The sphaleron rate at the electroweak crossover with 126 GeV Higgs mass

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Introduction

- Study on Baryogenesis, a mechanism for producing more matter than antimatter;
- Mechanism unknown, but suggested it might take place at the electroweak phase transition;
- The EWPT needs to be first order; **[Kuzmin, Rubakov, Shaposhnikov, 1985]**
- The order of the transition and the temperature it occurs depend on the value of Higgs mass;
- Baryon number is violated in the SM **['t Hooft, 1976]** through sphaleron transitions.

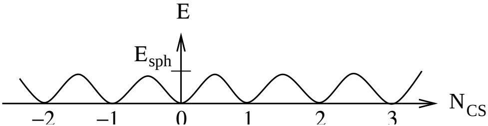
Motivation

Latest results (ATLAS/CMS) suggest Higgs (if it exists) might have a mass of $m_H \sim 126$ GeV. This would make the EWPT a crossover [Kajantie, Laine, Rummukainen, Shaposhnikov, 1996]. However, the sphaleron rate is independent of the transition order, being defined in terms of the topology of the EW theory, and Baryogenesis might still be viable through Leptogenesis, as $B - L$ is conserved. The sphaleron rate enters L number production as a parameter converting L to B.

Our work: We found out the sphaleron rate as a function of temperature across the electroweak crossover.

Vacuum structure of the electroweak theory

* In EW theory vacua are disconnected and degenerate



* Each vacuum is labeled by a different *Chern-Simons number N_{CS}*

$$N_{CS} = \int d^3x \; j^0_{CS} = -\frac{g^2}{64\pi}\int d^3x \; \epsilon^{ijk}\mathrm{Tr}\left(A_i F_{jk} + i\frac{g}{3}A_i A_j A_k\right).$$

* When gauge fields pass from one vacuum to another, N_{CS} changes by 1 unit and B by 3 *(through the EW anomaly)*

$$\partial_\mu j^\mu_B = n_g \; \partial_\mu j^\mu_{CS},$$

where n_g is the number of generations of fermions.

Moving between vacua: sphaleron transitions

* At zero temperature:

  - the process is classically forbidden;
  - quantum tunneling (instanton) is highly suppressed $\sim \exp(-16\pi^2/g^2) \sim 10^{-170}$

* At higher temperatures ($T \gtrsim$100 GeV):

  - transitions are possible by surmounting the potential barrier through *sphaleron transitions* (lower barrier, large thermal energy)

The sphaleron rate is

$$\Gamma \equiv \lim_{t\to\infty} \frac{\langle (N_{CS}(t) - N_{CS}(0))^2 \rangle}{V\ t}$$

Methods

We studied the baryon number violation rate:

* on a 3D-lattice, with $L = 32$;
* dimensional reduction $\rightarrow$ 3D effective theory;
* with fields: $SU(2)$ + Higgs;
* two different methods, depending on the T we are examining:
  - Canonical Monte Carlo *(heat bath)*
  - Multicanonical MC + Real-time

Dimensional reduction:

$4\text{D}_{\mathrm{continuum}} \rightarrow 3\text{D}_{\mathrm{continuum}}$

The thermodynamics of the 4D electroweak theory is studied in 3D by mean of dimensional reduction, which gives the correspondence between 4D and 3D parameters. **[Ginsparg, 1980; Appelquist, Pisarski, 1981]**

The SU(2)-Higgs effective theory is

$$L = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2,$$

where g_3^2, λ_3 and m_3^2 are 3D effective parameters. **[Kajantie, Laine, Rummukainen, Shaposhnikov, 1995]**

Lattice - continuum relations:

$3D_{\mathrm{continuum}} \leftrightarrow 3D_{\mathrm{lattice}}$

- The obtained 3D theory must be put on the lattice;
- In order to preserve the match between lattice and continuum actions, corrections must be applied to the parameters g_3, λ_3, m_3; **[Symanzik, 1983]**
- All corrections (except for m_3) have been calculated to O(a). **[Moore, 1997]**

Dynamics

The time evolution of this effective SU(2) theory follows Langevin dynamics, at leading order in $\log(1/g)$ **[Bödeker, 1998]**

However, since Langevin dynamics is slow on lattice, it can be substituted by any other dissipative method as long as we know the correspondence

$$\text{lattice-time step} \leftrightarrow \text{real-time step}$$

Heat bath evolution

In our case we use heat-bath update, with **[Moore, Rummukainen, 2001]**

$$\Delta t = \frac{n_t \ \sigma_{el} \ a^2}{4}$$

where:

