Lattice Flavor Physics with an eye to SuperB
An emblematic study showing the important role of Lattice QCD:
the Unitarity Triangle Analysis

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The **SM** turns out to be very successful in describing essentially all processes

**But**

It is expected to be an **effective theory** valid up to a cutoff scale as it has some important **limits**

- The **SM** is a quantum theory for strong and electroweak interactions but **NOT** for gravitation

- There is cosmological evidence of **Dark Matter** (not made up of **SM** particles) in the Universe

- The **SM** **CP**-violation due to the phase in the **Cabibbo-Kobayashi-Maskawa** matrix is **not enough** to explain the required amount for baryogenesis

- In order to have a Higgs mass of \( O(100 \, \text{GeV}) \) as expected, an **innatural fine-tuning** is required **(hierarchy problem)**
Moreover, the solution doesn’t seem to be trivial: the FLAVOR PROBLEM

“NP is expected at the TeV scale (in order to solve the hierarchy problem)

but in flavor processes NP effects are not observed (hinting for NP at higher scales)"

The flavor structure of the NP model cannot be generic

In order to reveal NP and understand its nature, Flavor Physics has a fundamental role, which is complementary to the direct production of NP particles

The study of clean and SM suppressed Flavor processes may reveal NP effects
It is crucial to have hadronic uncertainties well under control. **Lattice QCD has a primary role**

An emblematic study showing the important role of Lattice QCD is the determination of the parameters of the *Cabibbo-Kobayashi-Maskawa mixing matrix*. 
The CKM Matrix

$$\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix} = V_{CKM} \begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}$$

• 3x3 unitary matrix
• 4 parameters: 3 angles and 1 phase
• The phase is responsible for CP-violation

The Wolfenstein parameterization

$$(A, \lambda, \rho, \eta)$$

up to $O(\lambda^3)$ with $\lambda \approx \sin \theta_{\text{Cabibbo}} \approx 0.2$

$$\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix} \approx \begin{pmatrix}
    1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3 (\rho - \text{in}) \\
    -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
    A\lambda^3 (1 - \rho - \text{in}) & -A\lambda^2 & 1
\end{pmatrix}$$

($\eta \neq 0 \leftrightarrow$ CP-violation)

(O(\lambda^5) corrections are required by the present accuracy)
The expansion parameter $\lambda = V_{us}$ from Lattice QCD

- Unitarity ($V_{CKM}^\dagger V_{CKM} = 1$) provides 9 conditions on the CKM parameters

1st row: the most stringent unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Source: Nuclear $\beta$-dec. $K^{\pm}K^{\mp}$ b$\to$u semil.
Abs. error: $4 \cdot 10^{-4}$ $5 \cdot 10^{-4}$ $\sim 10^{-6}$

Pseudoscalar decay constant $f_K$ and vector form factor $f_+(q^2=0)$ from Lattice QCD

See Gilberto Colangelo's FLAG review (tomorrow)

FLAG 1011.4408

$|V_{us}| = 0.2254(9)$ $N_f = 2 + 1$

$|V_{us}| = 0.2251(18)$ $N_f = 2$

less than 0.5% uncert.
Isospin Breaking Effects

The lattice determinations are usually obtained in the limit of exact ISOSPIN SYMMETRY, i.e. $m_u = m_d$ and $Q_u = Q_d = 0$

Though small, isospin breaking effects are becoming important at the current level of precision in flavor physics. Their typical size is:

- $Q_u \neq Q_d : O(\alpha_{e.m.}) \approx 1/100$ "electromagnetic"
- $m_u \neq m_d : O[(m_d-m_u)/\Lambda_{QCD}] \approx 1/100$ "strong"

Recently, Lattice studies of (em and strong) isospin breaking effects have been performed (mainly for estimating mass splittings)

→ See Taku Izubuchi's review

Last year, the strong IB corrections to $f_K/f_\pi$ and to $f_+(0)$ have been calculated on the Lattice for the first time
A strategy for Lattice QCD: the \((m_d-m_u)\) expansion

**Roma123 Collaboration** 1110.6294 [hep-lat]


Expand the functional integral in powers of

\[
\langle O \rangle \propto \int D\phi \, O \, e^{-S_0 + \delta m \hat{S}} \approx \int D\phi \, O \, e^{-S_0} \left(1 + \delta m \hat{S}\right) \approx \langle O \rangle_0 + \delta m \langle O \hat{S} \rangle_0
\]

Computation of the (not small) slope

Very promising!
(exploratory study with modest statistics)

