Lattice Hadron Structure: QCD and Beyond

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Selected Topics

Apologies in advance to those I cannot fit in today…
Outline

§ The Old: Hadron Structure
    ☛ Selected latest updates

§ The New: Applications to BSM Physics
    ☛ Neutron electric dipole moment
    ☛ New tensor and scalar interactions in neutron $\beta$ decay

§ The Ugly: Nucleon Axial Charge
    ☛ Are we facing a new crisis?
Building a Picture of Hadrons
§ Structure function/distribution functions

✿ Deep inelastic scattering (DIS)

✿ $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$

✿ For example, OPE for unpolarized contribution

$$2 \int dx \, x^{n-1} F_1(x, Q^2) = \sum_{q=u,d} c_{1,n}^{(q)} \left( \frac{\mu^2}{Q^2}, g(\mu) \right) \langle x^n \rangle_q$$

$$\int dx \, x^{n-2} F_2(x, Q^2) = \sum_{q=u,d} c_{2,n}^{(q)} \left( \frac{\mu^2}{Q^2}, g(\mu) \right) \langle x^n \rangle_q$$
§ Structure function/distribution functions

- Deep inelastic scattering (DIS)
  \[ \langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q} \]

- TMDs (M. Engelhard)
- Disconnected $\langle x^n \rangle$ (M. Sun)
§ Exploratory stage: quenched results

\[ O_{\mu\nu} = -\operatorname{tr}_c F_{\mu\alpha} F_{\nu\alpha} \]

* Quenched, heavy pion masses, linear chiral extrapolation

§ QCDSF (’97) and LHPC (’07)

* Direct matrix-element calculation: \( \langle x \rangle_g = 0.53(23) \) QCDSF

* HYP-smearing, study pion: \( \langle x \rangle_g = 0.6(2)(1) \)

§ \( \chi \)QCD

K.F. Liu et al., 1203.6388

* Rewrite gluonic observables with massless overlap kernel

\[ \operatorname{tr}_s (\sigma_{\mu\nu} D_{ov}) \propto a^2 F_{\mu\nu} \]

§ QCDSF

R. Horseley et al., 1205.6410

* Feynman-Hellmann theorem with modification of the action

\[ S \rightarrow S + \lambda S_\theta \]

\[ \beta \lambda \frac{1}{3} \left( \sum_{\vec{x},i} \operatorname{Re} \operatorname{tr}_c [1 - P_{i4}(\vec{x})] - \sum_{\vec{x},i<j} \operatorname{Re} \operatorname{tr}_c [1 - P_{ij}(\vec{x})] \right) \]
Gluonic Momentum Fraction

§ $\chi$QCD  K.F. Liu et al., 1203.6388 & private communication

男神 rewrite gluonic observables $\text{tr}_s(\sigma_{\mu\nu}D_{\text{ov}}) \propto a^2 F_{\mu\nu}$

男神 Use $Z_4$ noise sources to estimate $D_{\text{ov}}$ stochastically

男神 $a \approx 0.1$ fm, $16^3$ Wilson + Wilson, $M_\pi \approx 480–650$ MeV, 500 confs, $\langle x \rangle_g = 0.313(56)$

男神 Expensive!
(but $D_{\text{ov}}$ reusable for disconnected contribution?)
**Gluonic Momentum Fraction**

§ QCDSF  
R. Horseley et al., 1205.6410

\( a \approx 0.1 \text{ fm}, \quad 24^3 \text{ Wilson + NP clover, } M_\pi \approx 1100–600 \text{ MeV} \)

\[ \langle x \rangle_g = 0.43(7) \]

Cheap, operator by operator; reweighting for dynamical lattices

\[ M_\pi \approx 1.1 \text{ GeV} \]

\[ M_\pi \approx 0.62 \text{ GeV} \]
Medium Modification


げる Structure function changes significantly between heavy nuclei and deuterium
げる Not only significant for heavy nuclei, also important for light-nuclear systems  J. Seely et al., Phys. Rev. Lett. 103, 202301 (2009)

