Lattice Artifacts in Strongly interacting theories

Pol Vilaseca Mainar
Stefan Sint
Trinity College Dublin

SEVENTH FRAMEWORK PROGRAMME
ITN STRONGnet
MARIE CURIE ACTIONS

Strongly interacting theories

The coupling constant **g** is big, perturbation theory is not valid.

Easily constructed with **Yang-Mills** fields coupled to **fermion fields**.

$$\mathcal{L} = \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \sum_{j=1}^{N_f} \left(i\overline{\psi}_j \gamma_\mu D^\mu \psi_j \right)$$

Starting with such Lagrangian, we specify a particular theory by fixing

The gauge group **SU(N)**
The number of flavours **Nf**.
The representation **R**.

Some examples

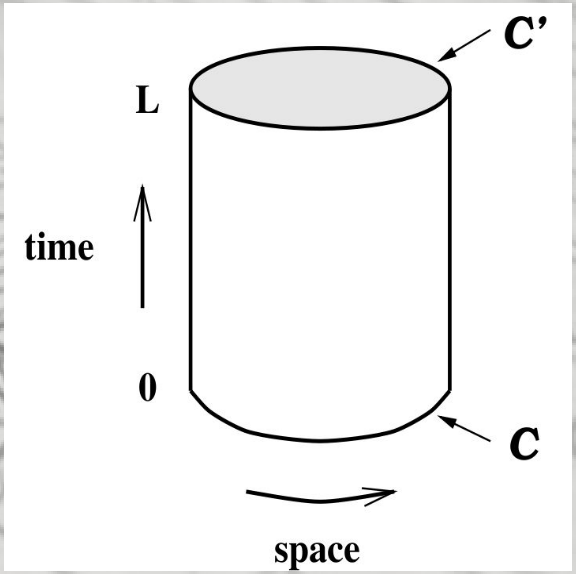
Quantum Chromodynamics
Dynamical EW Symmetry breaking (**Technicolor**).
Unparticle Physics
...

The Schrodinger Functional

Euclidean propagation amplitude from a field configuration in one boundary C to another field configuration in the oposite boundary C'.

[Luescher et al. '92]

$$\mathcal{Z}[C,C'] = \int \mathcal{D}[A,\psi,\bar{\psi}]e^{-S[A,\psi,\bar{\psi}]}$$

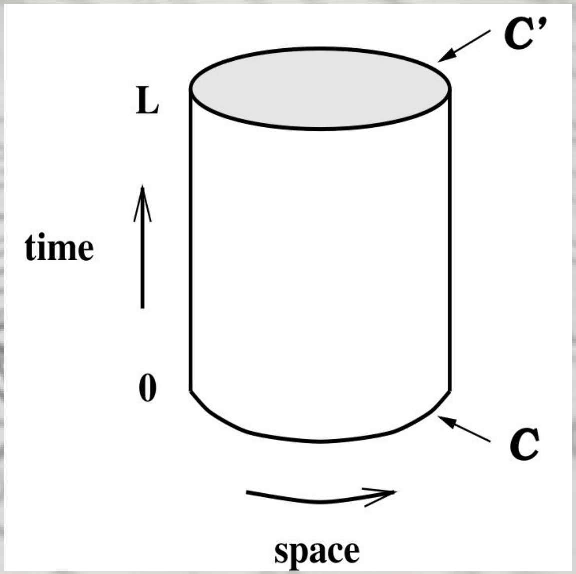


The Schrodinger Functional

Euclidean propagation amplitude from a field configuration in one boundary C to another field configuration in the oposite boundary C'.

[Luescher et al. '92]

$$\mathcal{Z}[C,C'] = \int \mathcal{D}[A,\psi,\bar{\psi}]e^{-S[A,\psi,\bar{\psi}]}$$



Boundary Conditions:

-Boundary gauge fields C and C' taken to be **abelian** and **spatialy constant**.

$$C_k = \frac{i}{L}\mathrm{diag}(\phi_{1k}, ..., \phi_{Nk})$$

- 2 Fermion regularizations:

SF [Sint '94]

$$\begin{aligned} P_+\psi|_{x_0=0} &= P_-\psi|_{x_T=0} = 0 \\ \overline{\psi}P_-|_{x_0=0} &= \overline{\psi}P_+|_{x_T=0} = 0 \end{aligned}$$

χ SF [Sint '05-'10]

$$\begin{aligned} \widetilde{Q}_+\psi|_{x_0=0} &= \widetilde{Q}_-\psi|_{x_T=0} = 0 \\ \overline{\psi}\widetilde{Q}_+|_{x_0=0} &= \overline{\psi}\widetilde{Q}_-|_{x_T=0} = 0 \end{aligned}$$

The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

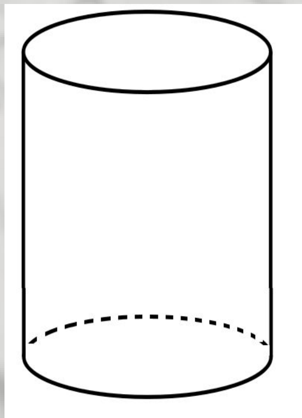
$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

$$\Gamma[B] \stackrel{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$

If **mq=0** and **T=L**, then **L** is the only scale.
B depends on a parameter **B(η)**.

Define a **renormalized coupling g(L)**

$$\overline{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$



The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

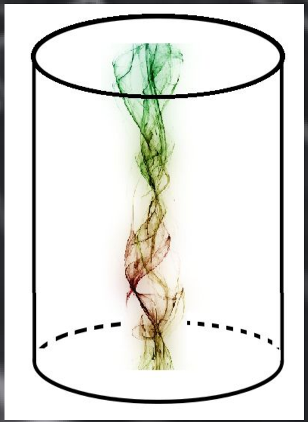
$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

$$\Gamma[B] \stackrel{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$

If mq=0 and T=L, then L is the only scale.
B depends on a parameter B(η).

Define a renormalized coupling g(L)

$$\overline{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$



The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

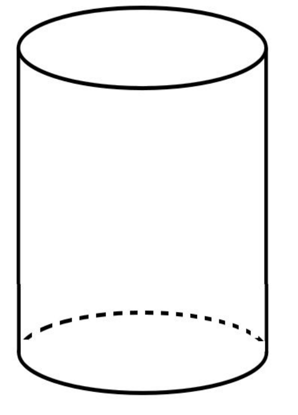
$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

$$\Gamma[B] \overset{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$

If **mq=0** and **T=L**, then **L** is the only scale.
B depends on a parameter **B(η)**.

