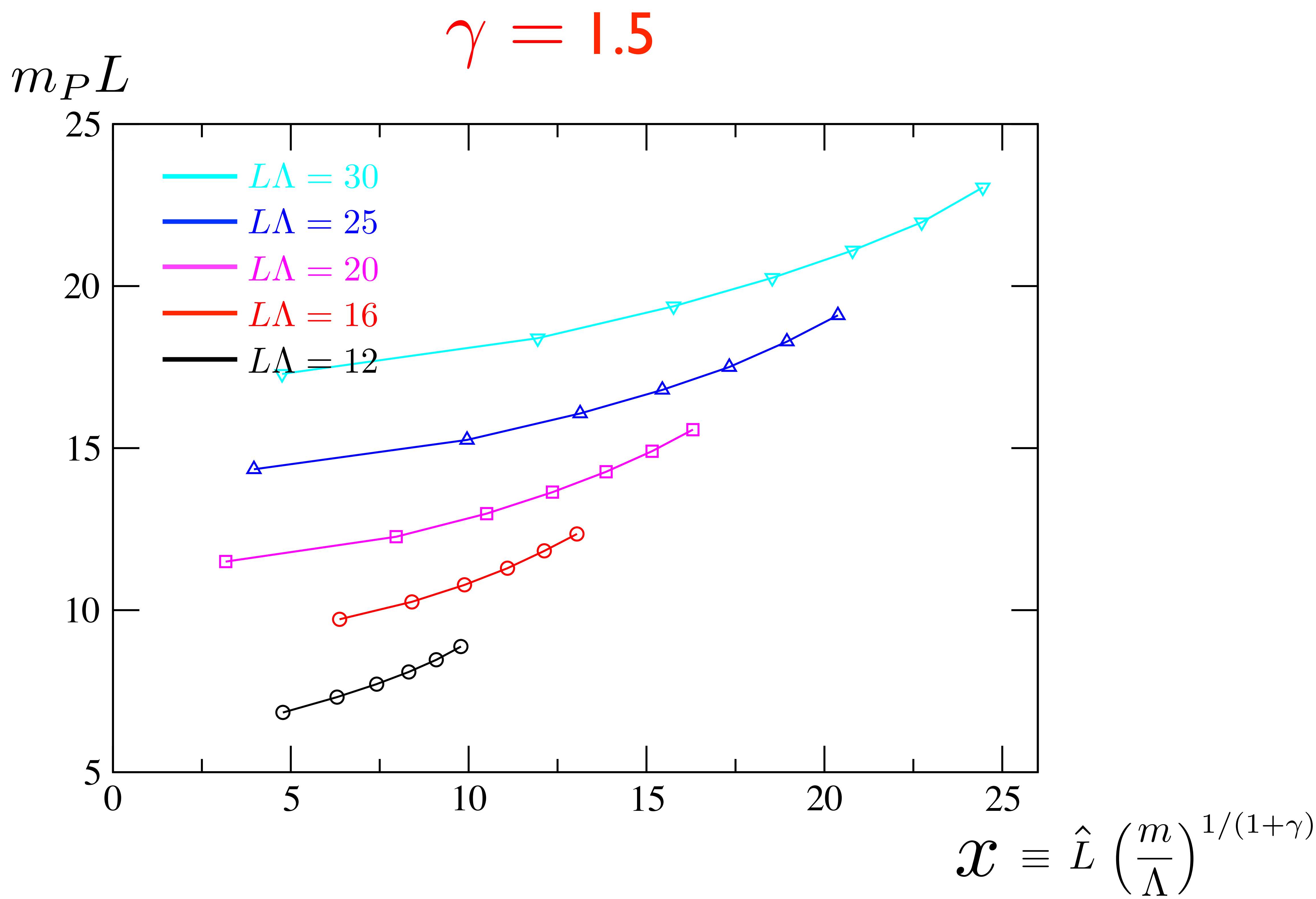

