A surprise with many-flavor staggered fermions in the strong coupling limit

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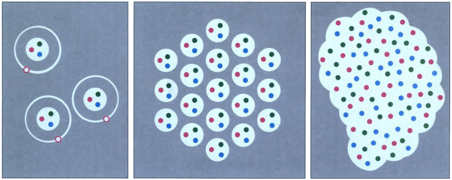

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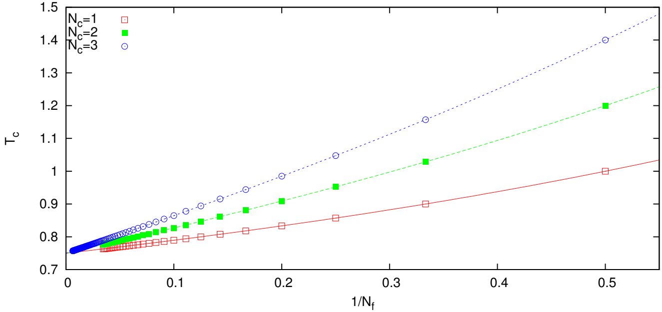
Outline

Mean Field: chiral symmetry is **always broken** in the strong-coupling limit of staggered fermions at $T = 0$ **for all values of N_f and N_c**

- chiral condensate well known to be independent of N_f and N_c, i.e. in d spatial dimensions: [Kluberg-Stern *et al.*, 1983]
$$\langle \bar{\psi}\psi \rangle (T = 0) = \frac{\left((1+d^2)^{1/2}-1)/2\right)^{1/2}}{d}$$
- we also found, following [Damgaard *et al.*, 1985]: chiral restoration temperature is $T_c = \frac{d}{4} + \frac{d}{8}\frac{N_c}{N_f} + \mathcal{O}(\frac{1}{N_f^2})$



- mean field expected to work well for large number of d.o.f. per site, e.g. exact results in the Gross-Neveu model for $N_f \to \infty$

On the other hand: loop expansion of the determinant shows that dynamical fermions induce a plaquette coupling $\propto N_f/m_q^4$, as studied numerically by [A. Hasenfratz, T. DeGrand PRD49 (1994)]
$\Rightarrow$ suggests **chiral symmetry restoration** for sufficiently large N_f **?**

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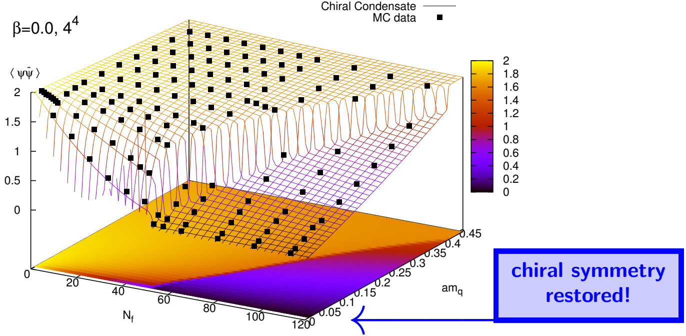
- **Answer from Monte Carlo: Surprise!** strong first order N_f-driven bulk transition for strong-coupling limit of staggered fermions found



- $N_f^c \simeq 52$ *continuum* flavors for $m_q = 0$, N_f^c increases with m_q
  (heavy fermions $\rightarrow$ less ordering)

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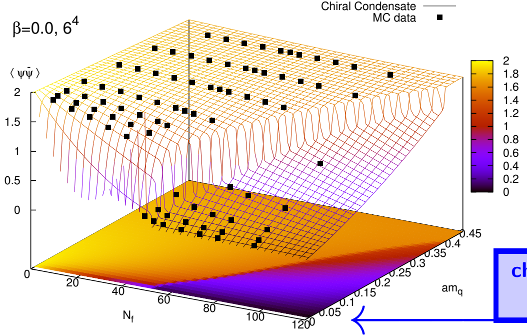

chiral symmetry restored!

- $N_\mathrm{f}^c \simeq 52$ *continuum* flavors for $m_q = 0$, N_f^c increases with m_q
  (heavy fermions $\rightarrow$ less ordering), almost no finite size effects

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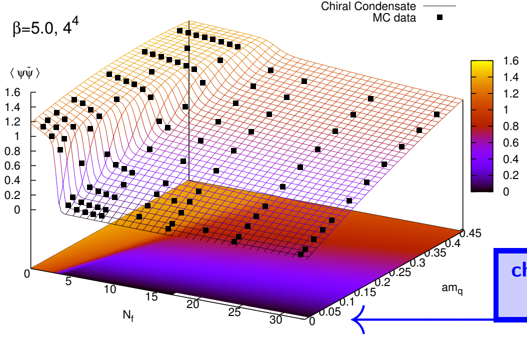

chiral symmetry restored!