§ Important for tests of SM; e.g. NuTeV anomaly
げる Weak mixing angle expt. 3 sigma away from SM

Medium Modification

§ The EMC effects

☞ Structure function changes significantly between heavy nuclei and deuterium
☞ Not only significant for heavy nuclei, also important for light-nuclear systems

§ Important for tests of SM;
e.g. NuTeV anomaly
☞ Evidence for medium modification effects?
First lattice-QCD attempt to measure EMC effects

- Pion momentum fraction in pion medium
  \[ O = O_{44} - \frac{1}{3} (O_{11} + O_{22} + O_{33}) \]

- With \( m_\pi \approx 290–490 \text{ MeV}, 2 \text{ lattice spacings} \)

\[ \langle x \rangle^{\pi,N} / \langle x \rangle^{\pi,0} \text{ with thermal-state degrees of freedom} \]

Multiple \( t_{\text{sep}} \) used

Preliminary

W. Detmold + HWL; and updated

Huey-Wen Lin — The XXX International Symposium on Lattice Field Theory
Form Factors

\section*{Structure function/distribution functions}

\begin{itemize}
  \item Deep inelastic scattering (DIS)
  \item $\langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}$
\end{itemize}

\section*{Form factors}

\begin{itemize}
  \item Elastic scattering
  \item $F_1(Q^2), F_2(Q^2), G_A(Q^2), G_P(Q^2)$
  \item For example, octet baryons
\end{itemize}

\begin{align*}
\langle B | V_\mu | B \rangle (q) &= \overline{u}_B(p') \left[ \gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M_B} \right] u_B(p) \\
\langle B | A_\mu (q) | B \rangle &= \overline{u}_B(p') \left[ \gamma_\mu \gamma_5 G_A(q^2) + \gamma_5 q_\nu \frac{G_P(q^2)}{2M_B} \right] u_B(p)
\end{align*}
Form Factors

§ Structure function/distribution functions

★ Deep inelastic scattering (DIS)

★ \langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}

§ Form factors

★ Elastic scattering

★ \text{\textit{F}}_1(Q^2), \text{\textit{F}}_2(Q^2), \text{\textit{G}}_A(Q^2), \text{\textit{G}}_P(Q^2)

★ Charges and radii (J. Green, S. Ohta, M. Lin, B. Owen, T. Rae, B. Menadue, V. Guelpers; C. Alexandrou, J. Zanotti)

★ Transition form factors

(X. Feng; B. Menadue; C. Alexandrou, S. Sasaki)
Fourier transform using large-$Q^2$ form factors to reveal transverse charge densities in a polarized nucleon
How does high-\(Q^2\) data affect the charge density?

- **Red** band uses lattice data \(\leq 2.0 \text{ GeV}^2\)
- **Blue** band uses lattice data \(\leq 4.0 \text{ GeV}^2\)

HWL et al., arXiv: 1005.0799

**Large-\(Q^2\) Caveats**

- Essential to get coordinate-space distribution in central region
- Further studies needed for discretization effects
- Possible improvement: step-scaling method, …
- Important for pion form factor
§ Charge radii

✿ Lattice data way too low; no help for the proton-radius puzzle
§ Charge radii

Lattice data way too low; no help for the proton-radius puzzle.
§ Charge radii

Form Factors
§ Charge radii

- Lattice data way too low; no help for the proton-radius puzzle

\[ g_P = \left[ m_\mu G_P (0.88 m_\mu^2) / 2 m_N \right] \]

- Poor constraints
- (DWF numbers so far)
- Important for muon physics
Generalized Parton Distribution

§ Structure function/distribution functions

≝ Deep inelastic scattering (DIS)
≝ \langle x^n \rangle_q, \langle x^n \rangle_{\Delta q}, \langle x^n \rangle_{\delta q}