Preliminary,
to be extrapolated to the chiral and continuum limit,
disconnected contributions to be included

\[
\left[\frac{F_{K^+}/F_{\pi^+}}{F_K/F_{\pi}} - 1\right]^{QCD} = -0.0039(3)(2) \times \frac{[M_{K^0}^2 - M_{K^+}^2]^{QCD}}{6.05 \times 10^3 \text{ MeV}^2}
\]

\[
\left[\frac{f_{K^0\pi^-}(0) - f_{K^+}(0)}{f_{K^+\pi}(0)}\right]^{QCD} = 0.85(18)(1) \times 10^{-4} \times \frac{[M_{K^0}^2 - M_{K^+}^2]^{QCD}}{6.05 \times 10^3 \text{ MeV}^2}
\]
The Unitarity Triangle Analysis (UTA)

- Unitarity ($V_{CKM}^+ V_{CKM} = 1$) provides 9 conditions on the CKM parameters.
- Among these it is of great phenomenological interest:
  \[ V_{ub} V_{ud} + V_{cb} V_{cd} + V_{tb} V_{td} = 0 \]

\[ \bar{\rho} = \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} = \eta \left(1 - \frac{\lambda^2}{2}\right) \]

It defines a triangle in the ($\rho, \eta$)-plane (with sides of similar size, so that CP-violation is visible).

Unitarity Triangle (UT)
UTA by UTfit
www.utfit.org : Summer2012 (post-Moriond12) fit
(conservative averages for the Lattice inputs:
simple (not-weighted) averages with the error
representing present typical uncertainties)

Other UT analyses exist, by: CKMfitter (http://ckmfitter.in2p3.fr/),
Laiho&Lunghi&Van de Water (http://krone.physik.unizh.ch/~lunghi/webpage/LatAves/page3/page3.htm),
Lunghi&Soni (1010.6069),...
**Collaboration of Theorists and Experimentalists**

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<th>Institution</th>
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<tr>
<td>Adrian Bevan</td>
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Great Accuracy achieved in the UTA

Experimental Constraints

<table>
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<th>Obs.</th>
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<td>$\varepsilon_K$</td>
<td>$\approx 0.5%$</td>
</tr>
<tr>
<td>$\Delta m_d$</td>
<td>$\approx 1%$</td>
</tr>
<tr>
<td>$\Delta m_d / \Delta m_s$</td>
<td>$\approx 1%$</td>
</tr>
<tr>
<td>$V_{ub} / V_{cb}$</td>
<td>$\approx 15%$</td>
</tr>
<tr>
<td>Br($B \rightarrow \tau \nu$)</td>
<td>$\approx 20%$</td>
</tr>
<tr>
<td>$\sin 2\beta$</td>
<td>$\approx 3%$</td>
</tr>
<tr>
<td>$\cos 2\beta$</td>
<td>$\approx 15%$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\approx 7%$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\approx 14%$</td>
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<tr>
<td>$(2\beta + \gamma)$</td>
<td>$\approx 50%$</td>
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For a significant comparison between exp. measurements and theor. predictions, hadronic uncertainties must be well under control.
THE UTA CONSTRANTS

Relying on LATTICE calculations

UT-ANGLES
The UTA within the Standard Model

The experimental constraints overconstrain the CKM parameters consistently

\[ \bar{\rho} = 0.139 \pm 0.021 \]

\[ \bar{\eta} = 0.353 \pm 0.014 \]

The UTA has established that the CKM matrix is the dominant source of flavor mixing and CP violation

SM analysis

\sim 15\%

\sim 4\%
From a closer look

From the UTA (excluding its exp. constraint)

<table>
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<th>Measurement</th>
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<td>$\sin 2\beta$</td>
<td>0.81±0.05</td>
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<tr>
<td>$\gamma$</td>
<td>68°±3°</td>
<td>76°±11°</td>
<td>&lt;1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>88°±4°</td>
<td>91°±6°</td>
<td>&lt;1</td>
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<tr>
<td>$</td>
<td>V_{cb}</td>
<td>\cdot 10^3$</td>
<td>42.3±0.9</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>\cdot 10^3$</td>
<td>3.62±0.14</td>
</tr>
<tr>
<td>$\hat{B}_K$</td>
<td>0.85±0.09</td>
<td>0.75±0.02</td>
<td>1.1</td>
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<td>$\text{BR}(B \rightarrow \tau \nu) \cdot 10^4$</td>
<td>0.82±0.08</td>
<td>1.67±0.30</td>
<td>-2.7</td>
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Can NP explain the enhancement?

- The NP contribution, for being visible, should be at tree-level too.
- It could come from a charged Higgs, as it couples significantly only to the $\tau$.

The charged Higgs can not explain the enhancement in simple models (due to other constraints, mainly $b \rightarrow s \gamma$).

