Higgs boson mass bounds in presence of a heavy fourth quark family

Attila Nagy[1,2]
in collaboration with:
John Bulava[3], Philipp Gerhold[1], Karl Jansen[2] and Jim Kallarackal[1]

[1]Humboldt University Berlin; [2]NIC, DESY Zeuthen; [3]Theory Division, CERN

June 28th, 2012

Outline

Higgs Boson Mass

- Higgs boson experimentally not found yet, but it is the simlest way to explain EW symmetry breaking
- No direct theoretical prediction for the Higgs boson mass available
  Only perturbative arguments for mass bounds
  - upper bound is given by the Landau Pole
  - lower bound is given by the requirement of a stable vacuum

Higgs Boson Mass

- Higgs boson experimentally not found yet, but it is the simlest way to explain EW symmetry breaking
- No direct theoretical prediction for the Higgs boson mass available
  Only perturbative arguments for mass bounds
  - upper bound is given by the Landau Pole
  - lower bound is given by the requirement of a stable vacuum
- A fourth generation of quarks is not excluded, but could provide a larger amount of CP violation which could have impact on the Baryogenesis of the universe [Holdom et al. 2009]
- We investigate non-perturbatively the Higgs boson mass bounds depending on the mass of a hypothetical fourth quark generation

Higgs-Yukawa model in the continuum

We consider: Fermion doublet $\psi = (t, b)^T$ and a complex scalar doublet φ:

$$S^{\text{cont}}[\bar{\psi}, \psi, \varphi] = \int d^4x \left\{ \frac{1}{2} \left(\partial_\mu \varphi\right)^\dagger \left(\partial^\mu \varphi\right) + \frac{1}{2} m_0^2 \varphi^\dagger \varphi + \lambda_0 \left(\varphi^\dagger \varphi\right)^2 \right\} + \int d^4x \left\{ \bar{t} \partial\!\!\!/ t + \bar{b} \partial\!\!\!/ b + y_{b_0} \bar{\psi}_L \varphi \, b_R + y_{t_0} \bar{\psi}_L \tilde{\varphi} \, t_R + h.c. \right\}$$

- No gauge fields (color or electroweak)
- Only the heaviest quarks are considered

Lattice action

Real four-vector Φ and N_f quark doublets ψ
Bosonic action:

$$S_B[\Phi] = -\kappa \sum_{x,\mu} \Phi_x^\dagger \left[\Phi_{x+\mu} + \Phi_{x-\mu}\right] + \sum_x \Phi_x^\dagger \Phi_x + \hat{\lambda} \sum_x \left[\Phi_x^\dagger \Phi_x - N_f\right]^2$$

The connection to continuum is given by:

$$\varphi = \sqrt{2\kappa} \begin{pmatrix} \Phi^2 + i\Phi^1 \\ \Phi^0 - i\Phi^3 \end{pmatrix}, \quad \lambda = \frac{\hat{\lambda}}{4\kappa^2}, \quad m^2 = \frac{1 - 2N_f\hat{\lambda} - 8\kappa}{\kappa}$$

Lattice action

Fermionic action:

$$S_F[\bar{\psi},\psi,\Phi] = \sum_x \bar{\psi}_x \left[D^{ov} + P_+ \phi_x^\dagger \mathrm{diag}(\hat{y}_t,\hat{y}_b)\hat{P}_+ + P_- \mathrm{diag}(\hat{y}_t,\hat{y}_b)\phi_x \hat{P}_- \right] \psi_x$$

with $y_{t/b} = \frac{\hat{y}_{t/b}}{\sqrt{2\kappa}}$

We use the overlap operator: $D^{ov} = \frac{\rho}{a}\left[1 + \frac{A}{\sqrt{AA^\dagger}}\right], \qquad A = D^{\mathrm{Wilson}} - \frac{\rho}{a}$

[Kaplan 1992; Neuberger, Lüscher, Hasenfratz 1998]

- D^{ov} has analytical eigenvalues and eigenvectors in momentum space due to the absence of gauge fields

In this work we use only one mass degener¡ate quark doublet ($N_f = 1$ and $\hat{y}_b = \hat{y}_t$)