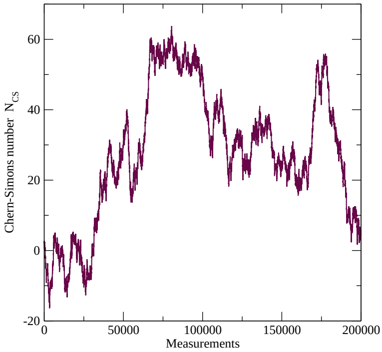
$$\begin{aligned} n_t &= \text{updates / measurement} \\ \sigma_{el} &= \frac{m_D^2}{3\gamma} \quad \text{non-abelian color conductivity} \\ m_D &= \sqrt{\frac{11}{6}}\, gT \quad \text{Debye mass} \\ \gamma &= \frac{Ng^2 T}{4\pi}\left[\ln\left(\frac{m_D}{\gamma}\right) + 3.041\right] \approx 0.66361688 \ g^2 T \\ &= \text{damping rate for the gauge boson} \end{aligned}$$

Method: canonical vs multicanonical

- We choose a Higgs mass;
- We start at high T in the symmetric phase and use canonical MC/heat bath simulations to find the sphaleron rate;
- We lower the T towards the broken phase and notice the number of transitions decreases;
- In the deep broken phase the rate is highly suppressed and canonical methods don't work anymore;
  $\rightarrow$ we need multicanonical methods to compensate for the high potential barrier.

Canonical MC in the *symmetric phase*

* The evolution follows a random walk in n_{CS} (canonical Monte Carlo) for, at high T:
  - Larger availability of thermal energy
  - Potential barriers become lower



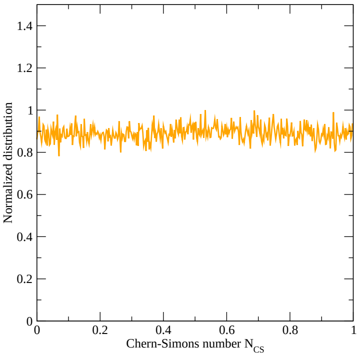


Figure: Left: n_{CS} evolution; Right: Probability distribution.

Canonical MC at the *crossover*

* The Hamiltonian evolution of a state wanders around one vacuum for an amount of time;

* After crossing to another vacuum, it doesn't promptly continue to the next vacuum: EITHER it settles around the new vacuum OR returns to the initial one;

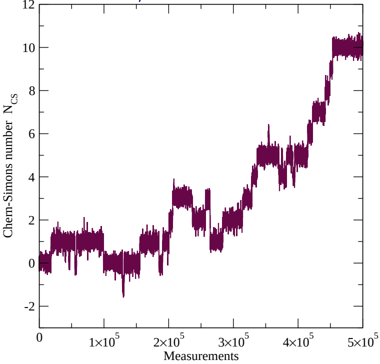


Figure: Left: n_{CS} evolution over time; Right: Probability distribution.

Canonical MC in the *deep broken phase*

* The configuration remains in the same vacuum, if we use canonical MC, because transitions are highly suppressed;

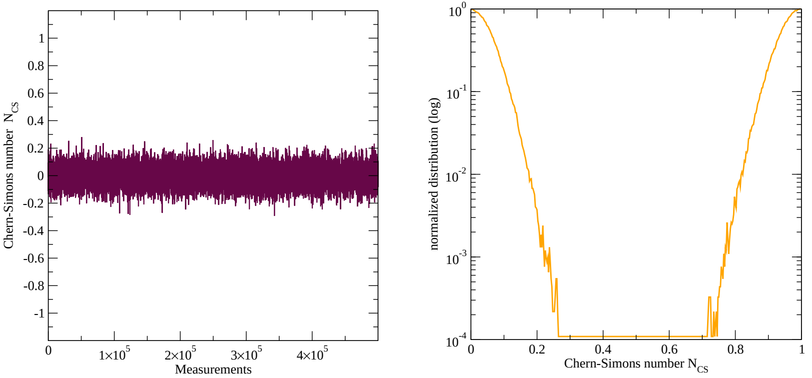


Figure: Left: n_{CS} evolution over time; Right: Probability distribution.