§ Form factors

≝ Elastic scattering
≝ \mathcal{F}_1(Q^2), \mathcal{F}_2(Q^2), \mathcal{G}_A(Q^2), \mathcal{G}_P(Q^2)

§ Generalized Parton Distribution

≝ Deeply virtual Compton scattering (DVCS)
≝ \langle x^{n-1} \rangle_q = A_{n0}(0), \langle x^{n-1} \rangle_{\Delta q} = \tilde{A}_{n0}(0),
\langle x^n \rangle_{\delta q} = A_{Tn0}(0)

≝ \mathcal{F}_1(Q^2) = A_{10}(Q^2), \mathcal{F}_2(Q^2) = B_{10}(Q^2),
\mathcal{G}_A(Q^2) = \tilde{A}_{10}(Q^2), \mathcal{G}_P(Q^2) = B_{10}(Q^2)

≝ Nucleon spin \mathcal{A}_{20}(0), \mathcal{B}_{20}(0)
Generalized Parton Distribution

\[ \langle N(p', s')|O_{\mu\nu}^{\mu\nu}|N(p, s)\rangle = \bar{u}_N(p', s') \left[ A_{20}(q^2) \gamma^{\mu P^\nu} + B_{20}(q^2) \frac{i\sigma^{\mu\alpha} q_\alpha P^\nu}{2m} + C_{20}(q^2) \frac{1}{m} q^{\mu q^\nu} \right] u_N(p, s), \]

\[ \langle N(p', s')|O_{\mu\nu}^{\mu\nu}\gamma_5|N(p, s)\rangle = \bar{u}_N(p', s') \left[ \tilde{A}_{20}(q^2) \gamma^{\mu P^\nu} \gamma^5 + \tilde{B}_{20}(q^2) \frac{q^{\mu P^\nu}}{2m} \gamma^5 \right] u_N(p, s). \]

§ Generalized Parton Distribution

- Deeply virtual Compton scattering (DVCS)

- \[ \langle x^{n-1} \rangle_q = A_{n0}(0), \langle x^{n-1} \rangle_{\Delta q} = \tilde{A}_{n0}(0), \]
  \[ \langle x^n \rangle_{\delta q} = A_{Tn0}(0) \]

- \[ F_1(Q^2) = A_{10}(Q^2), F_2(Q^2) = B_{10}(Q^2), \]
  \[ G_A(Q^2) = \tilde{A}_{10}(Q^2), G_P(Q^2) = B_{10}(Q^2) \]

- Nucleon spin \( A_{20}(0), B_{20}(0) \)
What is the makeup of the nucleon?

- The origin of the nucleon’s spin (the “spin crisis”)
- For example, LHPC + QCDSF dynamical results

$\Delta \Sigma$: spin

$L$: orbital angular momentum

$M^2_{\pi} (\text{GeV}^2)$

- Ignore disconnected diagram
- Gluon contribution estimated from sum rule
Origin of Proton Spin

§ What is the makeup of the nucleon?

χQCD, 1203.6388 [hep-ph] and private communication w/ Y. Yang

§ Breakdown:

\[ \Delta \Sigma_q = 50(2)\%, \quad L_q = 25(12)\% \text{ (mostly DI)}, \quad J_g = 25(8)\% \]

§ Looking forward to χQCD (overlap/DWF), QCDSF (clover)
Applications beyond QCD
Many opportunities to probe BSM with LQCD

§ Muon $g-2$

§ Strangeness and dark matter

§ Electric dipole moment
  ☞ CP-violating effect
  ☞ Extremely small in SM: $\approx 10^{-30}$ e-cm
  ☞ Best SUSY model killer (T. Bhattacharya; E. Shintani)