2HDM of type II
($H_u$ couples to up-quarks $H_d$ couples to down-quarks)

$$\frac{BR(B \rightarrow \tau \nu)}{\text{BR}(B \rightarrow \tau \nu)_{2HDM}} = \left(1 - \tan^2 \beta \frac{m_3^2}{m_{H^+}^2}\right)^2$$

Suppression factor for allowed $\tan \beta / m_{H^+}$ values.
More recent NP analyses have been motivated by the new (full data) BaBar results [1205.5442] for

\[ \mathcal{R}(D^*) = \frac{B(\bar{B} \to D^*(\tau^-\bar{\nu}_\tau))}{B(\bar{B} \to D(\ell^-\bar{\nu}_\ell))} \]

- They exceed the SM prediction by 2.0(2.7)σ (3.4 σ when combined!)
- A charged Higgs could contribute, but in 2HDM of type II the \( \tan\beta/m_{H^+} \) value which is able to explain the D enhancement cannot explain the D* measurement
  [based on Heavy Quark Symmetry + quenched form factors Kamenik&Mescia08 and Fajfer&Kamenik&Nisandzic12]

More elaborated NP models could provide an explanation for the BaBar results and for \( \text{Br}(B \to \tau \nu) \):

- 2HDM of type III (with \( H_u \) and \( H_d \) coupling to both up- and down-quarks) with flavor violation in the up sector [A.Crivellin, C.Greub, A.Kokulu, 1206.2634]

- Right-right vector and right-left scalar currents (effective field theory approach) that could exist in some 2HDM, leptoquarks or composite quarks and leptons Models (with non trivial flavor structure) [S.Fajfer, J.Kamenik, I.Nisandzic, J.Zupan, 1206.1872]
Last Friday: two papers with more accurate theoretical predictions for \( \text{Br}(B \to D \tau \nu) \)

No helicity suppression both \( f_+ \) and \( f_0 \) are relevant

Becirevic\&Kosnik\&Tayduganov 1206.4977: estimate with minimal theory input (from the Lattice (quenched and unquenched) \( f_+/f_0 \) and \( f_+(q^2 > 8 \text{ GeV}^2) \)) \( R(D)=0.310(20) \)

FNAL/MILC 1206.4992: using \( f_0(q^2) \) from Lattice (unquenched FNAL/MILC 1202.6346) \( R(D)=0.316(14) \), and a different constraint on \( \tan \beta / m_{H^+} \) (see A.Kronfeld’s talk)
Looking for an explanation for the $B \to \tau \nu$ excess within the Standard Model

$$BR(B \to \tau \nu) = \frac{G_F^2 m_B m_{\tau}^2}{8\pi} \left(1 + \frac{m_{\tau}^2}{m_B^2}\right)^2 \frac{f_B^2 |V_{ub}|^2}{f_B^2} \tau_B$$

• $BR(B \to \tau \nu)_{\text{exp}}$ prefers a large value for $|V_{ub}|$ ($f_B$ well under control)
• But a shift in the central value of $|V_{ub}|$ would not solve the (2.4$\sigma$) $\beta$ tension
  the debate on $V_{ub}$ (exclusive vs inclusive determination) is not enough to explain all
B-physics hadronic parameters on the Lattice: fundamental ingredient in the UTA and more in general for Flavor Physics
B-physics on the lattice has the difficulty of large discretization effects of \( O(\alpha_s m_b) \) the physical b-quark mass (\( \approx 4 \) GeV) cannot be directly simulated on present \( (a^{-1} \leq 4 \) GeV) lattices.

Several approaches have been investigated and used so far, either with relativistic heavy quark or effective theory based.

**RELATIVISTIC QCD** with simulated quark masses in the charm region (and higher)+some suitable technique:

- **Step-scaling** [Tor Vergata], matching several lattice simulations at different volumes and up to physical b-quark mass (at small volume)
- **Ratio method** [ETMC], suitable ratios with exactly known static limit
- **HISQ** [HPQCD], leading discretization terms of \( O(\alpha_s a^2 m_h^2) \), \( O(a^4 m_h^4) \) + small taste changing

**EFFECTIVE THEORY BASED:**

- **HQET** [Alpha], static quark limit (expansion in \( \Lambda_{QCD}/m_h \))
- **NRQCD** [HPQCD], expansion in the velocity \( v \)
- **FermiLab** [FNAL/MILC], removing key discretization errors by tuning 3 parameters (from exp. input + pert. theory)
- **Non-perturbatively tuned relativistic heavy-quark action** [RBC/UKQCD], (NEW! 1206.2554, see C.Lehner’s talk), a variant of the FermiLab approach with fully non-pert. tuning of the 3 parameters from the clean \( B_s^{(*)} \) system (exp. values of \( m_{B_s} \) and \( m_{B_s^*} \) and continuum energy-momentum relation for \( B_s \))
Decay constants: $f_{Bs}$ and $f_{Bs}/f_B$

($f_{Bs}$ has a smooth chiral limit, in the ratio some uncertainties cancel)

