Effect of the Schrödinger functional boundary conditions on the convergence of step scaling

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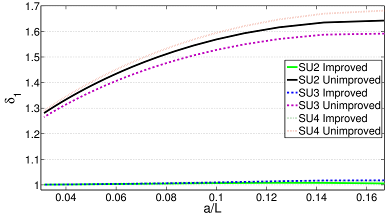
Outline

1. Motivation
2. Theory
3. Fundamental domain
4. Higher representations
5. Fundamental representation
6. Gauge part for $SU(2)$
7. Conclusion

Motivation



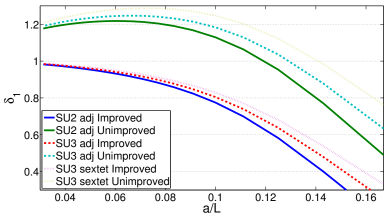


Figure: Fermionic parts of the Lattice step scaling function with fundamental (left) and higher representation (right) fermions.

- Step scaling converges extremely slowly with higher representation fermions even after improvement

$O(a)$ improved wilson action for $SU(N)$ in the Schrödinger functional scheme

$$S = S_G + \delta S_{G,b} + S_F + S_{gf} + S_{FP},$$

$$S_G = \frac{1}{g_0^2} \sum_p \mathrm{Tr}[1 - U(p)],$$

$$\delta S_{G,b} = \frac{1}{g_0^2}(c_t - 1) \sum_{p_t} \mathrm{Tr}[1 - U(p_t)],$$

$$S_F = a^4 \sum_x \left[\bar{\psi}(x) \left(D + m_0 + \frac{\mathrm{i} a c_{sw}}{4} \sigma_{\mu\nu} F_{\mu\nu}(x) \right) \psi(x) \right].$$

- The improvement coefficient c_t is set to its one loop perturbative value and $c_{sw} = 1$
- For the specific form of S_{gf} and S_{FP}, see[1]

[1] M. Lüscher, R. Narayanan, P. Weisz, U. Wolff, hep-lat/9207009v1

Theory: Schrödinger functional

Schrödinger functional boundary conditions

$$U_k(t=0,\vec{x}) = \exp[aC_k], \quad U_k(t=L,\vec{x}) = \exp[aC'_k]$$

$$C_k = \frac{\mathrm{i}}{L}\mathrm{diag}(\phi_1(\eta),\ldots,\phi_n(\eta)), \quad C'_k = \frac{\mathrm{i}}{L}\mathrm{diag}(\phi'_1(\eta),\ldots,\phi'_n(\eta))$$

- These boundary conditions induce a constant chromo-electric field

Effective action

$$\Gamma = -\ln\left\{\int D[\psi]D[\bar{\psi}]D[U]D[c]D[\bar{c}]e^{-S}\right\} = g_0^{-2}\Gamma_0 + \Gamma_1 + \mathcal{O}(g_0^2)$$

Theory: Step scaling

Running coupling

$$g^2 = \frac{\partial \Gamma_0}{\partial \eta} \Big/ \frac{\partial \Gamma}{\partial \eta} = g_0^2 - g_0^4 \frac{\partial \Gamma_1}{\partial \eta} \Big/ \frac{\partial \Gamma_0}{\partial \eta} + \mathcal{O}(g_0^6)$$

- The method used to calculate the running coupling perturbatively follows[2] and[3]

Definition of the Lattice step scaling function and its perturbative expansion to one-loop order

$$\begin{aligned}\Sigma(u, s, L/a) &= g^2(g_0, sL/a)\big|_{g^2(g_0, L/a)=u} \\ &= u + \left[\Sigma_{1,0}(s, L/a) + \Sigma_{1,1}(s, L/a) N_F\right] u^2,\end{aligned}$$

[2] S. Sint, R. Sommer, hep-lat/9508012

[3] M. Lüscher, R. Narayanan, P. Weisz, U. Wolff, hep-lat/9207009v1

Theory: Step scaling

Definition of δ_i

$$\delta_i = \frac{\Sigma_{1,i}(2, L/a)}{\sigma_{1,i}(2)} = \frac{\Sigma_{1,i}(2, L/a)}{2b_{0,i}\ln 2}, \qquad i = 0, 1.$$

$$b_{0,0} = 11N_c/(48\pi^2), \qquad b_{0,1} = N_f T_R/(12\pi^2).$$

Fundamental domain

Fundamental domain

$$\phi_1 < \phi_2 < \ldots < \phi_n, \quad |\phi_i - \phi_j| < 2\pi, \quad \sum_{i=1}^{N} \phi_i = 0.$$

- Boundary fields of this type lead to a unique (up to a gauge transformation) minimal action[4]

Boundary fields for SU(2)

$$\begin{aligned} \phi_1 &= -\eta & \phi'_1 &= \eta - \rho, \\ \phi_2 &= \eta, & \phi'_2 &= \rho - \eta. \end{aligned}$$

- Conventional choice $\eta = \pi/4$, $\rho = \pi$.