Define a **renormalized coupling g(L)**

$$\overline{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$

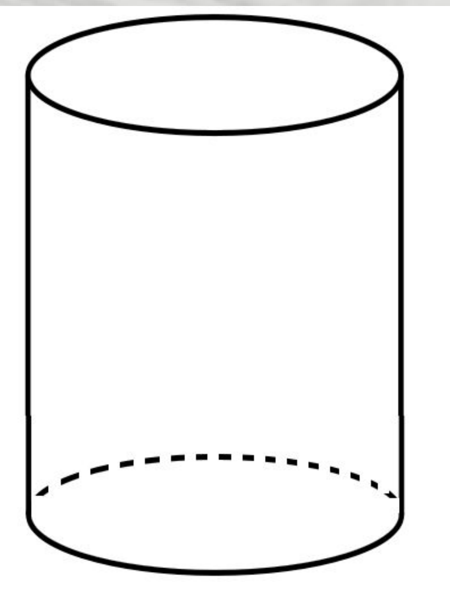


The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

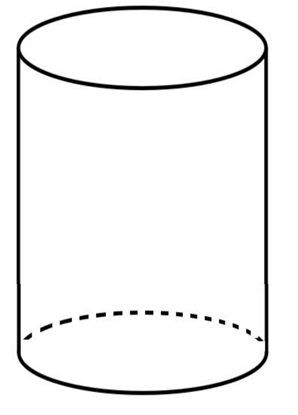
$$\Gamma[B] \xrightarrow{g_0 \to 0} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$



If **mq=0** and **T=L**, then **L** is the only scale.
B depends on a parameter **B(η)**.

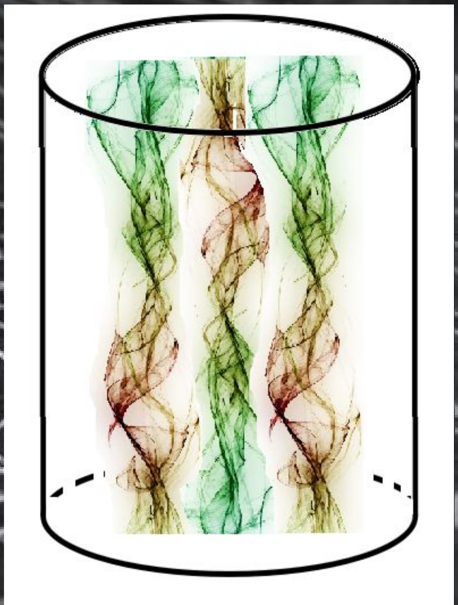
Define a **renormalized coupling g(L)**

$$\overline{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$



The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

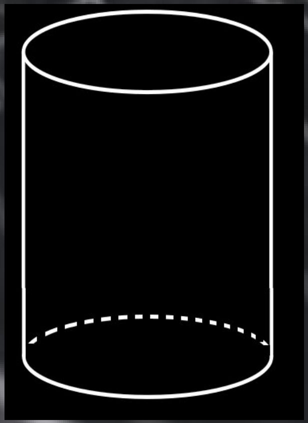


$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

$$\Gamma[B] \overset{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$

If mq=0 and T=L, then L is the only scale.
B depends on a parameter B(η).

Define a renormalized coupling g(L)



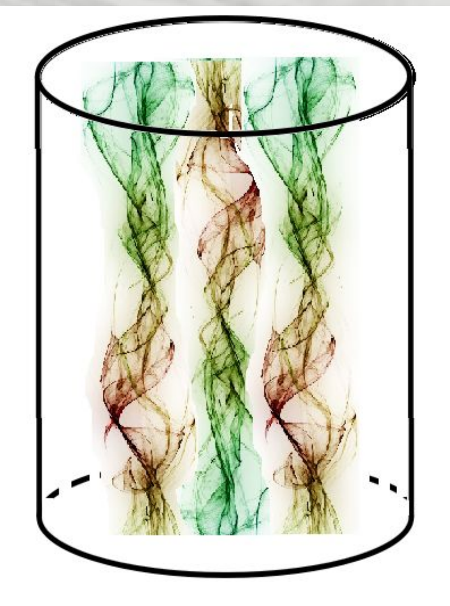
$$\overline{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$

The Schrodinger Functional

C and C', induce a background abelian chromoelectric field B.
The effective action of B

$$\Gamma[B] = -\ln \mathcal{Z}[C, C']$$

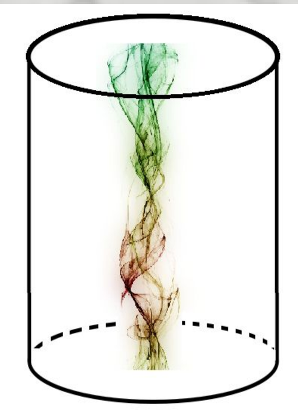
$$\Gamma[B] \stackrel{g_0 \to 0}{\longrightarrow} \frac{1}{g_0^2}\Gamma_0[B] + \Gamma_1[B] + g_0^2\Gamma_2[B] + \ldots$$



If **mq=0** and **T=L**, then **L** is the only scale.
B depends on a parameter **B(η)**.

Define a **renormalized coupling g(L)**

$$\bar{g}^2(L) = \frac{\partial\Gamma_0/\partial\eta|_{\eta=0}}{\partial\Gamma/\partial\eta|_{\eta=0}}$$



Monitoring cutoff effects

The **Step Scaling Function**, an integrated version of the beta function.

With a lattice counterpart:

$$\sigma(s,u) \equiv \bar{g}^2(sL)\Big|_{u=\bar{g}^2(L)}$$

$$\Sigma(s=2,u,L/a) = \bar{g}^2(2L)|_{u=\bar{g}^2(L)}$$

In perturbation theory:

$$\sigma(u) \xrightarrow{g_0\to 0} u + \sigma_1 u^2 + O(u^3)$$

$$\Sigma(u,L/a) \xrightarrow{g_0\to 0} u + \Sigma_1(L/a)u^2 + O(u^3)$$

$$\sigma_1 = 2b_0\ln(2)$$

Monitoring cutoff effects

The **Step Scaling Function**, an integrated version of the beta function. With a lattice counterpart:

$$\sigma(s,u) \equiv \bar{g}^2(sL)\Big|_{u=\bar{g}^2(L)}$$

$$\Sigma(s=2,u,L/a) = \bar{g}^2(2L)|_{u=\bar{g}^2(L)}$$

In perturbation theory:

$$\sigma(u) \xrightarrow{g_0\to 0} u + \sigma_1 u^2 + O(u^3)$$

$$\Sigma(u,L/a) \xrightarrow{g_0\to 0} u + \Sigma_1(L/a)u^2 + O(u^3)$$

Relative cuttoff effects:

$$\delta_1(a/L) = \frac{\Sigma_1(a/L) - \sigma_1}{\sigma_1}$$

$$\sigma_1 = 2b_0 \ln(2)$$

Monitoring cutoff effects

The **Step Scaling Function**, an integrated version of the beta function.

$$\sigma(s,u) \equiv \overline{g}^2(sL)\Big|_{u=\overline{g}^2(L)}$$

With a lattice counterpart:

$$\Sigma(s=2,u,L/a) = \overline{g}^2(2L)|_{u=\overline{g}^2(L)}$$

In perturbation theory:

$$\sigma(u) \xrightarrow{g_0 \to 0} u + \sigma_1 u^2 + O(u^3)$$

$$\Sigma(u,L/a) \xrightarrow{g_0 \to 0} u + \Sigma_1(L/a) u^2 + O(u^3)$$

Relative cuttoff effects:

$$\delta_1(a/L) = \frac{\Sigma_1(a/L) - \sigma_1}{\sigma_1}$$

$$\sigma_1 = 2b_0 \ln(2)$$

The coupling:

$$\overline{g}^2 \xrightarrow{g_0 \to 0} g_0^2 + p_1 g_0^4 + O\left(g_0^6\right)$$

$$p_1(L/a) = p_{1,0}(L/a) + n_f p_{1,1}(L/a)$$

Towards an optimal regularization.