§ Nucleon beta decay
  ☞ Non-$V-A$ (e.g. scalar and tensor) interactions
  ☞ Scalar and tensor charges R. Gupta (PNDME), J. Green (LHPC)
Lagrangian \( \mathcal{S} = \mathcal{S}^{\text{CP\ Even}}_{\text{QCD}} - i \Theta \frac{g^2}{16 \pi^2} \int d^4x \, G^{\mu\nu} \tilde{G}_{\mu\nu} \)

Leading contribution

- Nucleon EM form factor \( eF_3(0)/2M_N \) \( \text{RBC, J/E, CP-PACS(2005), ...} \)
  - E. Shintani (Improved algorithm to increase statistics)

- Energy shift in nucleon mass with external \( \vec{E} \) \( \text{CP-PACS(2006, 2010), QCDSF(2011), ...} \)

- Calculate \( \tan[2\alpha(\theta)] F_2(0) \)
nEDM

§ Lagrangian

\[ S = S_{\text{QCD}}^{\text{CP Even}} - i \Theta \frac{g^2}{16 \pi^2} \int d^4x G^{\mu \nu} \tilde{G}_{\mu \nu} \]

§ Leading contribution

Chiral extrapolation
K. Ottnad et al., 2010

\[ -0.015(5) \theta \, e \cdot \text{fm} \]

§ Plenty of room to make improvement

HWL, 1112.2435
\[ S = S_{\text{QCD}}^{\text{CP Even}} - i \Theta \frac{g^2}{16 \pi^2} \int d^4 x \, G^{\mu \nu} \tilde{G}_{\mu \nu} + \]

\[ \frac{ie d_\mu^\gamma}{\Lambda_{\text{BSM}}^2} \overline{Q} \sigma_{\mu \nu} \gamma_5 F_{\mu \nu} \tilde{H} \, U + \frac{ie d_\mu^\gamma}{\Lambda_{\text{BSM}}^2} \overline{Q} \sigma_{\mu \nu} \gamma_5 F_{\mu \nu} \, H \, D + \]

\[ \frac{ig_3 d_\mu^G}{\Lambda_{\text{BSM}}^2} \overline{Q} \sigma_{\mu \nu} \gamma_5 \lambda^A G^{\mu \nu} A \tilde{H} \, U + \frac{ig_3 d_\mu^G}{\Lambda_{\text{BSM}}^2} \overline{Q} \sigma_{\mu \nu} \gamma_5 \lambda^A G^{\mu \nu} A \, H \, D \]

\section{Leading contribution}

\section{Higher-order operators}

T. Bhattacharya

(this conference)

Effective field theory and proposed LQCD calculations

HWL, 1112.2435
Neutron beta decay could be related to new interactions: the scalar and tensor

\[ H_{\text{eff}} = G_F \left( J_{V-A}^{\text{lept}} \times J_{V-A}^{\text{quark}} + \sum_i \varepsilon_i^{\text{BSM}} \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right) \]

\[ \hat{O}_S = \bar{u}d \times \bar{e}(1 - \gamma_5)\nu_e \rightarrow g_S = \langle n|\bar{u}d|p\rangle \]

\[ \hat{O}_T = \bar{u}\sigma_{\mu\nu}d \times \bar{e}\sigma^{\mu\nu}(1 - \gamma_5)\nu_e \rightarrow g_T = \langle n|\bar{u}\sigma_{\mu\nu}d|p\rangle \]

- \( \varepsilon_S \) and \( \varepsilon_T \) are related to the masses of the new TeV-scale particles
- … but the unknown coupling constants \( g_{S,T} \) are needed

Given precision \( g_{S,T} \) and \( O_{\text{BSM}} \), predict new-physics scales

\[ O_{\text{BSM}} = f_0(\varepsilon_{S,T} g_{S,T}) \]

Precision LQCD input

\( (m_\pi \approx 140 \text{ MeV, } a \rightarrow 0) \)

\( \varepsilon_{S,T} \propto \Lambda_{S,T}^{-2} \)
Tensor and Scalar Charges

§ Tensor charge: the zeroth moment of the transversity
\[ g_T = \delta u - \delta d \]