[4] M. Lüscher, R. Narayanan, P. Weisz, U. Wolff, hep-lat/9207009v1

Fundamental domain

Boundary fields for SU(3)

$$\begin{aligned} \phi_1 &= \eta - \rho & \phi_1' &= -\phi_1 - 4\rho, \\ \phi_2 &= \eta(\nu - 1/2), & \phi_2' &= -\phi_3 + 2\rho, \\ \phi_3 &= -\eta(\nu + 1/2) + \rho, & \phi_3' &= -\phi_2 + 2\rho. \end{aligned}$$

- Conventional choice $\eta = 0$, $\rho = \pi/3$, $\nu = 0$.[5]

[5] M. Lüscher, R. Sommer, P. Weisz, U. Wolff, hep-lat/9309005

Higher representations

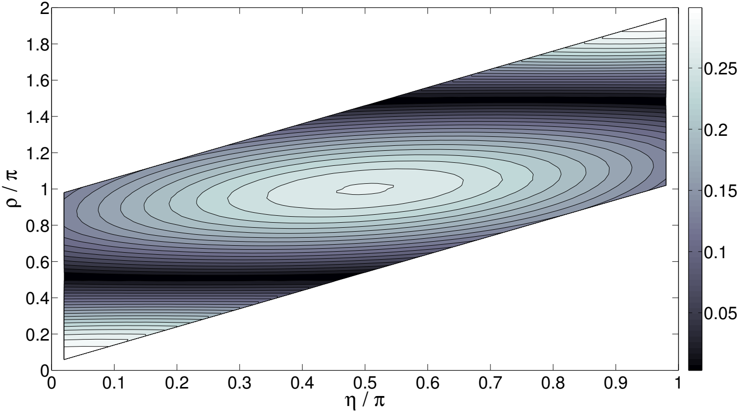


Figure: Fermionic part of the Lattice step scaling function for $SU(2)$ with adjoint fermions at $L = 10$. $|\delta_1 - 1|$ ploted as a function of η/π and ρ/π. Optimal choice $\rho = \frac{\pi}{2}$ and $\eta = \frac{\pi}{8}$.

Higher representations

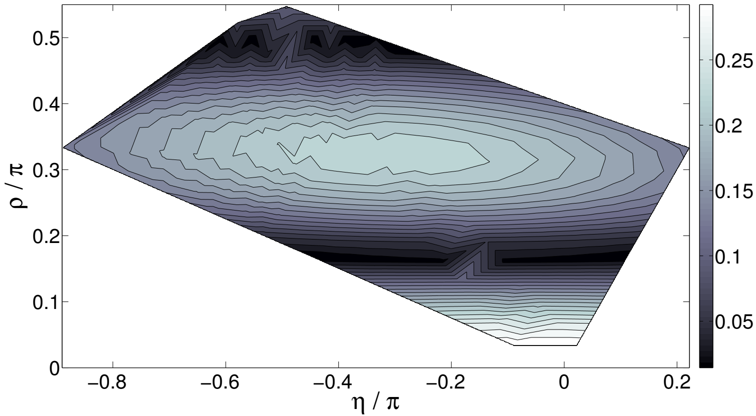


Figure: Fermionic part of the Lattice step scaling function for $SU(3)$ with adjoint fermions at $L = 10$. $|\delta_1 - 1|$ ploted as a function of η/π and ρ/π. Optimal choice $\rho = \frac{\pi}{6}$ and $\eta = -\frac{\pi}{9}$.

Higher representations

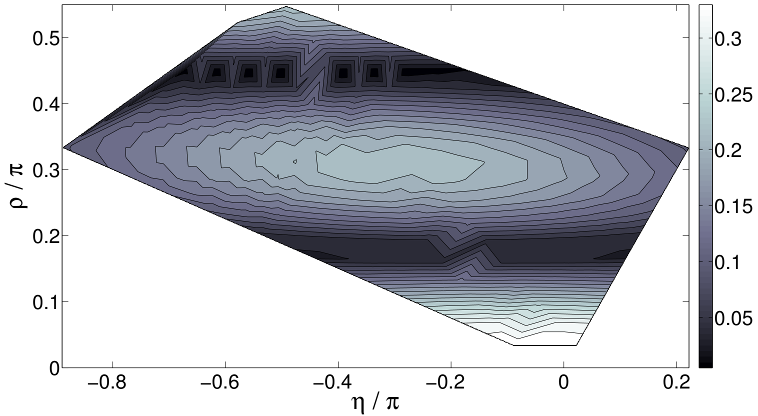


Figure: Fermionic part of the Lattice step scaling function for $SU(3)$ with sextet fermions at $L = 10$. $|\delta_1 - 1|$ ploted as a function of η/π and ρ/π. Optimal choice $\rho = \frac{67\pi}{150}$ and $\eta = -\frac{\pi}{3}$.