Multicanonical method in the broken phase

We use a weight function to compensate the potential barrier and obtain a flat probability distribution. **[Moore, 1998; Moore, Rummukainen, 2000]**

1. Set an order parameter to distinguish vacua: $n^*_{CS} = 1/2$, i. e. on top of the barrier;
2. Find the weight function W and calculate the P_ϵ to be in the small interval $n^*_{CS} \pm \epsilon/2$: -multicanonical run
3. Calculate the probability flux: need to know the mean inverse time for crossing ϵ: -real-time run

$$\frac{1}{\epsilon}\left\langle \left| \frac{dN_{CS}}{dt} \right| \right\rangle$$

Multicanonical method in the broken phase

4. Calculate the dynamical prefactor

$$d = \sum_{sample} \frac{\delta}{\text{nr crossings}}$$

which is the fraction of n^*_{CS} crossings that lead to a permanent change in N_{CS}. δ is either 0 or 1. -real-time simulation

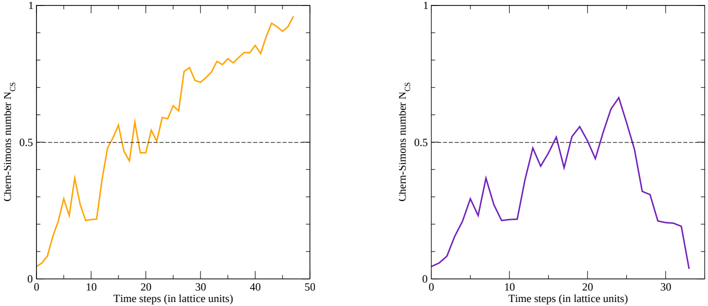


Figure: n_{CS} real-time trajectories.

The sphaleron rate

The sphaleron rate we obtain from the combined multicanonical and real-time simulations is [Moore, 1998]

$$\Gamma_s \equiv P(|N_{CS} - N_{CS}^*| < \epsilon/2) \times \frac{1}{\epsilon} \left\langle \left| \frac{dN_{CS}}{dt} \right| \right\rangle \times d$$

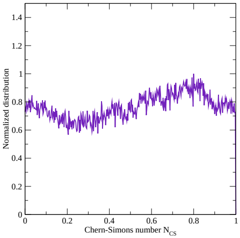
and is converted into physical units by:

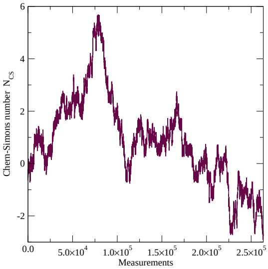
$$\Gamma = \frac{\Gamma_s}{\Delta t \ a^3 \ N_s^3}.$$

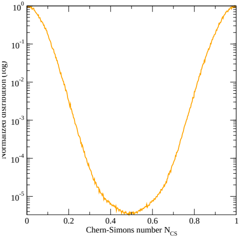
with $a =$ lattice spacing, $N_s =$ lattice sites.

Multicanonical MC in the *deep broken phase*

We need to use multicanonical methods to observe transitions. We sample with constant probability: $P_{\mathrm{muca}} \propto \exp[W]\ P_{\mathrm{can}}$

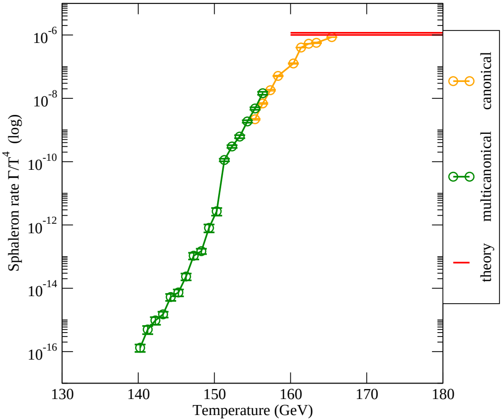






Left: P_{muca}; Centre: n_{CS} evolution; Right: P_{can}

The sphaleron rate for $m_H = 126$ GeV



Asymptotic limit: $\sim 5 \times 10^{-7}$ T^4. **[Burnier, Laine, Shaposhnikov, 2006]**

Conclusions

* We got the complete picture of the sphaleron rate through the electroweak crossover, improving previous estimates.

* Our results are in agreement with previous results, in the range where they exist.

* The two simulation methods are consistent with each other.

* The crossover temperature depends on the Higgs mass.

* Even if CP-violation is too small in SM, Baryogenesis might be viable through Leptogenesis, where the sphaleron rate enters as a parameter.