UTA Lattice inputs are (conservative) simple averages of unquenched ($N_f=2$ and $2+1$) results:

$$f_{Bs} = 233(10) \text{ MeV}$$

$$f_{Bs}/f_B = 1.20(2)$$

The HPQCD11 result for $f_{Bs}$ is very accurate (2%), thanks to heavy HISQ quark [see H.Na’s talk]

New accurate analyses are in progress [see E.Neil’s(FNAL/MILC), O.Witzel’s(RBC/UKQCD) and A.Shindler’s(ETMC) talks and F.Bernardoni’s(Alpha) poster]
The pseudoscalar decay constant $f_{Bs}$ also enters the important rare decays:

$$B_s \rightarrow \mu^+ \mu^-$$

\[
Br(B_s \rightarrow l^+l^-) = \tau(B_s) \frac{G_F^2}{\pi} \left( \frac{\alpha}{4\pi\sin^2\theta_W} \right)^2 F_{Bs}^2 m_t^2 m_{B_s} \sqrt{1 - 4 \frac{m_t^2}{m_{B_s}^2} |V_{tb}^* V_{ts}|^2 Y(x_t)}
\]

- Highly sensitive to NP (loop FCNC: Z-penguin dominated)
- Theoretically clean (purely leptonic)

Intervals at 95% CL for $Br(B_s \rightarrow \mu^+ \mu^-)$

- D0 (PLB 693 2010 539)
- CDF (H. Miyake, La Thuile 2012)
- ATLAS (arXiv:1204.0735)
- CMS (arXiv:1203.3976)
- LHCb (arXiv:1203.4493)

SM From the UTA: $Br(B_s \rightarrow \mu^+ \mu^-) = (3.5 \pm 0.3) \cdot 10^{-9}$

- Experimentally the fragmentation fraction $f_s/f_d$ of $b \rightarrow B_s X$ is a fundamental ingredient
- Through factorization $f_s/f_d$ can be related to the ratio of semileptonic form factors for $B^0 \rightarrow D^+ \ell^- \nu$ and $B_s^0 \rightarrow D_s^+ \ell^- \nu$
- FNAL/MILC has computed it ($N_f=2+1$, two lattice spacings), finding: $f_s/f_d=0.28(4)$ [1202.6346] (see A.Kronfeld’s talk)
- in good agreement with LHCb 1111.2357 (0.27(2)) and PDG (0.29(2)) and 12% higher than a previous QCD sum rule estimate (P.Blasi et al.93)
**B-parameters: $B_{Bs}$ and $B_{Bs}/B_B$**

UTA Lattice inputs coincide with the $N_f=2+1$ HPQCD09 results [0902.1815]:

- $\hat{B}_{Bs} = 1.33(6)$
- $B_{Bs}/B_B = 1.05(7)$

Very recently FNAL/MILC12 has obtained a very well compatible result [1205.7013]:

- $B_{Bs}/B_B = 1.06(11)$ [combining $\xi$ and $f_{Bs}/f_B \rightarrow$ overestimated error]

New Lattice analyses are in progress:
- ETMC, with $N_f=2$, see N.Carrasco Vela's talk
- FNAL/MILC, see E.Freeland’s talk
  (direct computation of $B_{Bs}/B$ and first unquenched results for the B-parameters of the complete NP basis)
Theoretically clean Lattice calculations but only two modern results exist so far. Experimental cuts introduce some model dependence in treating long-distance contributions at threshold.

Combining it with Belle 2010 instead of Babar, $|V_{ub}|$ is found to be 15% higher.

$$|V_{ub}|_{\text{excl}} = (32.8 \pm 3.1) \cdot 10^{-4}$$

$$|V_{ub}|_{\text{incl}} = (44.1 \pm 2.8) \cdot 10^{-4}$$
Conservative combination for the UTA

- $|V_{ub}|_{input} = (38.2 \pm 5.6) \cdot 10^{-4}$
- The UTA output is close to the (lower) exclusive result: $|V_{ub}|_{UTA} = (36.2 \pm 1.4) \cdot 10^{-4}$
- Further Lattice calculations are looked forward and are in progress [see talks by T.Kawanai(RBC/UKQCD), F.Bernardoni(Alpha) and C.Bouchard(HPQCD] or under investigation [see Steven Gottlieb's talk (FNAL/MILC)]
**$V_{cb}$:**

**exclusive (Lattice form factors) vs inclusive (OPE based global fit)**

Theoretically clean Lattice calculations but only one *modern* result exists so far

Some model dependence affects the global fit

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- **Conservative combination for the UTA:** $|V_{cb}|_{\text{input}} = (41.0 \pm 1.0) \cdot 10^{-3}$
- The UTA output is close to the (higher) inclusive result: $|V_{cb}|_{\text{UTA}} = (42.3 \pm 0.9) \cdot 10^{-3}$
- Further Lattice calculations are looked forward and are in progress