- Experimentally, probed through SIDIS: \[ g_T(Q^2=0.8 \text{ GeV}^2) = 0.77^{+0.18}_{-0.24} \]
- Model estimate: 0.8(4)

§ Scalar charge \[ \langle n | \bar{u}d | p \rangle \] Prior model estimate: \[ 1 \gtrsim g_S \gtrsim 0.25 \]

\[ g_T^{\text{LQCD}} = 0.988(42)(?) \]
\[ m_\pi^2 \text{ (GeV}^2) \]
\[ g_S^{\text{LQCD}} = 0.761(88)(?) \]
Combined with Experiments

§ Given precision $g_{S,T}$ and $O_{BSM}$, predict new-physics scales

Nuclear Exp.

$$O_{BSM} = f_{O}(\varepsilon_{S,T} g_{S,T})$$

Model input

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Nuclear beta decays
- $0^+ \rightarrow 0^+$ transitions
- $\beta$ asym in Gamow-Teller $^{60}$Co
- polarization ratio between Fermi and GT in $^{114}$In
- positron polarization in polarized $^{107}$In
- $\beta$-ν correlation parameter $a$

Nuclear Exp + Model $g_{S,T}$
Combined with Experiments

§ Given precision $g_{S,T}$ and $O_{BSM}$, predict new-physics scales

New UCN Exp. $O_{BSM} = f_o(\epsilon_{S,T} g_{S,T})$

Model input

$\epsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

LANL UCN neutron decay exp’t

$\frac{d\Gamma}{d\epsilon} \propto F(E_e) \left[ 1 + \right.$

\begin{align*}
-\frac{b}{E_e} & + \left( B_0 + B_1 \frac{m_e}{E_e} \right) \frac{\tilde{\sigma}_n p_\nu}{E_\nu} + \ldots
\end{align*}

Expect by 2013:

$|B_1 - b|_{BSM} < 10^{-3}$

$|b|_{BSM} < 10^{-3}$

Similar proposal at ORNL by 2015
Combined with Experiments

§ Given precision $g_{S,T}$ and $O_{BSM}$, predict new-physics scales

New UCN Exp.

$$O_{BSM} = f_{o}(\xi_{S,T} g_{S,T})$$

Precision LQCD input

($m_\pi \to 140$ MeV, $a \to 0$)

$$\xi_{S,T} \propto \Lambda_{S,T}^{-2}$$

LANL UCN neutron decay exp’t

$$d\Gamma \propto F(E_e) \left[ 1 + \left( \frac{B_0 + B_1 m_e}{E_e} \right) \frac{\vec{\sigma}_n p_\nu}{E_\nu} + \ldots \right]$$

Expect by 2013:

$$|B_1 - b|_{BSM} < 10^{-3}$$

$$|b|_{BSM} < 10^{-3}$$

Similar proposal at ORNL by 2015
Constraints from high-energy experiments?
LHC current bounds and near-term expectation

Estimated though effective $L$

$$\mathcal{L} = -\frac{\eta_S}{\Lambda_S^2} V_{ud}(\bar{u}d)(\bar{e}P_L\nu_e)$$
$$-\frac{\eta_T}{\Lambda_T^2} V_{ud}(\bar{u}\sigma^{\mu\nu}P_Ld)(\bar{e}\sigma_{\mu\nu}P_L\nu_e)$$

Looking at high transverse mass in $e\nu+X$ channel

Compare with $W$ background

Estimated 90% C.L. constraints on $\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

HWL, 1112.2435; 1109.2542
T. Bhattacharyya et al, 1110.6448
$g_A$ Crisis!

The ugly: when should we panic about it?
A fundamental measure of nucleon structure
Important to the rate of $pp$ fusion
Axial-vector–current matrix element
and $g_A = G_A^{u-d} (Q^2=0)$
Benchmark for nucleon structure
Survey of lattice studies (2011 Nov)

**Nucleon Axial Charge**

**2f**

**2+1f**
What’s the Deal with $g_A$?

§ No longer gold-plated?
“Welcome to the lattice and its dangerous animals.”