- $\beta = 5$ similar but N_f^c smaller

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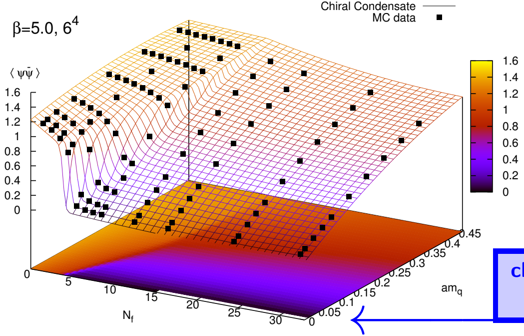

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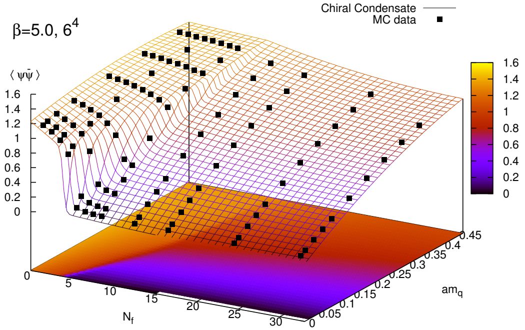
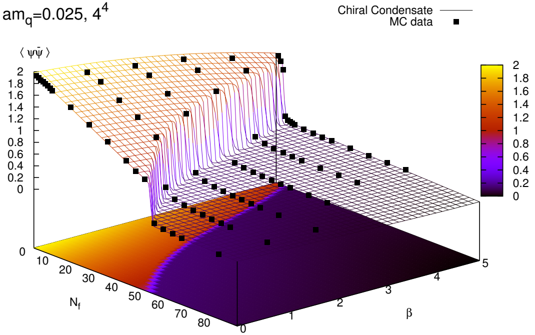


- Explanation for **failure of mean field:** terms of $\mathcal{O}(\frac{N_{\mathrm{f}}}{N_{\mathrm{c}}}, \frac{N_{\mathrm{f}}}{d^2})$ are neglected

The Chirally Restored Phase for large β

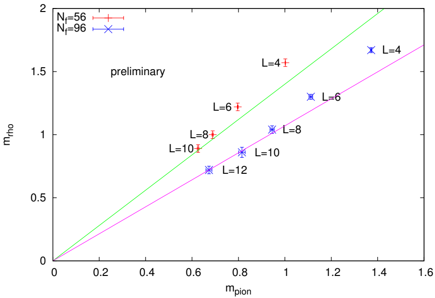
- smooth variation with $\beta \;\rightarrow\; N_f$-driven transition extends to weak coupling
- $N_f^c \simeq \mathcal{O}(10)$ at weaker coupling
- connection with N_f-driven transition to **conformal window?**

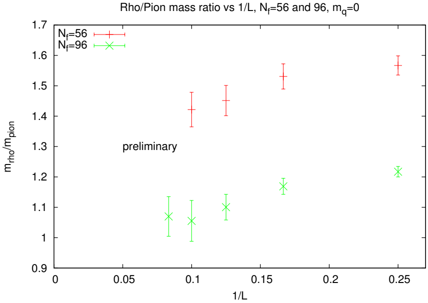


Characterizing the chirally restored phase: hadron masses

Hadron spectrum obtained from simulations with $N_\mathrm{f} = 56$ and $N_\mathrm{f} = 96$ at **zero quark mass**

- hadron masses measured for $m_q = 0$ are non-zero
- but masses decrease as the lattice size L is increased
- parity partners degenerate (c.f. chiral symmetry restoration)
- mass ratios $\sim$ **independent of L**:





Characterizing the chirally restored phase: Dirac Spectrum

Dirac eigenvalue spectrum, measured at **zero quark mass**, $\beta = 0$:

- **integrated eigenvalue density:** $\int_0^\lambda \rho(\bar{\lambda}) d\bar{\lambda} = \frac{\text{rank}(\lambda)}{\text{rank(Dirac matrix)}} \in [0, 1]$
- derivative gives $\rho(\lambda)$