[see C. De Tar's talk (FNAL/MILC)]
The UTA beyond the Standard Model

Update of UTfit 0909.5065

Model-independent UTA: bounds on deviations from the SM (+CKM)

• Parametrize generic NP in DF=2 processes
• Use all available experimental info
• Fit simultaneously the CKM and NP parameters

Results for the $B_s$ mixing amplitude:

$\Delta m_{q/K} = C_{B_q/K} \Delta m_{q/K}^{SM}$
$A_{CP}^{B_s \to J/\psi \phi} \sim \sin 2(-\beta_s + \phi_{B_s})$

=1 in SM
=0 in SM

Neutral mesons are not eigenstates of the Weak Interactions:

⇒ “particle-antiparticle oscillations”:

highly sensitive to NP
In 2009, CDF and D0 results for $\phi_{Bs}$

More than $2.5\sigma$ deviation from the SM!

Summer 2011: Bad news for NP in $B_s$!
New CDF data do not show any deviation
Still, the dimuon charge asymmetry (measured by D0) $a_{\mu\mu}$ points to a large value of $\phi_{Bs}$.

$$a_{3l}^b = \frac{\Gamma(B_d \rightarrow B_q \rightarrow \mu^+X) - \Gamma(B_d \rightarrow B_q \rightarrow \mu^-X)}{\Gamma(B_d \rightarrow B_q \rightarrow \mu^+X) + \Gamma(B_d \rightarrow B_q \rightarrow \mu^-X)}$$

$$A_{3l}^b = \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}} = a_{3l}^b$$

CDF: full data analysis (compatibility with the SM within less than 1 $\sigma$)

LHCb: finds compatibility as well

Further confirmations from experiments are looked forward!
Flavour Physics is highly sensitive to NP: Bounds on the NP scale $\Lambda$

\[ H_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i O_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{O}_i \]

The high scale coefficients $C_i(\Lambda)$ can be extracted from the data (switching on one operator per time).

Tree/strong inter. NP: $L \sim 1$
Perturbative NP: $L \sim \alpha_s^2, \alpha_W^2$

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**SM/MFV**

- e.g. for $K - \bar{K}$

**Beyond SM/MFV**

**MFV**

- $F_1 = F_{SM} \sim (V_{tq} V_{tb}^*)^2$
- $F_i \neq 1 = 0$

**next-to-MFV**

- $|F_i| \sim F_{SM}$
- arbitrary phases

**generic**

- $|F_i| \sim 1$
- arbitrary phases
Updated lower bound on the NP scale w.r.t. 0707.0636

From (the most constraining) K-K sector, with the unquenched Lattice results for the NP B-parameters, by P. Dimopoulos et al. [ETMC, with N_f=2, three lattice spacings] (forthcoming paper, see N. Carrasco Vela's talk)

\[ R_i = \frac{\langle \bar{K}^0 | O_i | K^0 \rangle}{\langle \bar{K}^0 | O_1 | K^0 \rangle}, \quad i = 2, \ldots, 5 \quad (\overline{\text{MS}} \text{ at } 2 \text{ GeV}) \]

**ETMC**

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<th>R_4</th>
<th>R_5</th>
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<tr>
<td>NEW</td>
<td>-14.7(06)</td>
<td>6.2(04)</td>
<td>25.7(11)</td>
<td>6.8(05)</td>
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**RBC/UKQCD**

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<tr>
<td>NEW (this week) results by RBC/UKQCD exist (N_f=2+1 domain-wall and ONE lattice spacing) 1206.5737 (see N. Garron's talk)</td>
<td>-16.1(17)</td>
<td>7.7(08)</td>
<td>28.0(29)</td>
<td>9.0(09)</td>
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Preliminary results have been also obtained by SWME (see H. J. Kim's talk and S. Sharpe's talk on staggered ChPT)
**D-\bar{D} mixing: B_D parameters**

- At variance with K and B systems, the first evidence for D-\bar{D} mixing is quite recent, 2007 (BaBar & Belle)

- It is sensitive to a different sector of New Physics (NP) with respect to K and B, being the charm an up-type quark

- D-\bar{D} mixing is affected by large long-distance effects (internal d and s quarks) which dominate over the short-distance contribution

- Only order of magnitude estimates exist for the long-distance contributions and are at the level of the experimental constraints, preventing from revealing and unambiguous sign of NP

- Still, barring accidental cancellations between SM and NP contributions, significant constraints can be put on the NP parameter space

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Update of the $D \bar{D}$ mixing analysis of M. Ciuchini et al. hep-ph/0703204

$$A = A_{SM} + A_{NP} e^{i\phi_{NP}}$$

With $A_{SM}$, due to large long-distance uncertainties, taken as flatly distributed in $[-0.01, 0.01]$ ps$^{-1}$