Spatial bc for fermion fields are periodic up to a phase **θ** .

$$\psi(x + L\hat{k}) = e^{i\theta/L}\psi(x),$$
$$\overline{\psi}(x + L\hat{k}) = e^{-i\theta/L}\overline{\psi}(x),$$

Condition number:

$$\kappa(\Delta_2) = \left(\frac{\lambda_{max}}{\lambda_0}\right)^{-1/2}$$

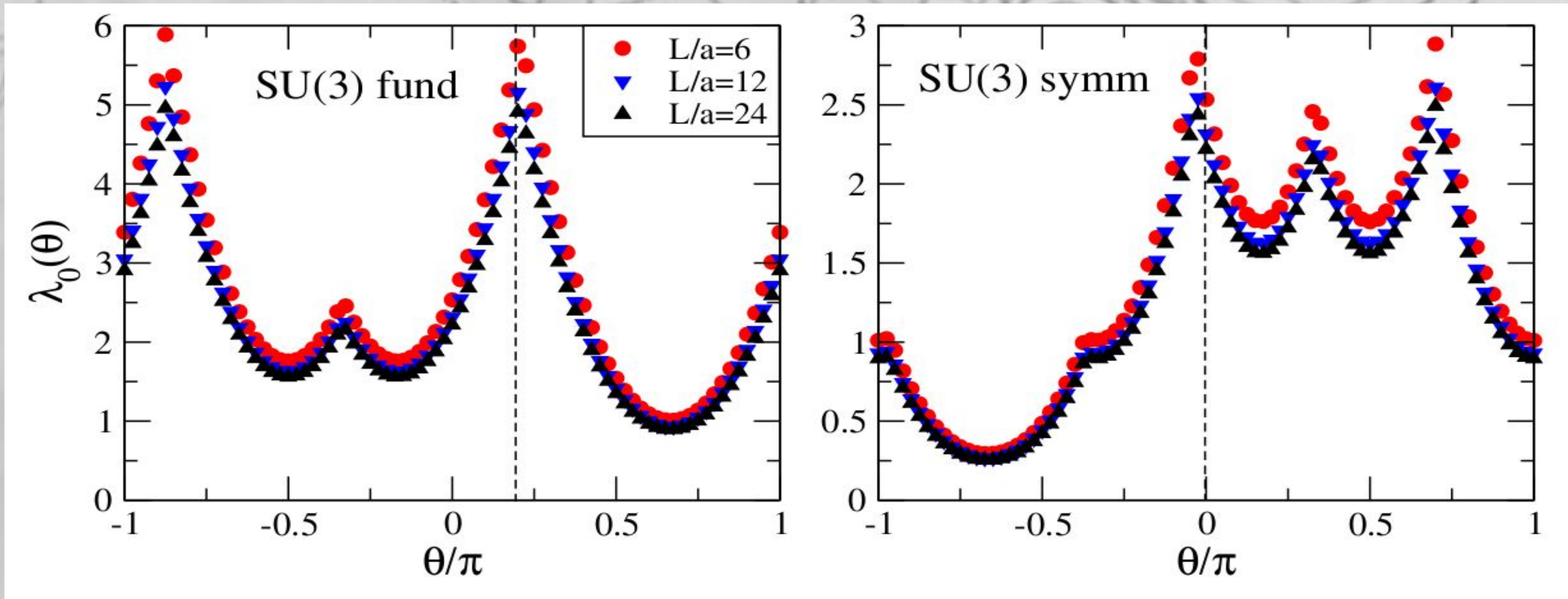
Towards an optimal regularization.

Spatial bc for fermion fields are periodic up to a phase **θ** .

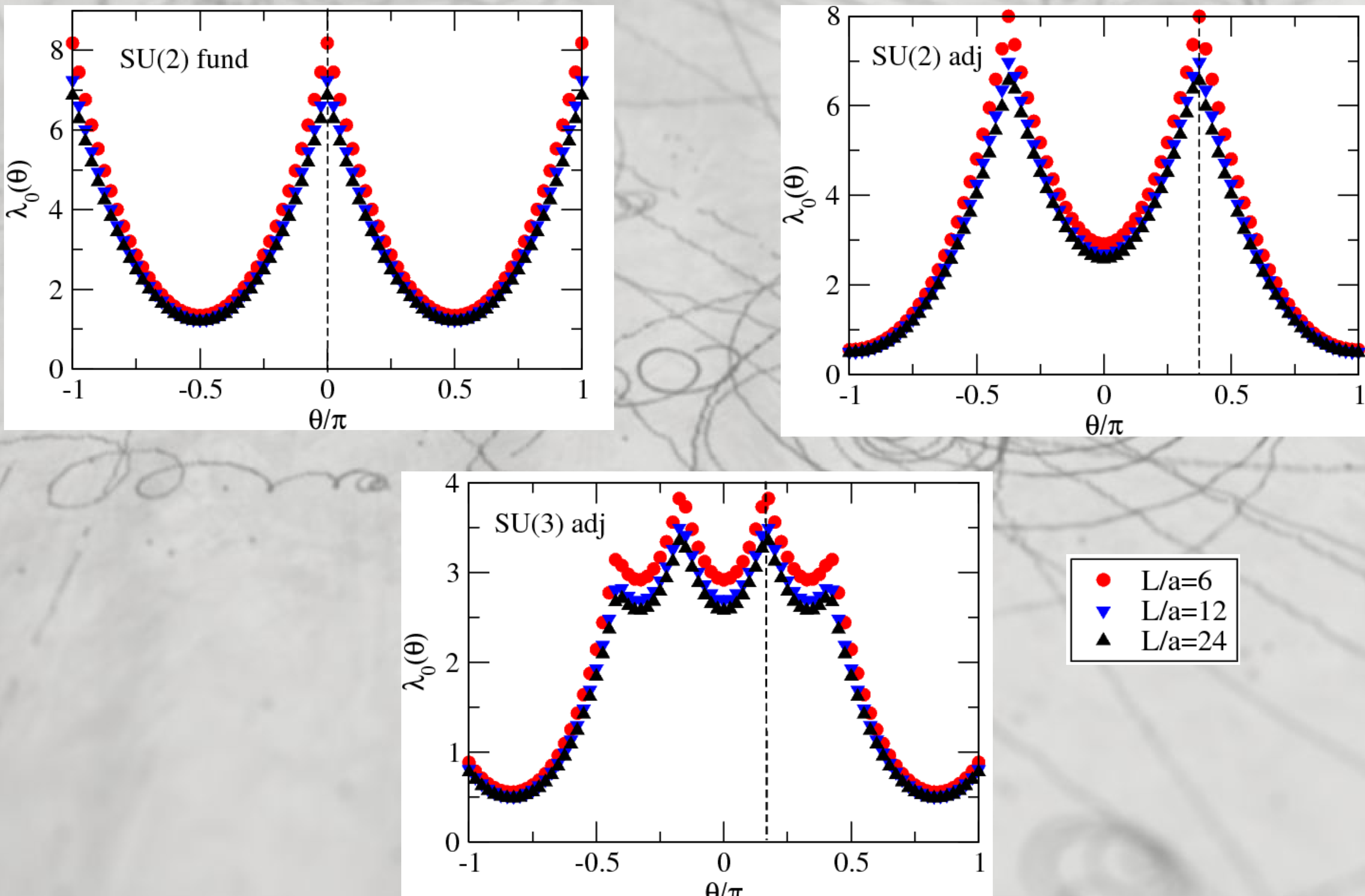
$$\psi(x + L\hat{k}) = e^{i\theta/L}\psi(x),$$
$$\overline{\psi}(x + L\hat{k}) = e^{-i\theta/L}\overline{\psi}(x),$$