Implementation

- polynomial Hybrid Monte Carlo algorithm [Frezzotti & Jansen 1997-1999]
- various algorithmic improvements [Gerhold 2010]
  - Fourier acceleration
  - Exact reweighting
  - Preconditioning of the fermion matrix
- Scale is set over renormalized vacuum expectation value of the scalar field: $\frac{v_r}{a} = 246$ GeV
- Definition of the cutoff: $\Lambda = \frac{1}{a} = \frac{246 \text{ GeV}}{v_r}$

Observables

- $v_r = \frac{v}{\sqrt{Z_G}}, \quad \bar{\varphi}^{rot} = \begin{pmatrix} 0 \\ \bar{v} \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2\kappa}\bar{m} \end{pmatrix}$

Observables

- $v_r = \frac{v}{\sqrt{Z_G}}, \quad \bar{\varphi}^{rot} = \begin{pmatrix} 0 \\ \bar{v} \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2\kappa}\bar{m} \end{pmatrix}$
- Field renormalization constants:

$$Z_{G/H}^{-1} = \frac{d}{d(p^2)} \Re \left(\left[G_{G/H}(p^2) \right]^{-1} \right) \Big|_{p^2 = -m_{G/H}^2}$$

Observables

- $v_r = \frac{v}{\sqrt{Z_G}}, \quad \bar{\varphi}^{rot} = \begin{pmatrix} 0 \\ \bar{v} \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{2\kappa}\bar{m} \end{pmatrix}$
- Field renormalization constants:

$$Z^{-1}_{G/H} = \frac{d}{d(p^2)} \Re \left(\left[G_{G/H}(p^2) \right]^{-1} \right) \Big|_{p^2=-m^2_{G/H}}$$

- The fermion masses are computed via the time slice correlator:

$$C_f(\Delta t) = \frac{1}{L_t L_s^6} \sum_{t=0}^{L_t-1} \sum_{\vec{x}\vec{y}} \left\langle 2\Re \mathrm{Tr} \left\{ f_{L,t+\Delta t,\vec{x}} \bar{f}_{R,t,\vec{y}} \right\} \right\rangle$$

$$C_f(1 \ll \Delta t \leq L_t/2) \propto \cosh\left(m_f \left[L_t/2 - \Delta_t\right]\right)$$

Observables

- Higgs-/Goldstone boson masses from propagators:

$$0 = \Re \left(\left[G_{G/H}(p^2) \right]^{-1} \right) \Big|_{p^2 = -m^2_{G/H}}$$

Observables

- Higgs-/Goldstone boson masses from propagators:

$$0 = \Re\left(\left[G_{G/H}(p^2)\right]^{-1} \right)\Big|_{p^2=-m^2_{G/H}}$$

- The propagators are fitted according a one-loop motivated LPT-expression and an analytical continuation is performed to extract the pole

Observables

- Higgs-/Goldstone boson masses from propagators:

$$0 = \Re\left(\left[G_{G/H}(p^2) \right]^{-1} \right)\Big|_{p^2=-m^2_{G/H}}$$

- The propagators are fitted according a one-loop motivated LPT-expression and an analytical continuation is performed to extract the pole
- This procedure is only reliable, if the decay width is small compared to the mass

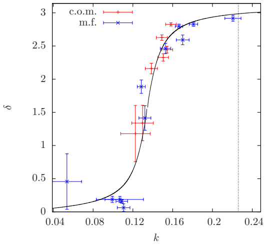
Cross check of the Higgs boson propagator mass

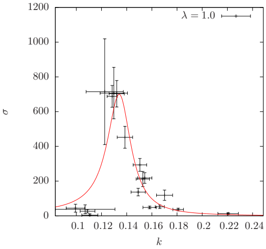
- Perform a rigorous resonance analysis for the Higgs boson at physical top quark mass
- Connection between infinite volume scattering phases and finite volume two particle energies [Lüscher, 1991]
- Two particle energies computed in rest frame and moving frame

[Rummukainen & Gottlieb, 1995]

Cross check of the Higgs boson propagator mass

- Perform a rigorous resonance analysis for the Higgs boson at physical top quark mass
- Connection between infinite volume scattering phases and finite volume two particle energies [Lüscher, 1991]
- Two particle energies computed in rest frame and moving frame [Rummukainen & Gottlieb, 1995]
- Example result:

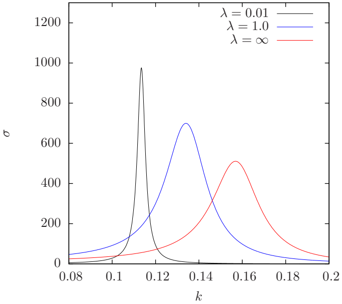




Results of cross check ($m_t \approx 175 GeV$) [Gerhold, Jansen, Kallarackal 2011]

$\hat{\lambda}$	Cutoff Λ	aM_H^R	$a\Gamma_H^R$	aM_H^P
0.01	883(1) GeV	0.2811(6)	0.007(1)	0.278(2)
1.0	1503(5) GeV	0.374(4)	0.033(4)	0.386(28)
∞	1598(2) GeV	0.411(3)	0.040(4)	0.405(4)

- very good agreement of the masses
- decay width smaller than 10% of the mass



Determination of mass bounds

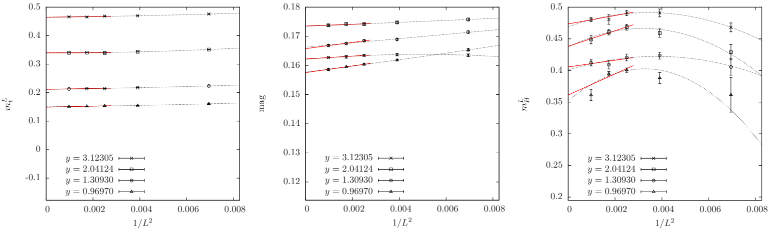
Basic procedure:

- Set the couplings to the desired values to obtain the desired quark masses and cutoff
- lower bound: $\hat{\lambda} = 0$, upper bound: $\hat{\lambda} = \infty$
- (Almost) massless Goldstone bosons induce $\mathcal{O}(L_s^{-2})$ finite volume effects [P. Hasenfratz et al. 1990, A. Hasenfratz et al. 1991]

Determination of mass bounds

Basic procedure:

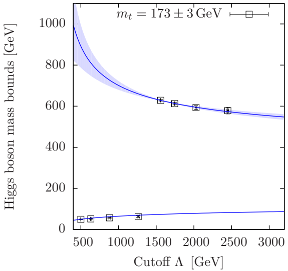
- Set the couplings to the desired values to obtain the desired quark masses and cutoff
- lower bound: $\hat{\lambda} = 0$, upper bound: $\hat{\lambda} = \infty$
- (Almost) massless Goldstone bosons induce $\mathcal{O}(L_s^{-2})$ finite volume effects [P. Hasenfratz et al. 1990, A. Hasenfratz et al. 1991]

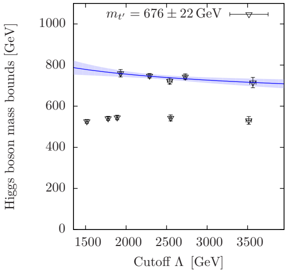


(a) quark mass (b) magnetization (c) Higgs boson mass

Cutoff Dependence

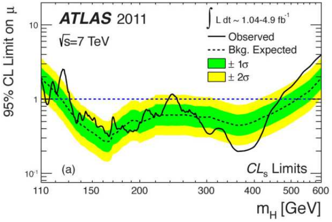
[Gerhold 2010; Kallarackal 2011]

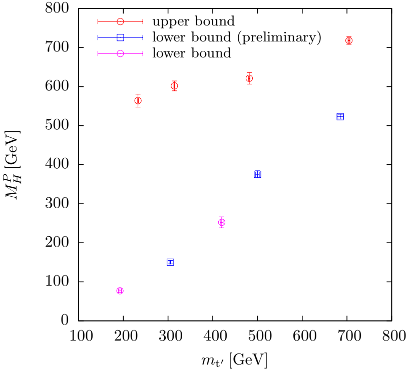




- Blue band for the upper bound shows a fit according to:
  $m_H^{\text{up}} = A \cdot \left\{\log\left(\Lambda^2/\mu^2\right) + B\right\}^{-1/2}$
- Blue curve for the lower bound corresponds to a leading oder LPT prediction
- Significant change in the lower Higgs boson mass bound when increasing the quark mass

Dependence on the quark mass at $\Lambda = 1500$ GeV





- Consequences for the fourth generation. . .