Higher representations

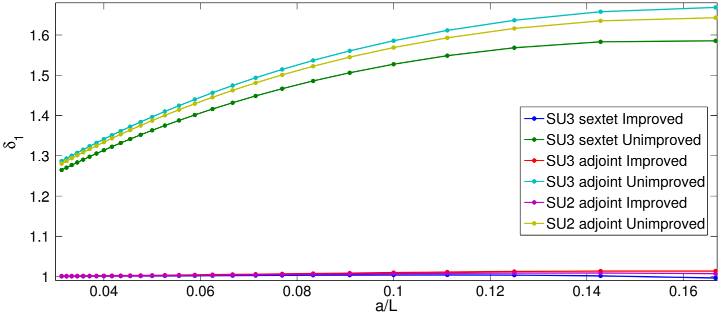


Figure: Fermionic part of the Lattice step scaling function for higher representation fermions with new boundary conditions.

Fundamental representation

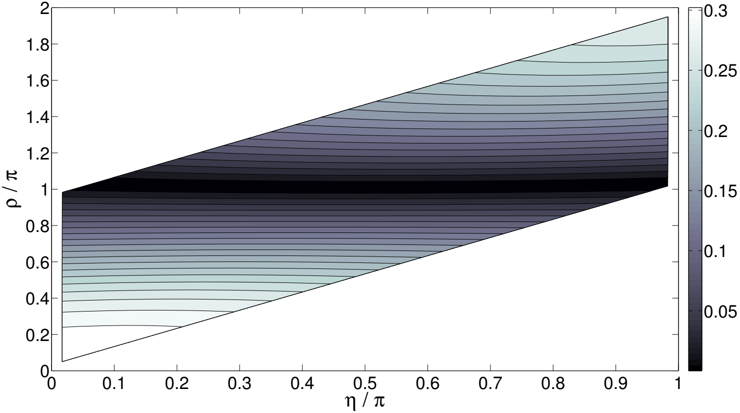


Figure: Fermionic part of the Lattice step scaling function for $SU(2)$ with fundamental fermions at $L = 10$. $|\delta_1 - 1|$ ploted as a function of η/π and ρ/π. Conventional choice $\rho = \pi$ and $\eta = \frac{\pi}{4}$.

Gauge part for $SU(2)$

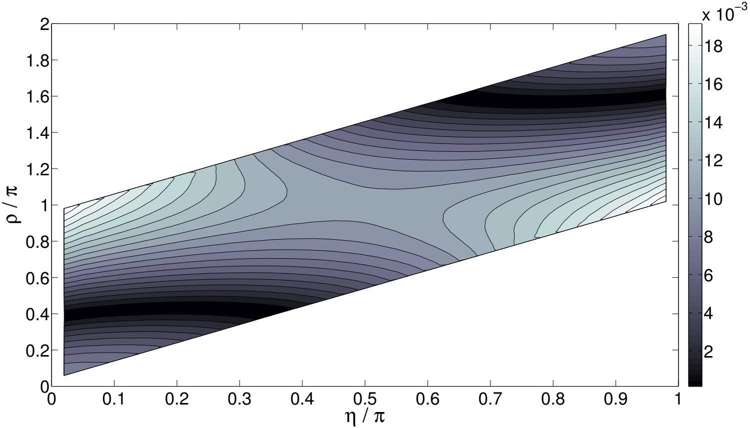


Figure: Gauge part of the Lattice step scaling function for $SU(2)$ at $L = 10$. $|\delta_0 - 1|$ ploted as a function of η/π and ρ/π.

Gauge part for $SU(2)$

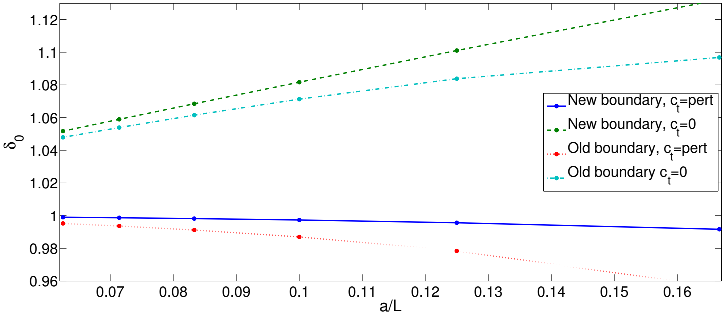


Figure: Gauge part of the Lattice step scaling function for $SU(2)$ with old and new boundary conditions for adjoint fermions.

Conclusions

- Improvement needed to remove $\mathcal{O}(a)$ terms from perturbative step scaling
- Careful choice of the boundary fields as important to minimize the higher order effects

Thank you!