Condition number:

$$\kappa(\Delta_2) = \left(\frac{\lambda_{max}}{\lambda_0}\right)^{-1/2}$$



Towards an optimal regularization.



Towards an optimal regularization.

SF: O(a) effects coming from bulk and boundaries.

- Boundary counterterms: c_t, $\tilde{c}_t$
- Bulk counterterm, clover term: C_{sw}

χ **SF**: Only boundary counterterms : c_t, d_s, z_f (dim(z_f)= 3)

SF

$$
\begin{aligned}
c_{SW} &= c_{SW}^{(0)} + c_{SW}^{(1)} g_0^2 + O(g_0^4) \\
c_t &= c_t^{(0)} + c_t^{(1)} g_0^2 + O(g_0^4) \\
\widetilde{c}_t &= \widetilde{c}_t^{(0)} + \widetilde{c}_t^{(1)} g_0^2 + O(g_0^4)
\end{aligned}
$$

χ **SF**

$$
\begin{aligned}
c_t &= c_t^{(0)} + c_t^{(1)} g_0^2 + O(g_0^4) \\
z_f &= z_f^{(0)} + z_f^{(1)} g_0^2 + O(g_0^4) \\
d_s &= d_s^{(0)} + d_s^{(1)} g_0^2 + O(g_0^4)
\end{aligned}
$$

Towards an optimal regularization.

SF: O(a) effects coming from bulk and boundaries.

- Boundary counterterms: c_t, $\tilde{c}_t$
- Bulk counterterm, clover term: C_{SW}

χ SF: Only boundary counterterms : c_t, d_s, z_f (dim(z_f)= 3)

SF

$$
\begin{aligned}
c_{SW} &= c_{SW}^{(0)} + c_{SW}^{(1)} g_0^2 + O(g_0^4) \\
c_t &= c_t^{(0)} + c_t^{(1)} g_0^2 + O(g_0^4) \\
\tilde{c}_t &= \tilde{c}_t^{(0)} + \tilde{c}_t^{(1)} g_0^2 + O(g_0^4)
\end{aligned}
$$

χ SF

$$
\begin{aligned}
c_t &= c_t^{(0)} + c_t^{(1)} g_0^2 + O(g_0^4) \\
z_f &= z_f^{(0)} + z_f^{(1)} g_0^2 + O(g_0^4) \\
d_s &= d_s^{(0)} + d_s^{(1)} g_0^2 + O(g_0^4)
\end{aligned}
$$

Tree level values

$$c_t^{(0)} = 1, \tilde{c}_t^{(0)} = 1, C_{SW}^{(0)} = 1$$

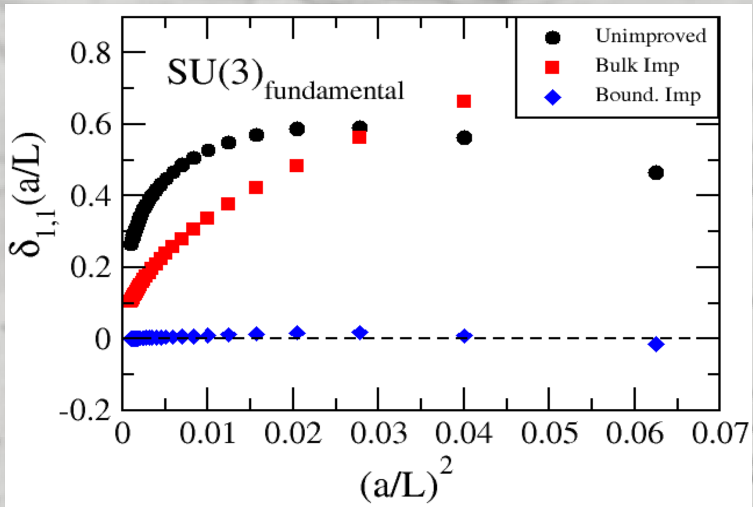
$$z_f^{(0)} = 1, d_s^{(0)} = 1/2$$

$$p_1(L/a)$$

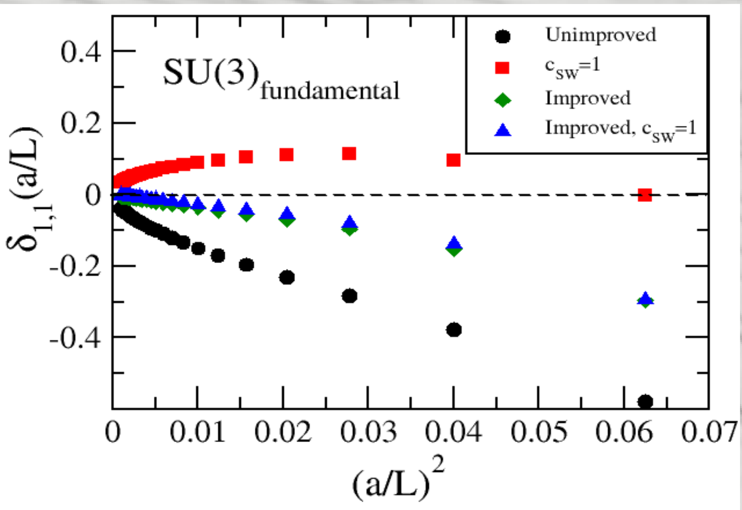
Cuttoff effects:

Fundamental

SF



χ SF

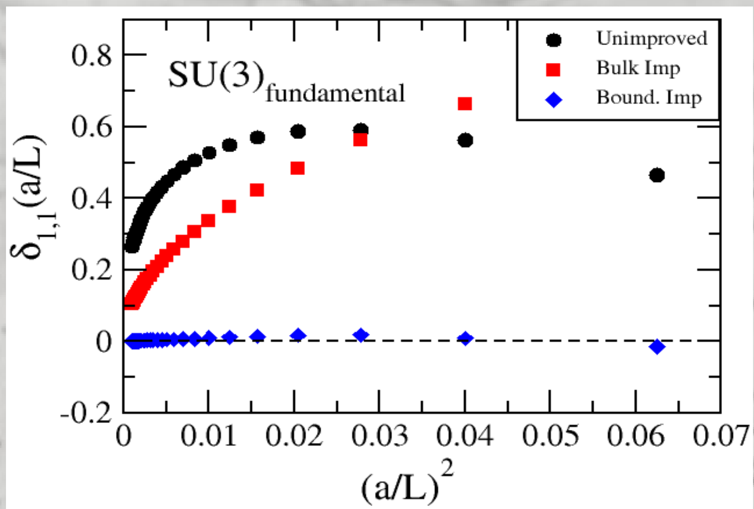


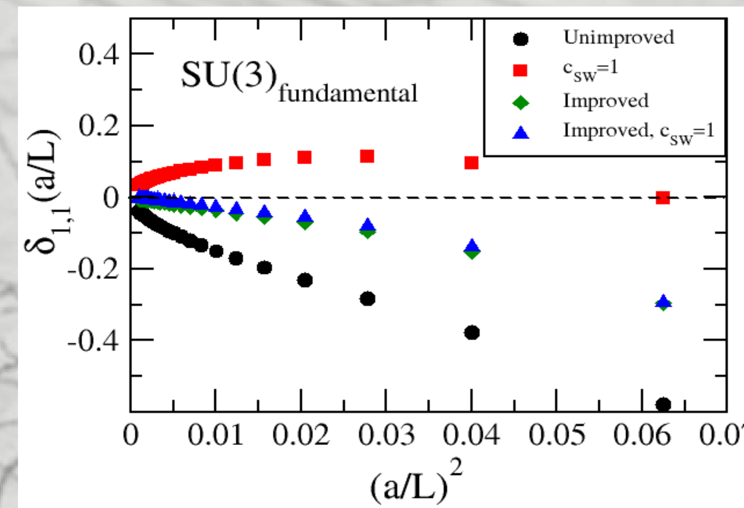
Cuttoff effects:

Fundamental

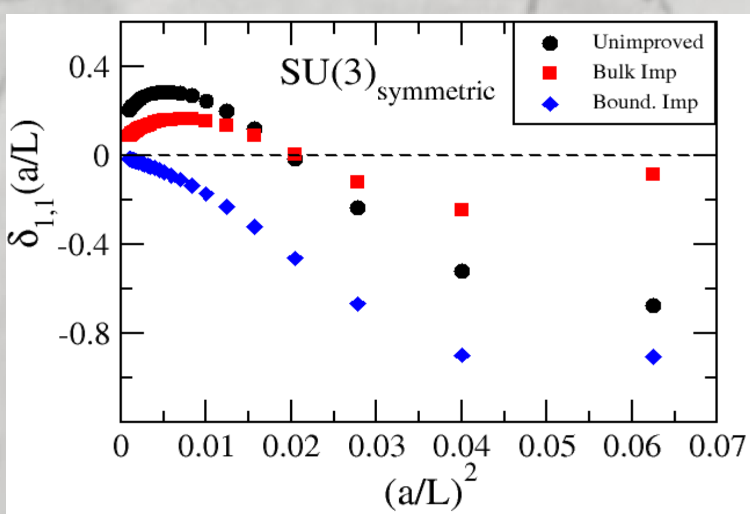
SF

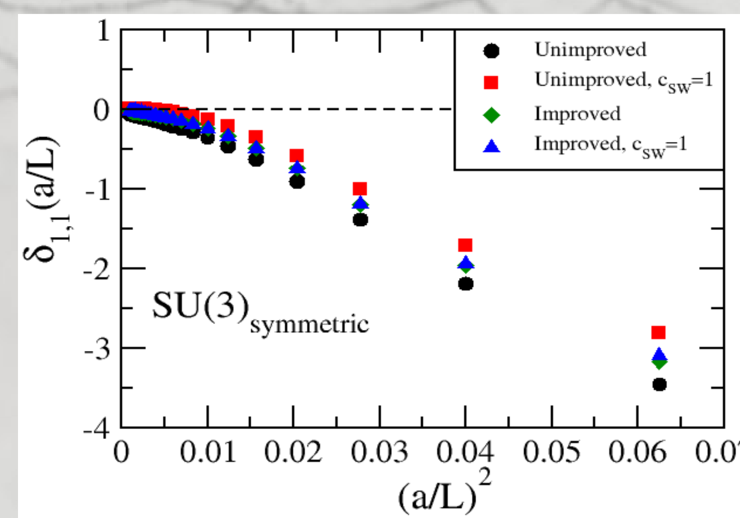
χ SF





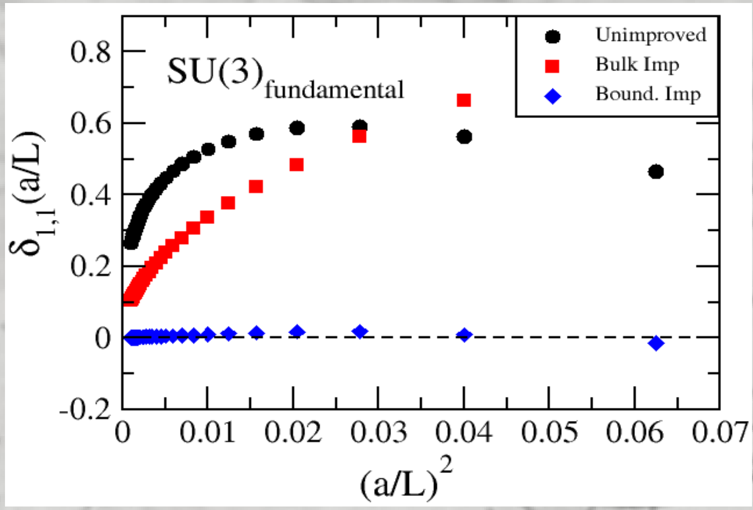
Symmetric

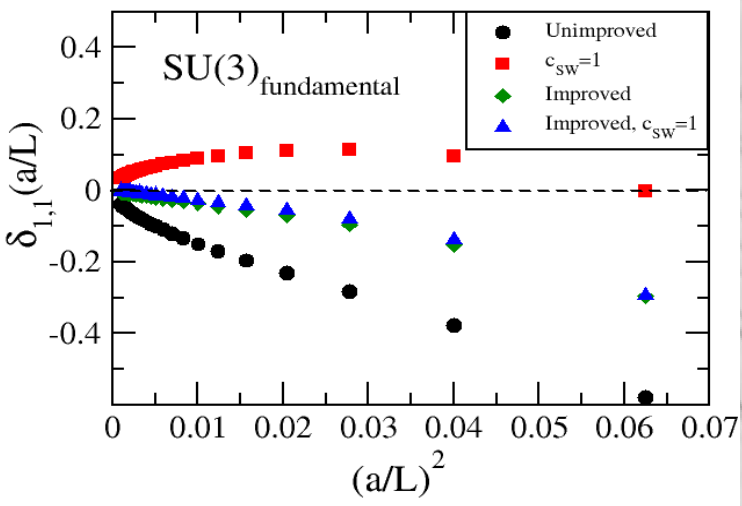




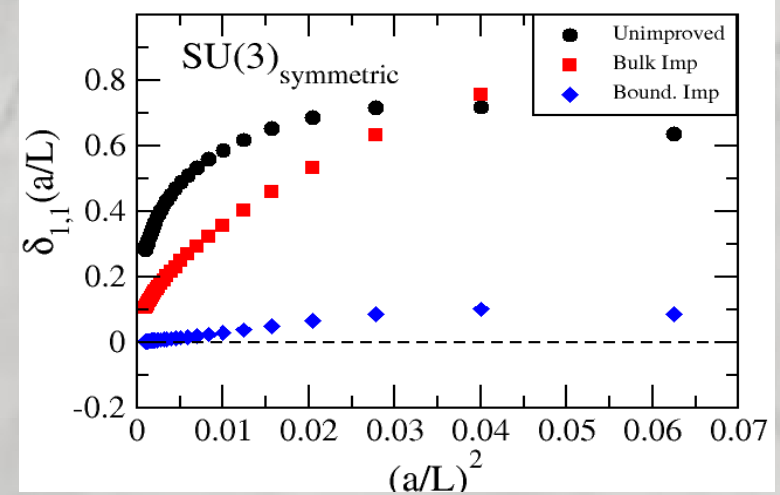
Cuttoff effects:

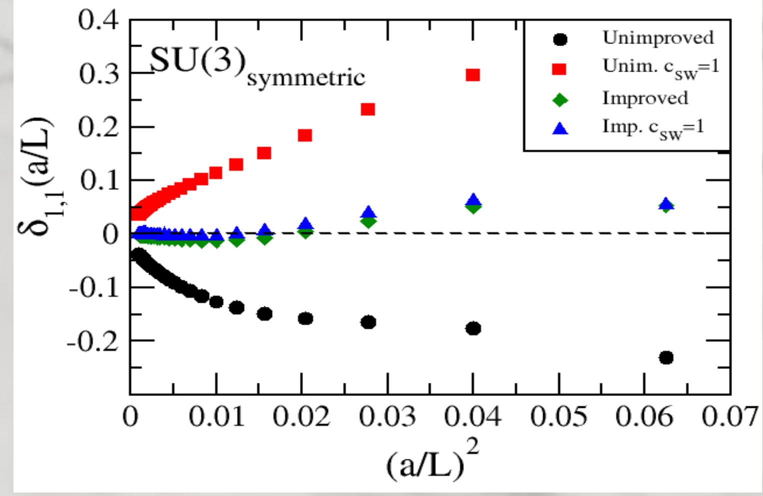
Fundamental **SF** **χ SF**





Symmetric BF → BF/2 [Sint, V., Lattice 2011]





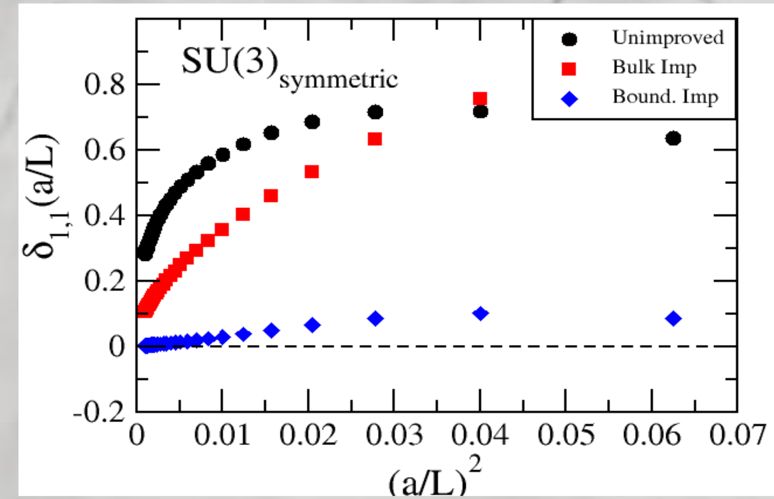
Cutoff effects:

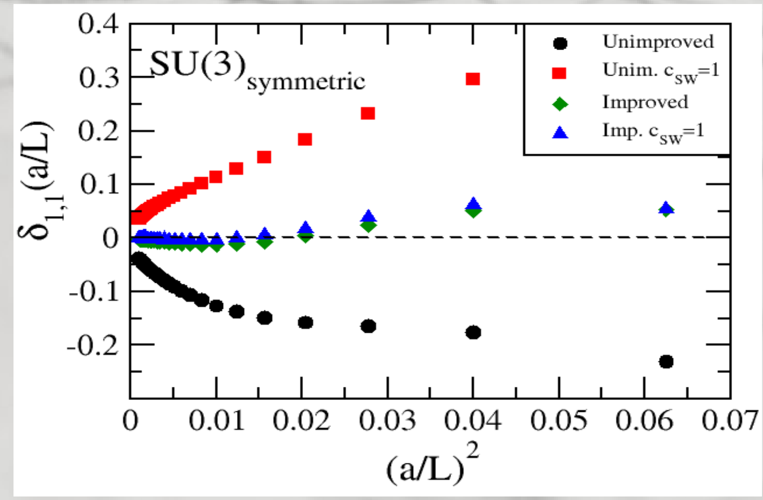
Problems alleviated, but...

Modified BF ⟶ pure gauge part must be recomputed.

Non-symmetric fields induce big statistical fluctuations.