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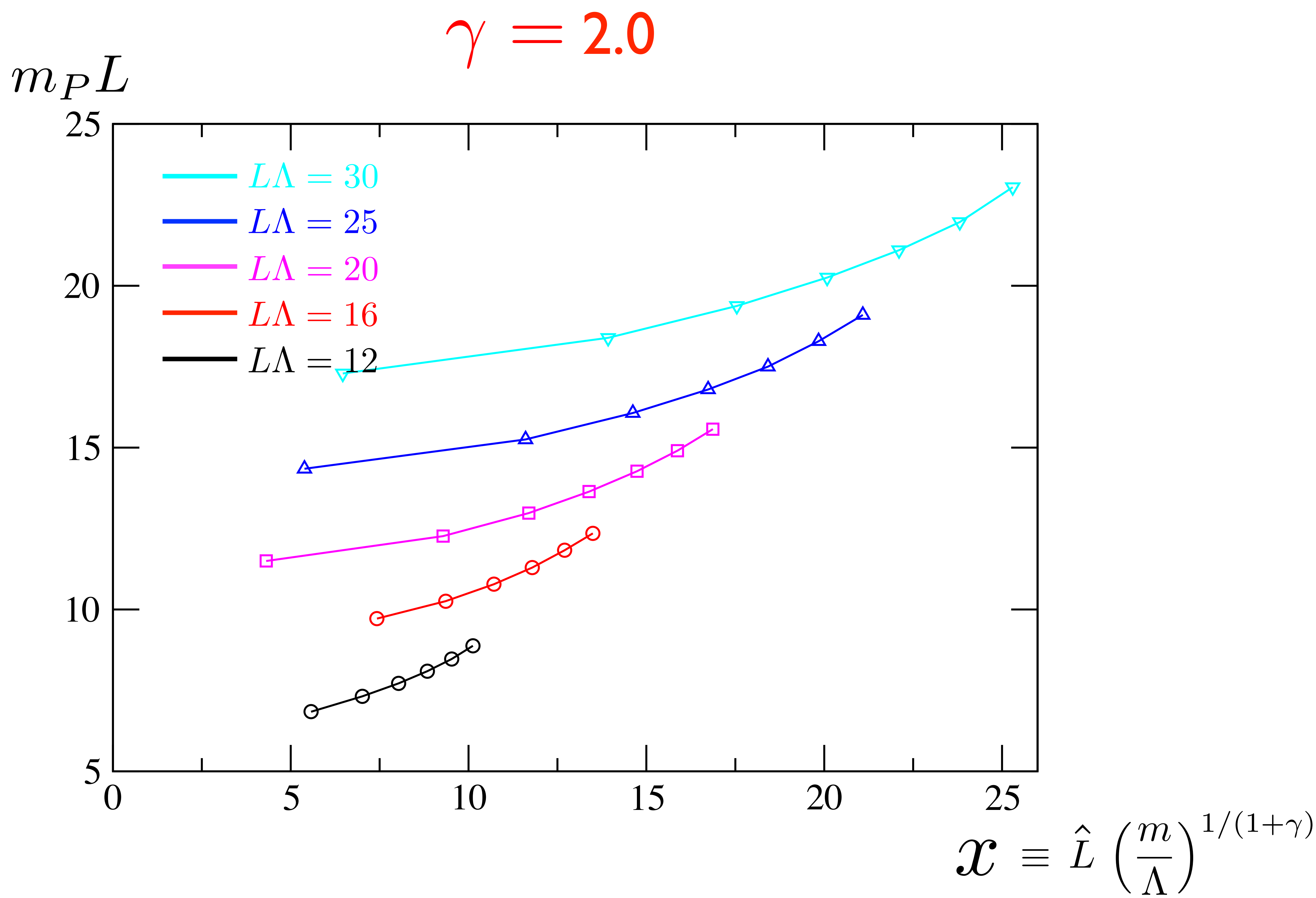