Karl Jansen
PROCEED WITH CAUTION

§ Re-examine all the systematics
Nucleons are more complicated than mesons because...

§ Noise issue

⁻ Large $t_{\text{sep}}$ loses signal

§ Excited-state contamination

⁻ Nearby excited-state Roper(1440)

§ Hard to extrapolate

⁻ $\Delta$ resonance nearby; multiple expansions, poor convergence…
⁻ May not be an issue in the physical pion-mass era

§ Requires large volume and statistics

⁻ Ensembles are not always generated with nucleons in mind
§ Trade off: signal-to-noise versus contamination

Noise issue (P. Lepage; D. Kaplan 2011)

For example, CLS/Mainz

2f NP clover,

\[ M_\pi \approx 320 \text{ MeV} \]

\[ a \approx 0.063 \text{ fm} \]

Fix \( N_{\text{meas}} = 200 \)

1205.0180 & private communication

\( t_{\text{sep}} 0.69-1.07 \text{ fm} \)
§ Trade off: signal-to-noise versus contamination

Noise issue (P. Lepage; D. Kaplan 2011)

§ Options

Stay at large $t_{\text{sink}}$: RBC/UKQCD (need to check smaller pion mass)

Include excited-state degrees of freedom

Multistate fitting or variational method from 3pt correlator matrix

H WL (Lat 2008); ETMC/LHPC/Mainz-CLS (2011); CSSM 2012 (mesons)

Extend to small $t_{\text{sink}}$ to pick up better signal and apply “summation” method

$$S(t_s) := \sum_{t=0}^{t_s} R(t, t_s) \xrightarrow{t_s \gg 0} c + t_s \left\{ g_A^{\text{bare}} + O(e^{-\Delta t_s}) \right\}$$

$g_A$ obtained from slope
ETMC C. Alexandrou et al. & private communication

- 2+1+1f, $M_\pi \approx 380$ MeV, APE + Gaussian increase $N_{\text{meas}}$ to $O(3000)$ at largest $t_{\text{sep}}$.
- No effect due to $t_{\text{sep}}$ from 0.31–1.01 fm
- Summation [5,13] is consistent with $t = 12a$ plateau fit

0.31 fm, really? Would be interested to see other works
Check on lighter pion mass
Excited-State Contamination

§ CLS/Mainz

2f, NP Clover ("summation" vs "plateau" $t_{\text{sep}} = 1.1$ fm)

§ LHPC (J. Green):
consistent results using largest $t_{\text{sep}}$ and summation

§ My two cents: Not clearly superior
Including excited-states in the analysis is the way to go

§ HWL (2008): simultaneous fit and 3pt correlator matrix

§ PNDME

R. Gupta (Tuesday)

☞ 2+1+1f, clover/HISQ; $t_{\text{sep}} = 0.96–1.44$ fm

☞ Sim. fit gives consistent results with largest $t_{\text{sep}}$ at each ensemble

☞ Will be more aggressive to try our smaller $t_{\text{sep}}$

§ ETMC: variational method

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R. Gupta (Tuesday)

☞ 2f, 300 MeV pion, $t_{\text{sep}} = 1.07$ fm

§ CSSM: successful in meson cases

B. Owen (Wednesday)

☞ nucleon next

May still have an optimal $t_{\text{sep}}$ but won’t lose as much signal
§ How big $M_\pi L$ is required?
§ How big $\pi L$ is required?
§ ChPT volume correction/used to estimate systematics

 мер ETMC, QCDSF, CLS/Mainz: possibly underestimated?
Finite-Volume Effects

§ How big $M_\pi L$ is required?

§ ChPT volume correction/used to estimate systematics

☞ ETMC, QCDSF, CLS/Mainz: possibly underestimated?