Symmetric BF ⟶ BF/2 [Sint, V., Lattice 2011]





Removing higher order effects:

The parameter η is added to define the coupling

$$C_k = \frac{i}{L}\mathrm{diag}(\phi_{1k}, ..., \phi_{Nk}) + \frac{i}{L}\eta\lambda_8|_{\eta=0}$$

The BF can depend on an extra parameter ν .

$$\eta\lambda_8 \longrightarrow \eta\left(\lambda_8 + \nu\lambda_3\right)$$

A whole familly of renormalized couplings can be defined

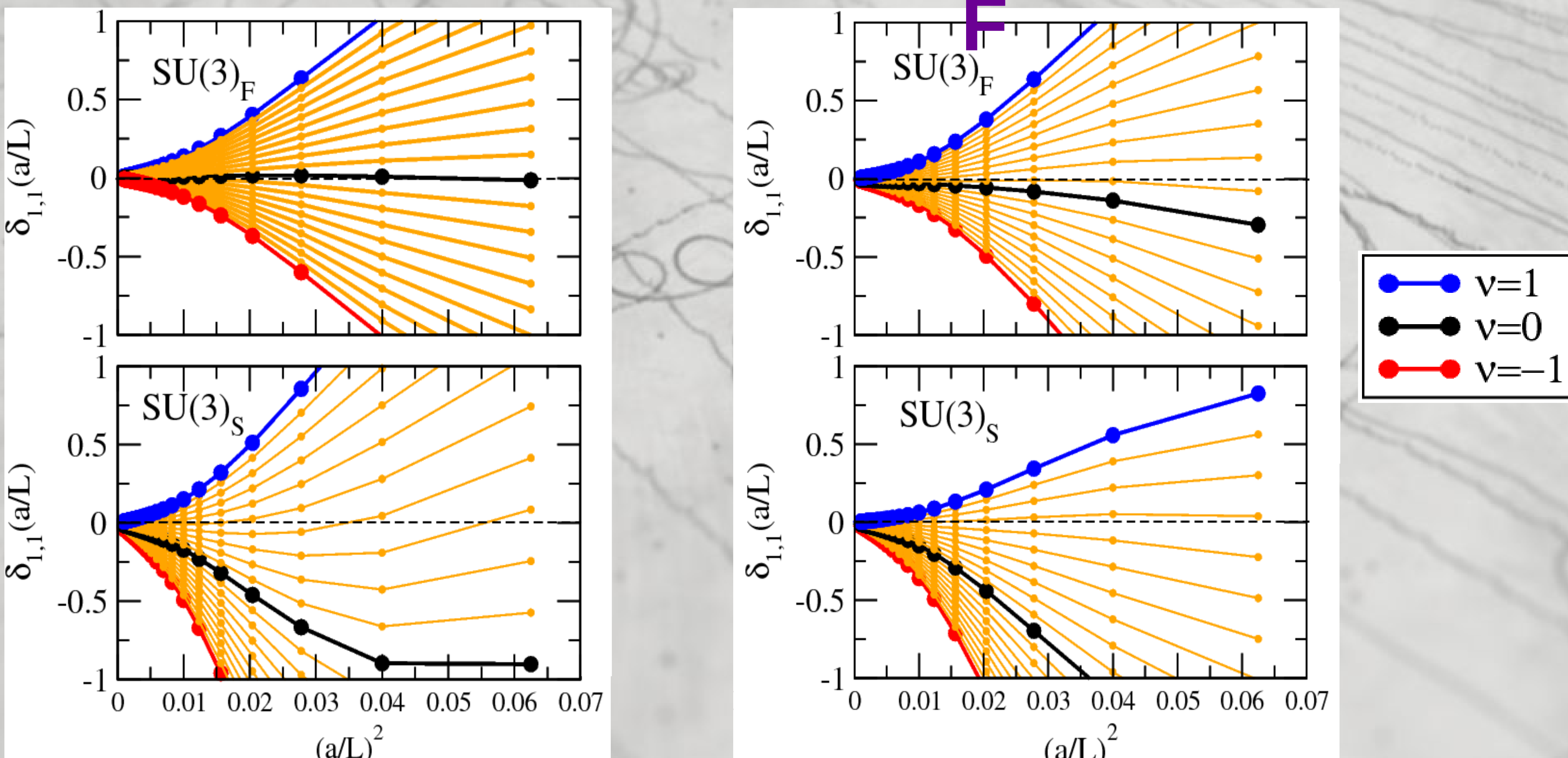
$$\frac{1}{\overline{g}_\nu^2(L)} = \frac{1}{\overline{g}^2(L)} - \nu\overline{v}(L)$$

$$\overline{v}(L) = \frac{1}{\kappa}\frac{\partial}{\partial\nu}\left\{\frac{\partial\Gamma}{\partial\eta}\Big|_{\eta=0}\right\}\Big|_{\nu=0}$$

Removing higher order effects:

Modifying **ν** all the familly of couplings can be explored

SF **χ SF**



Removing higher order effects:

Modifying **ν** all the familly of couplings can be explored

Cutoff effects **dramatically** reduced.
The **BF** is **not modified** (no extra calculations needed).
The quantities **g** and ν can be computed once, and the optimal **ν** chosen a posteriori.

SU(2): Only one **abelian** direction.
Consider **non-abelian** directions in the algebra.

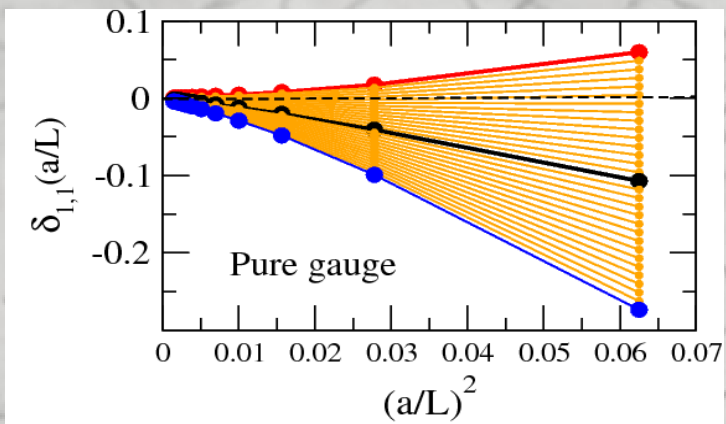
$$\eta\tau_3 \longrightarrow \eta\left(\tau_3 + \sum_{i=1}^{2}\nu_i\tau_i\right)$$