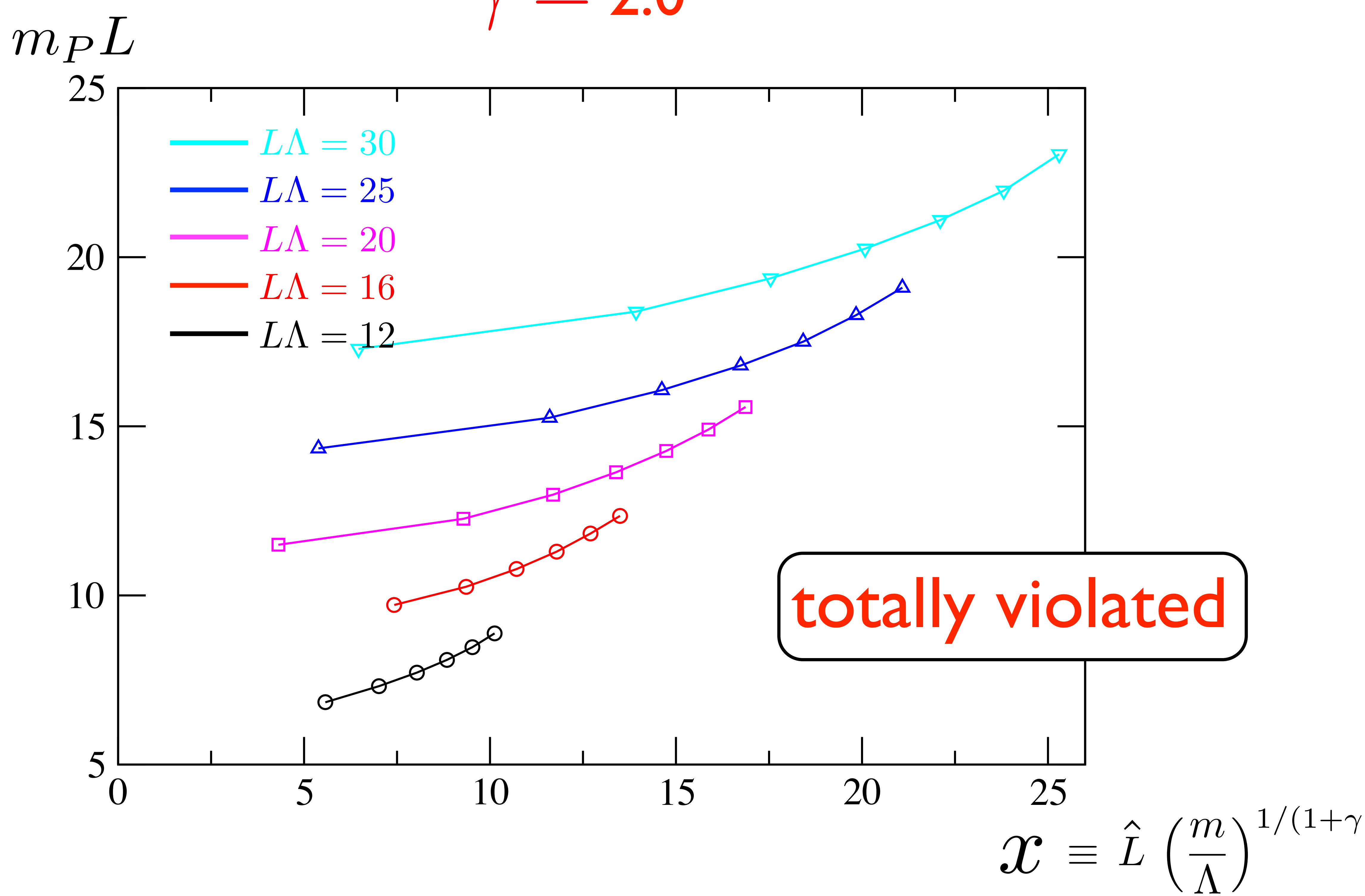