§ Example study (RBC/UKQCD)

\[ A + B m_\pi^2 + C f_V(m_\pi L) \]

§ How big $M_\pi L$ is required?
§ How global data changes with a cut

$$A + B m_\pi^2 + C f_V(m_\pi L)$$

$$f_V \sim e^{-m_\pi L}$$

$$f_V \sim (m_\pi L)^{-3}$$

$$f_V \sim m_\pi^2 e^{-m_\pi L} (m_\pi L)^{-0.5}$$
§ How big $M_{\pi}L$ is required?

§ How global data changes with a cut

Cut by $m_{\pi}L > 4$

\[ A + B m_{\pi}^2 + C f_V(m_{\pi}L) \]

\[ f_V \sim e^{-m_{\pi}L} \]

\[ f_V \sim (m_{\pi}L)^{-3} \]

\[ f_V \sim m_{\pi}^2 e^{-m_{\pi}L}(m_{\pi}L)^{-0.5} \]
Finite-Volume Effects

§ How big $M_\pi L$ is required?

§ How global data changes with a cut

Cut by $m_\pi L > 4$

$$A + B m_\pi^2 + C f_V(m_\pi L)$$

$f_V \sim e^{-m_\pi L}$

6-fm box

4-fm box
Chiral Extrapolation

§ Small shift matters?

CLS/Mainz, 1205.0180

§ Blind analysis?

§ More precise studies are needed

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§ Answers?

☞ I have more questions than when I started the review:

Q: Is large $t_{\text{sep}}$ needed for summation method? Do we gain anything? ETMC: “no”, CLS/Mainz: “yes”

Q: Can we get reliable ground-state signal from small $t_{\text{sep}}$? Current excited-state analysis (PNDME/LHPC) performed at larger $t_{\text{sep}}$, O(1) fm and larger errorbar too big to tell

§ More VM basis analyses (all-to-all, AMA)

§ High-statistics studies are needed!

§ Disappointment?

☞ Certainly not.

   We are just entering into the precision era to explore these issues…

§ Difficulties = opportunities

§ Affects the whole community! When we fail at $g_A$…
Summary

Exciting time to explore

§ LQCD is building a picture of hadrons
  ➔ Revealed proton spin components, hadron impact-space distribution
  ➔ New techniques for gluonic, disconnected and in-medium quantities shine new light for more calculations

§ Application of LQCD input to probe BSM
  ➔ Opportunities combining high- (TeV) and low- (GeV) energies
  ➔ Vital input when experiment is limited (e.g. $g_s$)

§ Aim at high precision and understand/quote systematics!

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Backup Slides
§ QCDSF hypothesis: $Z_A$ might be a problem?
§ QCDSF hypothesis: $Z_A$ might be a problem?

§ Residual $O(a)$ artifacts in

Clover: $A^R_\mu = Z_A (1 + b_A a m_q) (A_\mu + ac_A \partial_\mu P)$

Chiral fermions: $m_{\text{res}}$

§ Other systematic cancellations (such as volume, ...)

Need higher statistics to be conclusive
Nucleon Axial Charge

\[ g_A \]

\[ m_{\pi}^2 \text{ (GeV}^2) \]

- Experiment
- LHPC 2+1f Clover
- QCDSF 2f Clover
- RBC/UKQCD 2+1f DSDR
- PNDME 2+1+1f Mixed
- ETMC 2+1+1f TM
- HSC 2+1f Anisoclover
- ETMC 2f TM
- CLS/Mainz 2f Clover
Nucleon Axial Charge

![Graph showing the relationship between \( t_{\text{sep}} \) and \( m_\pi^2 \) for various fermion actions in lattice QCD simulations.](image)

- LHPC 2+1f Clover
- RBC/UKQCD 2+1f DSDR
- PNDME 2+1+1f Mixed
- ETMC 2+1+1f TM
- ETMC 2f TM
- CLS/Mainz 2f Clover