By using the experimental results

$$A_{NP} = (0.022 \pm 0.016)\%$$

$$\phi = (-0.035 \pm 0.19 \pm 0.09) \quad 0.216 \quad -0.280 \quad 0.128$$

http://www.utfit.org/UTfit/DDbarMixing

TABLE I. Experimental data used in the analysis of $D \bar{D}$ mixing, from ref. [38], $\alpha = (1 + |q/p|)^2/2$ and $\Delta A_{CP} = A_{CP}(\bar{D} \rightarrow K^{-} \pi^{+}) - A_{CP}(D \rightarrow K^{+} \pi^{-})$. Asymmetric errors have been symmetrized. We do not use measurements that do not fulfill the condition $\epsilon < 5\%$. $\Delta q$.
By using the Lattice results for the $B_D$-parameters strong constraints can be put on the parameter space of some NP models.

NEW Preliminary unquenched ($N_f=2$) results by ETMC

First accurate results:
unquenched, improved operators, non-perturbative renormalization, continuum limit, chiral extrapolation with $m_\pi \geq 260$ MeV

In the MSSM with a generic Flavour Structure

It is useful to work in the SuperCKM basis where gluino couplings are flavour diagonal and to expand (non-diagonal) sfermion mass matrices

Mass Insertion Approximation

$$M_{ij}^2 = \begin{pmatrix} m_{ij}^2 & (m_{ij}^2)_{LR} \\ (m_{ij}^2)_{LR}^\dagger & m_{ij}^2 \\ 
\end{pmatrix}$$

3x3 non-diagonal flavour matrices expanded in small off-diagonal entries:
e.g., $(\delta_{LL})_{ij} \equiv (m_{ij}^2)_{LL} / \tilde{m}^2$
Constraints on the $\delta s$ from $D-\bar{D}$ mixing

Mass insertions turn out to be more constrained than in hep-ph/0703204 by a factor $\approx 5$ due to the increased lattice accuracy.

Further Lattice results for the $B_D$-parameters are looked forward.
D_{(s)} leptonic decays: $f_D$ and $f_{Ds}$

\[
\Gamma(P \rightarrow \ell \nu) = \frac{G_F^2}{8\pi} f_P^2 \frac{m_\ell m_P}{m_\ell^2} \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 |V_{q_1q_2}|^2
\]

The past (2008) $f_{Ds}$ puzzle has been solved!

Tension between lattice determination and experimental measurement, mainly due to the 3 $\sigma$ deviation between:

- **HPQCD 2007** $f_{Ds} = 241 \pm 3$ MeV (by 2.3 $\sigma$)
- **PDG 2008** $f_{Ds} = 273 \pm 10$ MeV (by 1.5 $\sigma$)

Update of HPQCD 10 for $f_D$ with improved $r_1$

- $f_D = 208(3)$ MeV, see H. Na’s talk (HPQCD)

New preliminary $N_f=2+1+1$ result (HISQ on HISQ) by FNAL/MILC with accuracy similar to HPQCD

- $f_D = 210(5)$ MeV
- $f_{Ds} = 245(4)$ MeV

(see D. Toussaint’s talk)

+$E. Neil’s talk (FNAL/MILC)$
Other interesting $B$ and $D$ semileptonic form factors

$D \to K/\pi \ell \nu$: $V_{cs}$ and $V_{cd}$: at present the lattice uncertainty dominates (the most accurate unquenched result is by HPQCD11)
FNAL/MILC improved analysis is in progress [see J. Bailey’s poster]

First unquenched results for:

$B \to K^*\ell^+\ell^-$ (BaBar, Belle, CDF, LHCb) significant constraints on the Wilson coefficients $C_7, C_9, C_{10}$ of the NP effective Hamiltonian (C. Bobeth et al. 1006.5013, Hambrock & Hiller 1204.4444) [M. Wingate’s talk (Horgan & Liu & Meinel & Wingate on MILC confs.)]

$\Lambda_b \to \Lambda\ell^+\ell^-$ NP sensitive (baryonic analogue) first observation by CDF (1107.3753) [S. Meinel’s talk (Detmold & Lin & Meinel on RBC/UKQCD 2+1 flavor domain-wall ensembles)]

$B \to Kl^+l^-$, recently measured by BaBar (1204.3933)
complementary constraints to $B_s \to \mu^+\mu^-$ (Becirevic & Kosnik & Mescia & Schneider 1205.5811)
Lattice unquenched results for the three form factors $f_+, f_0$ and $f_T$ are looked forward [S. Gottlieb’s talk (FNAL/MILC)]
An eye to the SuperB Era

Present and next decades will see a great experimental activity, not only in the direct NP search at LHC, but also in the Flavor Sector.