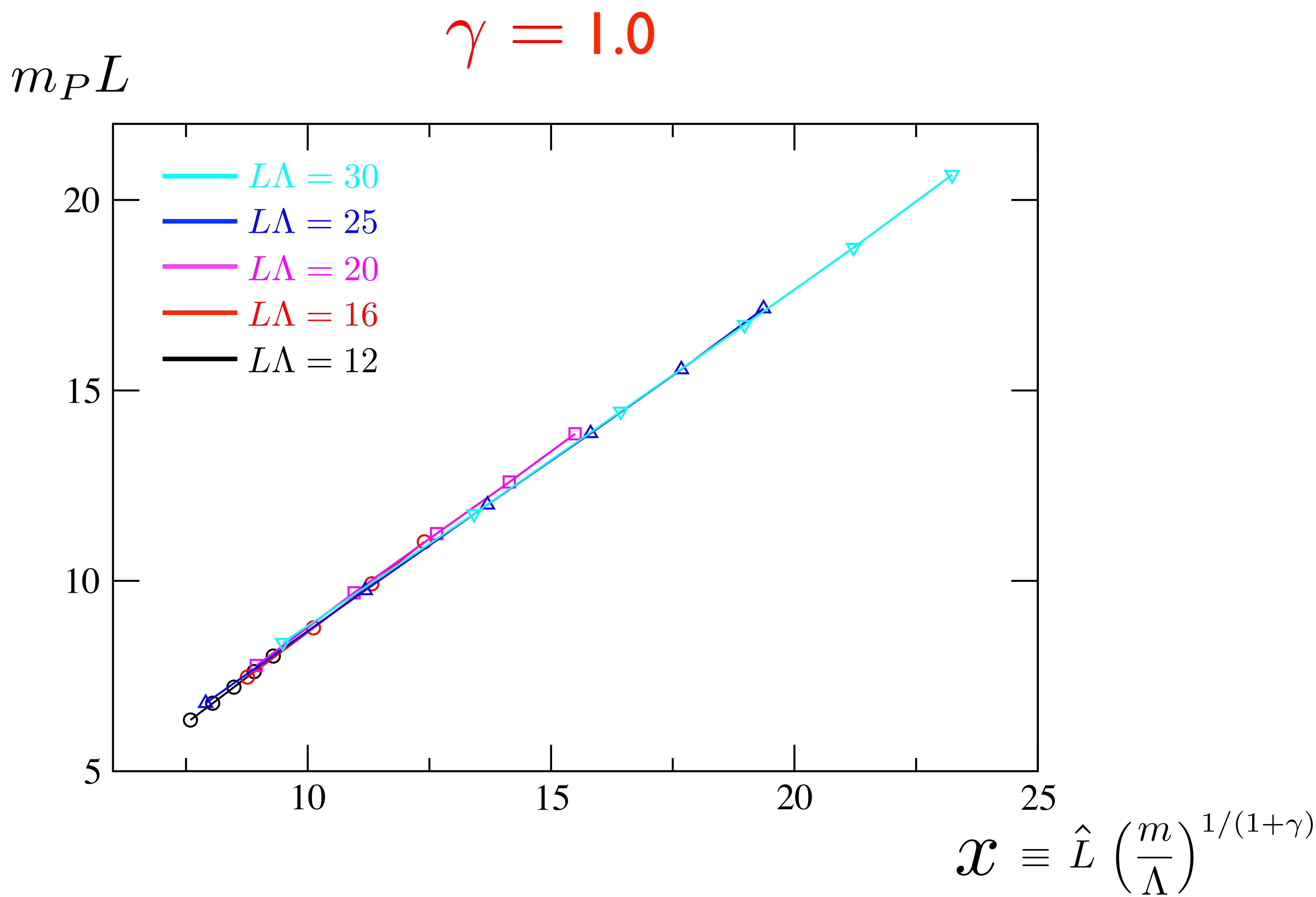
SU(3) gauge theory with 9 fundamental fermion