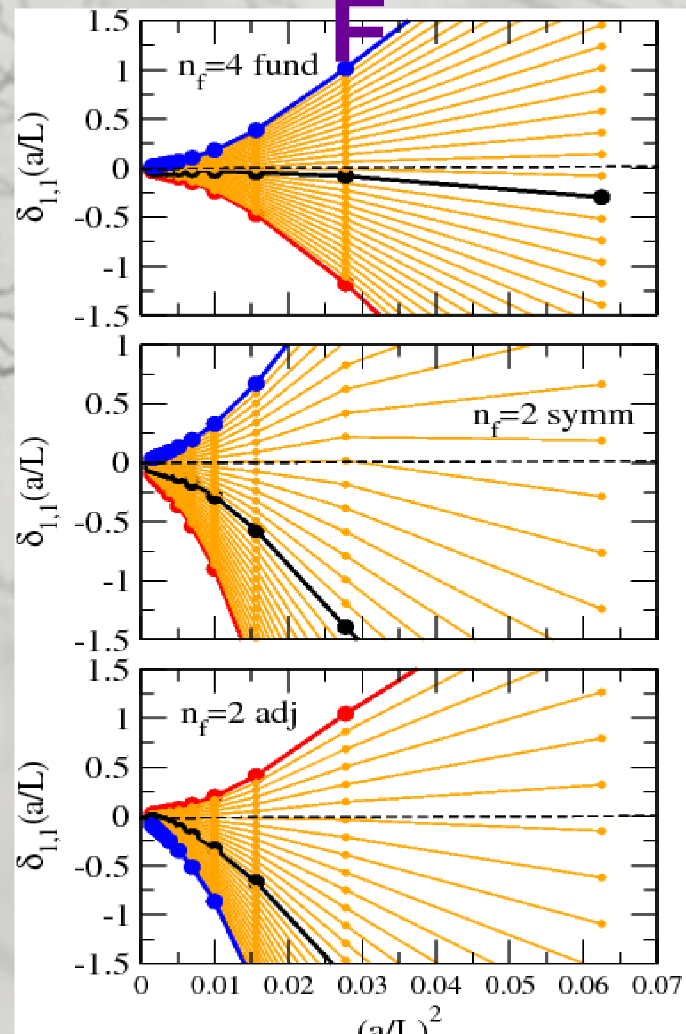
$$\frac{1}{\overline{g}^2_{\vec{V}}(L)} = \frac{1}{\overline{g}^2(L)} - \sum_{i=1}^{2}\nu_i\overline{v}_i(L)$$

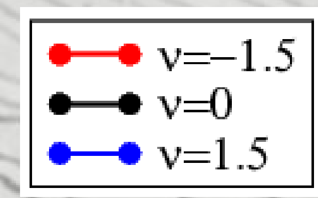
Removing higher order effects:

The full theory, for fundamental nf=4, and adjoint and symmetric nf=2.

χ S F







Conclusions:

We want to study the coupling **g** of strongly interacting theories.

We put them in the **lattice**.

Cutoff effects are the **enemy**. They must be **terminated**.

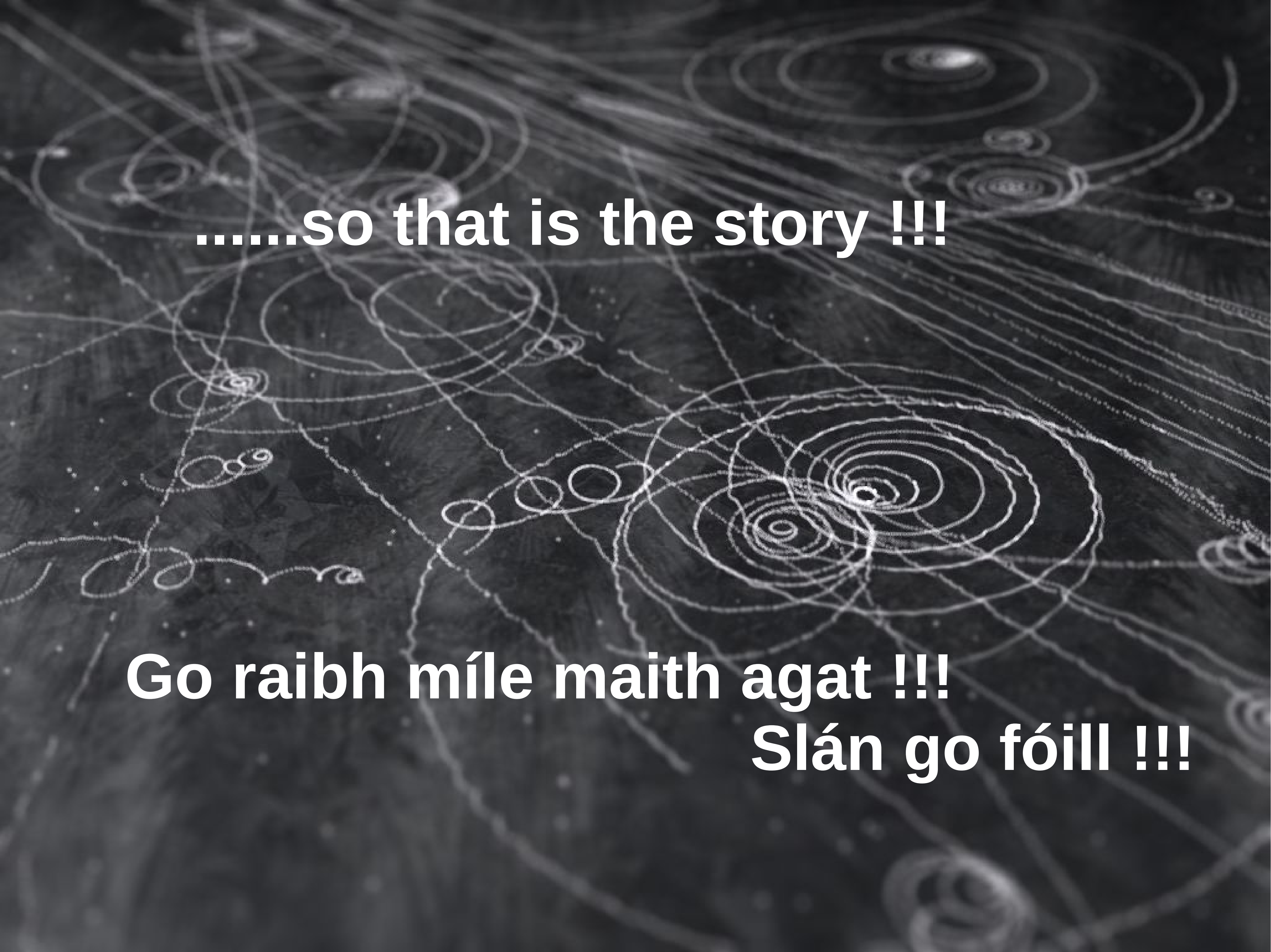
They are **large**, hence we develop alternative strategies.

We **remove** them

At **O(a)** through **Symanzik's improvement**.
At **higher orders** through a **redefinition of the coupling**.

The χ **SF** regularization works particularly well.

The **condition number** is **minimized** by a choice of spatial BC.



......so that is the story !!!

Go raibh míle maith agat !!!

Slán go fóill !!!