In the quark sector
The SuperB and Belle II projects have been approved!
(Ireland and Japan)

- $e^+ - e^-$ collider with the appropriate energy to produce couples of B and anti-B mesons, in a clean environment (like BaBar and Belle, but with ~100 times higher luminosity)

- It aims at improving the accuracy of the B-factories by a factor 5-10

- It will test the CKM matrix at 1% level

- It will increase the sensitivity for several channels sensitive to NP by one order of magnitude (e.g. $B \rightarrow \tau V$, but also beyond B-physics: $\tau$ decays which violate lepton flavor, CP-violation in the D-sector,...)
Role of B-factories in constraining the UT

Before B-factories

After B-factories

After SuperB-factories?

The CKM matrix will be tested at 1% level
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>( \hat{B}_K )</td>
<td>0.86(15) [17%]</td>
<td>0.75(2) [3%]</td>
</tr>
<tr>
<td>( f_{Bs} )</td>
<td>238(31) MeV [13%]</td>
<td>233(10) MeV [4%]</td>
</tr>
<tr>
<td>( f_{Bs}/f_B )</td>
<td>1.24(7) [6%]</td>
<td>1.20(2) [1.5%]</td>
</tr>
<tr>
<td>( \hat{B}_{Bs} )</td>
<td>1.34(12) [9%]</td>
<td>1.33(6) [5%]</td>
</tr>
<tr>
<td>( B_{Bs}/B_B )</td>
<td>1.00(3) [3%] ( \text{(quenched, } \mu_i &gt; m_s/2, \ldots) )</td>
<td>1.05(7) [7%]</td>
</tr>
<tr>
<td>( F_{D^*}(1) )</td>
<td>0.91(3) [3%]</td>
<td>0.92(2) [2%]</td>
</tr>
<tr>
<td>( F_+ )</td>
<td>-- [20%]</td>
<td>-- [11%]</td>
</tr>
</tbody>
</table>

- The last 10 years teach us that Lattice QCD has made important progresses (higher computational power, better algorithms, quenched \( \rightarrow \) unquenched).
- More recently further improvements are being realized: simulations at the physical point, discretization effects well under control (in the light and heavy sectors), \( N_f=2+1+1 \), ...
Conclusion:
Flavor Lattice QCD is on the right way to the 1% accuracy target
backup
Buras&Guadagnoli (0805.3887)+Buras&Guadagnoli&Isidori (1002.3612): decrease of the SM prediction of $\epsilon_K$ by $\sim$6%  

More recently:  
Brod&Gorbahn (1007.0684, 1108.2036): NNLO QCD analysis of the charm-top and charm-charm contributions in box diagrams (further 2% suppression of $\epsilon_K$)  

Improved accuracy in $B_K$ from Lattice QCD, thanks to the continuum limit in unquenched studies (smaller though compatible values w.r.t $\sim$5 years ago)
UTfit Lattice input: $B_K = 0.750(20)$

- very well compatible with FLAG10:
  $B_K^{N_f=2+1} = 0.738(20)$ and $B_K^{N_f=2} = 0.729(30)$,

- a bit higher than FLAG10, to take into account 2011 results

[BMW 1106.3230, Laiho&VandeWater 1112.4861, RBC/UKQCD 1201.0706, SWME 1111.5698] (see Gilberto Colangelo's talk)

Simulation with pion masses down to the physical value (and more) thanks to the 2-step HEX smeared clover-improved Wilson action
Experimental Sensitivities for SuperB golden modes

For several golden modes the sensitivity will be improved from 2 to 10 times.

The theoretical predictions, for a significant comparison, should improve by 2–5 times.