$\gamma = 2.0$



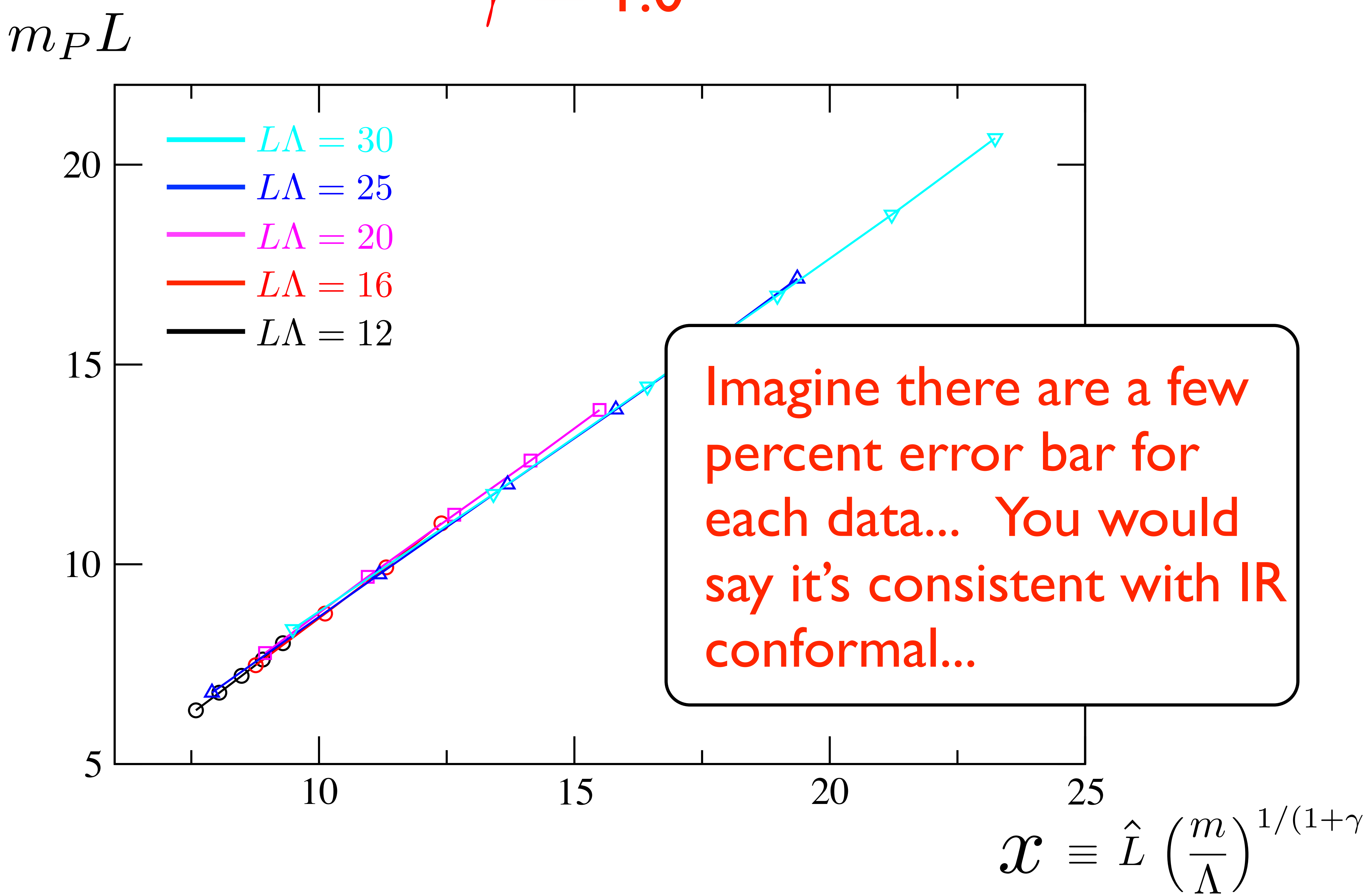
totally violated

SU(3) gauge theory with 11 fundamental fermion



SU(3) gauge theory with 11 fundamental fermion

$\gamma = 1.0$



Imagine there are a few percent error bar for each data... You would say it's consistent with IR conformal...

Summary

- Hyperscaling relation was investigated through the SD equation.

- Importance of the mass correction was discussed.

  It is very possible that the shift of γ due to the mass correction differs for different observables. When the lattice data show the good scaling for each physical quantity, but inconsistent values of γ (which means global fit with universal γ gives bad chi^2/dof), it is worth doing fitting with a universal γ + SD-inspired mass correction term for each physical quantity. It could significantly reduce chi^2/dof, which actually happened in the case of our Nf=12 lattice data. (talk by Ohki)

- Even after lattice studies of the large flavor QCD became popular, the SD equation is still a useful tool to give information from different point of view.

Backups

Effective anomalous dimension