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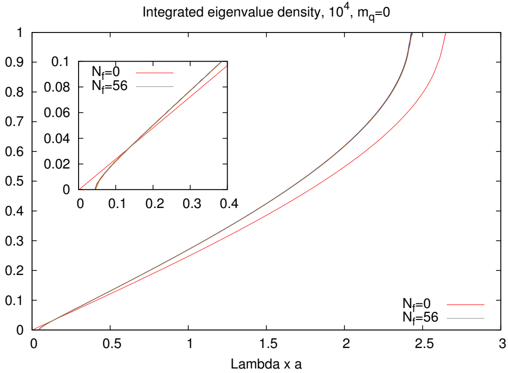
Compare $N_f = 0$ (quenched configurations) and $N_f = 56$ (chirally symmetric phase)

- similar for large eigenvalues (UV)
- the $N_f = 56$ curve shows a gap for small eigenvalues (IR), consistent with chiral symmetry restoration: $\rho(0) = 0$



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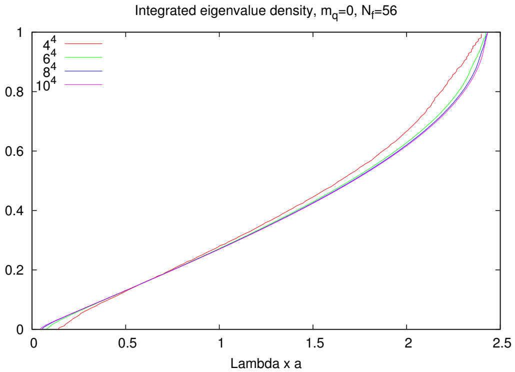
Compare different volumes for $N_f = 56$:

- large eigenvalues (UV) are L-independent,
- the IR spectral gap shrinks as L increases



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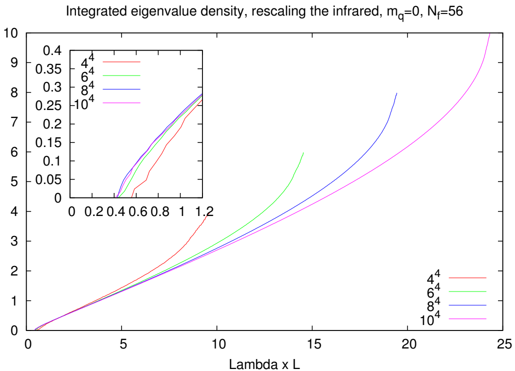
Compare different volumes for $N_f = 56$:

- IR spectrum invariant after rescaling by L: spectral gap $\propto 1/L$
- IR physics only depends on L, while the UV physics depends on a
- no other scale in the system $\Rightarrow$ we have an **IR-conformal theory!**



Conjecture: $\beta = 0$ IR-conformal phase is analytically connected with the weak-coupling, continuum IR-conformal phase

Study of **continuum limit** is much more difficult:

- for a given lattice size L^4, the scales are ordered as $a \ll 1/\Lambda_{QCD} \ll L$
- at strong-coupling the hierarchy is $a \simeq 1/\Lambda_{QCD} \ll L$
- range of conformal invariance $(L\Lambda_{QCD})$ maximized at $\beta = 0$ for given lattice size L/a

weak coupling:

strong coupling:

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Summary

Shown: for $\beta = 0$, a strong first order **bulk transition** exists which is N_f**-driven** to a chirally symmetric phase

- in the chiral limit: $N_\mathrm{f}^c = 52(4)$ *continuum* flavors
- finding in contrast to meanfield prediction
- chirally restored phase extends to weak coupling

Also shown: for $\beta = 0$, **“large-N_f QCD” is IR-conformal:**

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- simulations at large N_f and zero quark mass can be performed without too much computer effort

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no evidence for additional $T = 0$ phase transition as β is increased

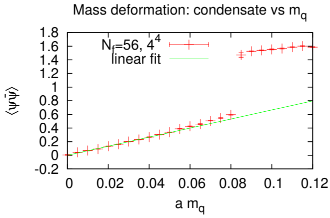
$\Downarrow$

Conjecture: strong coupling chirally symmetric, IR-conformal phase is analytically connected with the **continuum IR-conformal phase**

Discussion

Outlook

- quark mass breaks conformality; chiral condensate is proportional to $m_q \quad \Rightarrow \quad \gamma^* = 1$
- check for IRFP ?
- glueballs, static potential, ...