Perturbative estimate

- Compare our results for the lower Higgs boson mass bound to lattice perturbation theory
- Influence of a $\lambda_6\phi^6$ term?
- Leading order effective potential description: [Fodor et al. 2007; Gerhold 2010]

$$U(\bar{\phi}) = \frac{1}{2}m_0^2\bar{\phi}^2 + \lambda\bar{\phi}^4 + \lambda_6\bar{\phi}^6 + U_F(\bar{\phi}) + 6\lambda\bar{\phi}^2(P_H + P_G) + \lambda_6\bar{\phi}^4(15P_H + 9P_G) + \lambda_6\bar{\phi}^2(45P_H^2 + 54P_HP_G + 45P_G^2)$$

Perturbative estimate

- Compare our results for the lower Higgs boson mass bound to lattice perturbation theory
- Influence of a $\lambda_6\phi^6$ term?
- Leading order effective potential description: [Fodor et al. 2007; Gerhold 2010]

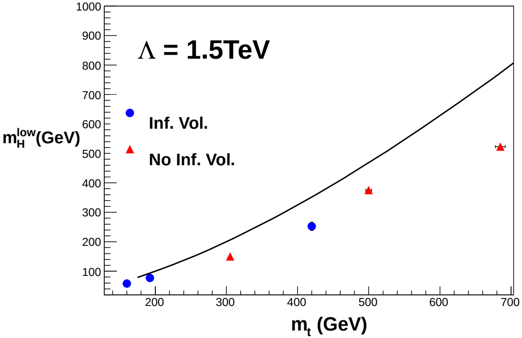
$$U(\bar\phi) = \frac{1}{2}m_0^2\bar\phi^2 + \lambda\bar\phi^4 + \lambda_6\bar\phi^6 + U_F(\bar\phi) + 6\lambda\bar\phi^2(P_H + P_G) + \lambda_6\bar\phi^4(15P_H + 9P_G) + \lambda_6\bar\phi^2(45P_H^2 + 54P_HP_G + 45P_G^2)$$

- vev and Higgs boson mass are given by:

$$U'(v) = 0, \quad U''(v) = M_H$$

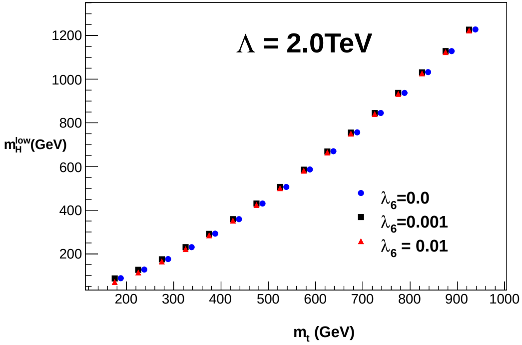
- Constraints: Only one local minimum of U in the scaling region ($|\bar\phi| < 0.5$): $U''(\bar\phi) > 0, \quad \bar\phi > v$

Comparison of the lower mass bound in the effective potential and the simulations



Qualitative agreement of the simulations with PT up to very large couplings

Lower mass bound with and without the ϕ^6 term
(only effective potential!)



No measurable effect on the lower bound due to the ϕ^6 term

Summary

- We provide Higgs boson mass bounds for a wide range of quark masses
- We give arguments for the stability of those bounds

Summary

- We provide Higgs boson mass bounds for a wide range of quark masses
- We give arguments for the stability of those bounds
- However, we will perform non-perturbative analysis of:
  - An additional ϕ^6 term
  - The influence of a mass splitting between the quarks

Summary

- We provide Higgs boson mass bounds for a wide range of quark masses
- We give arguments for the stability of those bounds
- However, we will perform non-perturbative analysis of:
  - An additional ϕ^6 term
  - The influence of a mass splitting between the quarks
  - Effect of an additional scalar singlet [Espinosa et al. 2012]