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§ Structure

☞ TMDs (M. Engelhardt); disconnected $\langle x^n \rangle$ (M. Sun)

§ Form Factors

☞ Charges and radii (J. Green, S. Ohta, M. Lin, B. Owen, T. Rae, B. Menadue, V. Guelpers; C. Alexandrou)
☞ Transition form factors (X. Feng; B. Menadue; C. Alexandrou, S. Sasaki)

§ Generalized Parton Distribution

☞ $\chi_{QCD}$ (private communication)

§ Electric Dipole Moments

☞ T. Bhattacharya; E. Shintani

§ Scalar and Tensor Charges

☞ R. Gupta (PNDME), J. Green (LHPC)
§ Excited-State Contamination

§ ETMC  C. Alexandrou et al. & private communication

\( 2+1+1, \, M_\pi \approx 380 \text{ MeV}, \, N_{\text{meas}} \) increase to \( O(3K) \), APE+Gaussian,

\( \sim \) Dramatic effects, \( t_{\text{sep}} \) from 0.39-1.01 fm for \( <x> \)

\( \sim \) Summation [6,13], \( t=13a \) plat. Fit

\[ \sum_t \]

\( t_{\text{sep}} \) from 0.93-1.39 fm

§ Similar results with LHPC
§ How big $M_\pi L$ is required?

§ Example study (RBC/UKQCD)

RBC/UKQCD arXiv:1003.3387[hep-lat]
Transversely Polarized Neutron

\[ \rho^N_T(b) = \rho^N_0(b) \]

\[ = \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M_N} J_1(bQ) F_2(Q^2) \]

Fourier transform using large-\(Q^2\) form factors to reveal transverse charge densities in a polarized nucleon

Huey-Wen Lin — The XXX International Symposium on Lattice Field Theory
A Tale of Two Scales

§ LHC strikes out onto the high-energy frontier (8 TeV)
☞ Direct measurement of Higgs and BSM particles

§ Many experiments refine low-energy measurements
☞ Discern small discrepancies from the Standard Model
    Muon $g-2$, $Q_{\text{weak}}$, CKM matrix…
☞ Probe small signals that are suppressed in the SM
    $0\nu\beta\beta$, nEDM, dark matter, non-$V-A$ interactions in $\beta$ decay…
Ultra-Cold Neutrons

§ Worldwide UCN sources

✧ A rapidly expanding resource for nuclear physics
✧ Many facilities available and under construction

Stay tuned for exciting low-energy precision data
A fundamental measure of nucleon structure

Important to the rate of $pp$ fusion

Axial-vector–current matrix element

and $g_A = G_A^{u-d} (Q^2=0)$

Benchmark for nucleon structure

Survey of lattice studies (2011 Nov)

$g_A^{\text{LQCD}} = 1.16(3)$
§ Chiral extrapolation

§ Small shift matters?

CLS/Mainz, 1205.0180

\[ m_{\pi,\text{phys}}^2 \]

\[ m_{\pi}^2 \text{ [GeV}^2\text{]} \]

\( \alpha = 0.079\text{fm} \)
\( \alpha = 0.063\text{fm} \)
\( \alpha = 0.050\text{fm} \)
§ Focus on $N_f = 2, 2+1, 2+1+1$

☞ Those who sent me the numbers to establish a database

§ Excited-state contamination

☞ Lots of studies in the past few years

§ Finite-volume effects

§ Lattice actions, renormalization and $O(a)$-improved operators

§ Chiral extrapolation
§ Trade off: signal-to-noise versus contamination

Noise issue (P. Lepage; D. Kaplan 2011)

What you want: \[ N \rightarrow \pi \]

What you get: \[ N^\dagger \rightarrow \pi \]

- Signal falls exponentially as \( e^{-m_N t} \)
- Noise falls as \( e^{-(3/2)m_{\pi} t} \)
- Problem worsens with decreasing quark (pion) mass