<table>
<thead>
<tr>
<th>Observable/mode</th>
<th>Current now</th>
<th>LHCb (2017) 5 fb⁻¹</th>
<th>SuperB (2021) 75 ab⁻¹</th>
<th>Belle II (2021) 50 ab⁻¹</th>
<th>LHCb upgrade (10 years of running) 50 fb⁻¹</th>
<th>theory now</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau \to \mu \gamma \ (x10^{-9})$</td>
<td>&lt; 44</td>
<td>&lt; 2.4</td>
<td>&lt; 5.0</td>
<td>&lt; 2.3 – 8.2</td>
<td>&lt; 10</td>
<td>&lt; 24</td>
</tr>
<tr>
<td>$\tau \to e \gamma \ (x10^{-9})$</td>
<td>&lt; 33</td>
<td>&lt; 3.0</td>
<td>3.7 (est.)</td>
<td>2.3 – 8.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau \to \ell \ell \ (x10^{-10})$</td>
<td>&lt; 150 – 270</td>
<td>&lt; 244 $^{a}$</td>
<td>&lt; 10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B_{u,d}$ Decays</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BR}(B \to \pi \nu) \ (x10^{-4})$</td>
<td>1.64 ± 0.34</td>
<td>0.05</td>
<td>0.04</td>
<td>1.1 ± 0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(B \to \mu \mu) \ (x10^{-6})$</td>
<td>&lt; 1.0</td>
<td>0.02</td>
<td>0.03</td>
<td>0.47 ± 0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(B \to K^+\pi^-) \ (x10^{-6})$</td>
<td>&lt; 80</td>
<td>1.1</td>
<td>2.0</td>
<td>6.8 ± 1.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(B \to K^+\pi^-) \ (x10^{-6})$</td>
<td>&lt; 160</td>
<td>0.7</td>
<td>1.6</td>
<td>3.6 ± 0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(B \to X_{\gamma}\gamma) \ (x10^{-4})$</td>
<td>3.55 ± 0.26</td>
<td>0.11</td>
<td>0.13</td>
<td>0.23</td>
<td>3.15 ± 0.23</td>
<td></td>
</tr>
<tr>
<td>$A_{CP}(B \to X_{(\gamma\gamma)})$</td>
<td>0.060 ± 0.060</td>
<td>0.02</td>
<td>0.02</td>
<td>1.19 ± 0.39</td>
<td>10⁻⁶</td>
<td></td>
</tr>
<tr>
<td>$B \to K^*\mu^+\mu^−$ (events)</td>
<td>250 $^c$</td>
<td>8000</td>
<td>10-15k$^d$</td>
<td>7-10k</td>
<td>100,000</td>
<td>-</td>
</tr>
<tr>
<td>$\text{BR}(B \to K^*\mu^+\mu^-) \ (x10^{-6})$</td>
<td>1.15 ± 0.16</td>
<td>0.06</td>
<td>0.07</td>
<td>1.19 ± 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B \to K^*\ell^+\ell^- (events)$</td>
<td>165</td>
<td>400</td>
<td>10-15k$^d$</td>
<td>7-10k</td>
<td>5,000</td>
<td>-</td>
</tr>
<tr>
<td>$\text{BR}(B \to K^*\ell^+\ell^-) \ (x10^{-6})$</td>
<td>1.09 ± 0.17</td>
<td>0.05</td>
<td>0.07</td>
<td>1.19 ± 0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{FB}(B \to K^*\ell^+\ell^-)$</td>
<td>0.27 ± 0.14$^e$</td>
<td>0.040</td>
<td>0.03</td>
<td>-0.089 ± 0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B \to X_{\ell^+\ell^-} (events)$</td>
<td>280</td>
<td>8,600</td>
<td>7,000</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{BR}(B \to X_{\ell^+\ell^-}) \ (x10^{-6})$</td>
<td>3.66 ± 0.77$^b$</td>
<td>0.08</td>
<td>0.10</td>
<td>1.59 ± 0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ in $B \to K_s^0 \ell^+\ell^-$</td>
<td>−0.15 ± 0.20</td>
<td>0.03</td>
<td>0.03</td>
<td>-0.1 to 0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ in $B \to \eta' K^0$</td>
<td>0.59 ± 0.07</td>
<td>0.01</td>
<td>0.02</td>
<td>±0.015</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S$ in $B \to \phi K^0$</td>
<td>0.56 ± 0.17</td>
<td>0.15</td>
<td>0.02</td>
<td>0.03</td>
<td></td>
<td>±0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$B_s^0$ Decays</th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BR}(B_s^0 \to \gamma\gamma) \ (x10^{-6})$</td>
<td>&lt; 8.7</td>
<td>0.3</td>
<td>0.2 – 0.3</td>
<td>0.4 – 1.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{SL}^S (x10^{-3})$</td>
<td>−7.87 ± 1.96$^i$</td>
<td>4.</td>
<td>5. (est.)</td>
<td>0.02 ± 0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$D$ Decays</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>(0.63 ± 0.20)%</td>
<td>0.06%</td>
<td>0.02%</td>
<td>0.04%</td>
<td>0.02%</td>
<td>~ 10⁻²$^k$</td>
</tr>
<tr>
<td>$y$</td>
<td>(0.75 ± 0.12)%</td>
<td>0.03%</td>
<td>0.01%</td>
<td>0.03%</td>
<td>0.01%</td>
<td>~ 10⁻² (see above).</td>
</tr>
<tr>
<td>$y_{CP}$</td>
<td>(1.11 ± 0.22)%</td>
<td>0.02%</td>
<td>0.03%</td>
<td>0.05%</td>
<td>0.01%</td>
<td>~ 10⁻² (see above).</td>
</tr>
<tr>
<td>$</td>
<td>q/p</td>
<td>$, $\arg{q/p}$ (°)</td>
<td>(0.91 ± 0.17)%</td>
<td>8.5%</td>
<td>2.7%</td>
<td>3.0%</td>
</tr>
<tr>
<td>$</td>
<td>q/p</td>
<td>$</td>
<td>−10.2 ± 9.2</td>
<td>4.4</td>
<td>1.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>