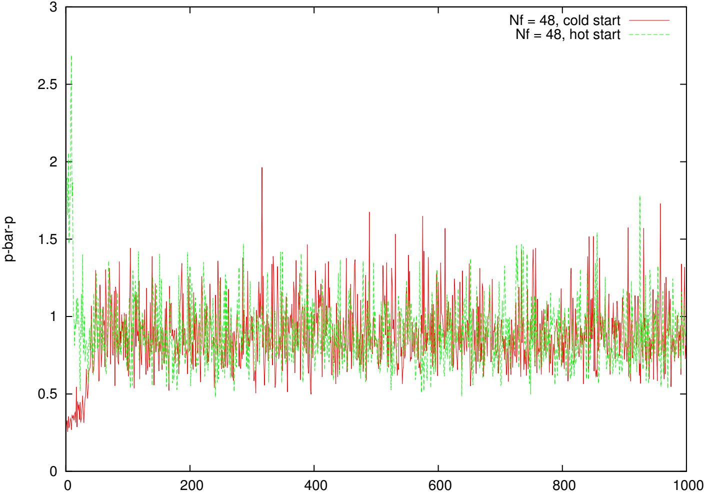
Single parameter of this IR-conformal theory is N_f:

- increasing N_f increases the magnitude of the spectral gap,
- decreasing N_f brings us back to a chirally broken phase, via a first-order transition as in Sannino's "Jumping dynamics" [hep-ph/1205.4246] $\rightarrow$ no walking dynamics

Our findings are **consistent with the literature:**

- Kogut, Sinclair, Nucl. Phys. B295 (1985) bulk transition for $N_\mathrm{f} = 12$
- Damgaard, *et al.*, Phys. Lett. B400 (1997): bulk transition for $N_\mathrm{f} = 16$
- Jin, Mawhinney, PoS Lattice2011: bulk transition for $N_\mathrm{f} = 12$ with improved action
- A. Hasenfratz, hep-lat/1111.2317 (2012): bulk transition for $N_\mathrm{f} = 8, 12$

Backup Slide: Time Histories for Chiral Condensate



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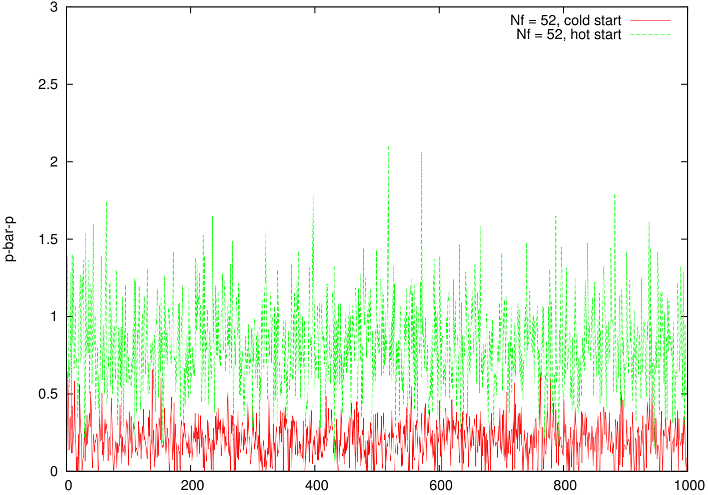

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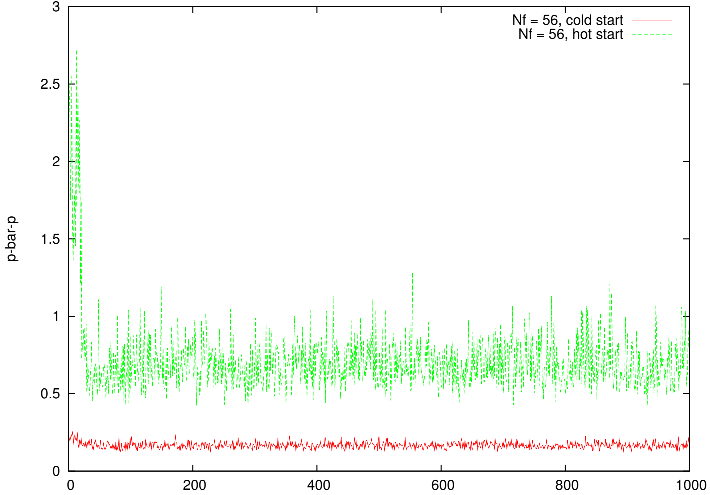

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