Four fermion operators and BSM Physics

Lattice 2012

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Motivations

$$\delta\mathcal{L} = \frac{G^2}{\Lambda^2}\left(\overline{\psi}\psi\right)^2$$

Can mock up effects of (new) strong interactions

- Fermi theory weak interactions
- NJL model - chiral symmetry breaking in QCD
- Extended technicolor models - fermion masses

$$\overline{Q}Q\overline{\psi}\psi \rightarrow \langle\overline{Q}Q\rangle\,\overline{\psi}\psi$$

- General composite Higgs eg top quark condensation
- Arrange that such ops are chiral but not conformal invariant - tunable coupling for walking ?

One such model: gauged NJL

- 2 flavors Dirac fermion in fundamental SU(N)
- Coupled to auxiliary scalar preserving SU(2)xSU(2) chiral symmetry
- Integrate over scalar yields four fermion interaction

$$S = \int \overline{\psi}\gamma.D\psi + \frac{G}{\Lambda}\overline{\psi}\left(\phi_0 + i\gamma_5\tau^i\phi_i\right)\psi + \frac{1}{2}\phi.\phi + F_{\mu\nu}F^{\mu\nu}$$

- SD analysis indicates critical line $\alpha_c(G)$ with $1 < \gamma(\alpha_c) < 2$

Critique of Schwinger-Dyson analysis

- Gauge coupling does not run - confining theories will spontaneously break chiral symmetry even for zero G
- What is nature of ``critical line" in this case ?
- New anomalous dimensions imply new fixed points - here arising from a naively irrelevant operator ...?

The lattice - reduced staggered fermions

- Continuum model may be rewritten in matrix variables

$$S_F = \int \mathrm{Tr}\left(\overline{\Psi}\gamma.D\Psi + \overline{\Psi}\Psi\Phi\right)$$

with

$$\Psi = \begin{pmatrix} 0 & \psi_R \\ \psi_L & 0 \end{pmatrix} \qquad \overline{\Psi} = \begin{pmatrix} \overline{\psi}_L & 0 \\ 0 & \overline{\psi}_R \end{pmatrix} \qquad \Phi = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix}$$

and $\phi = \phi_0 I + i\tau_i\phi_i$

- This facilitates transfer to lattice: matrix representation is equivalent to spin-taste basis for staggered fermions

$$\Psi(x) = \sum_{b=1}^{16} \gamma^{x+b}\lambda(x+b) \quad \text{with} \quad \gamma^{x+b} = \gamma_1^{x_1+b_1}\dots\gamma_4^{x_4+b_4}$$

Reduced staggered fermions

- Block structure of continuum matrices implies only 2 Dirac flavors
- Equivalent to restriction of staggered fields to even/odd parity

$$\lambda(x) = (1+\epsilon(x))\lambda(x) \quad \overline{\lambda}(x) = (1-\epsilon(x))\overline{\lambda}(x)$$

- Alternatively single reduced staggered field $\chi = (\lambda_+, \overline{\lambda}_-)$

$$S = \sum_{x,\mu} \chi^T(x)\, \mathcal{U}_\mu(x)\, \chi(x+\mu)\, [\eta_\mu(x) + G\, \overline{\phi}_\mu(x)\, \epsilon(x)\, \xi_\mu(x)].$$

with

$$\mathcal{U}_\mu(x) = \frac{1}{2}[1+\epsilon(x)]\, U_\mu(x) + \frac{1}{2}[1-\epsilon(x)]\, U^*_\mu(x)$$

$$\eta_\mu(x) = (-1)^{\sum_{i=1}^{\mu-1} x_i}$$

Symmetries

- Lattice theory invariant under discrete subset of continuum chiral syms

$$\chi(x) \to \xi_\rho(x)\,\chi(x+\rho),$$
$$U_\mu(x) \to U^*_\mu(x+\rho),$$
$$\phi_\mu(x) \to (-1)^{\delta_{\mu\rho}}\phi_\mu(x+\rho).$$

- where

$$\xi_\rho(x) = (-1)^{\sum_{i=\rho+1}^{4} x_i}$$

- No single site mass term possible.

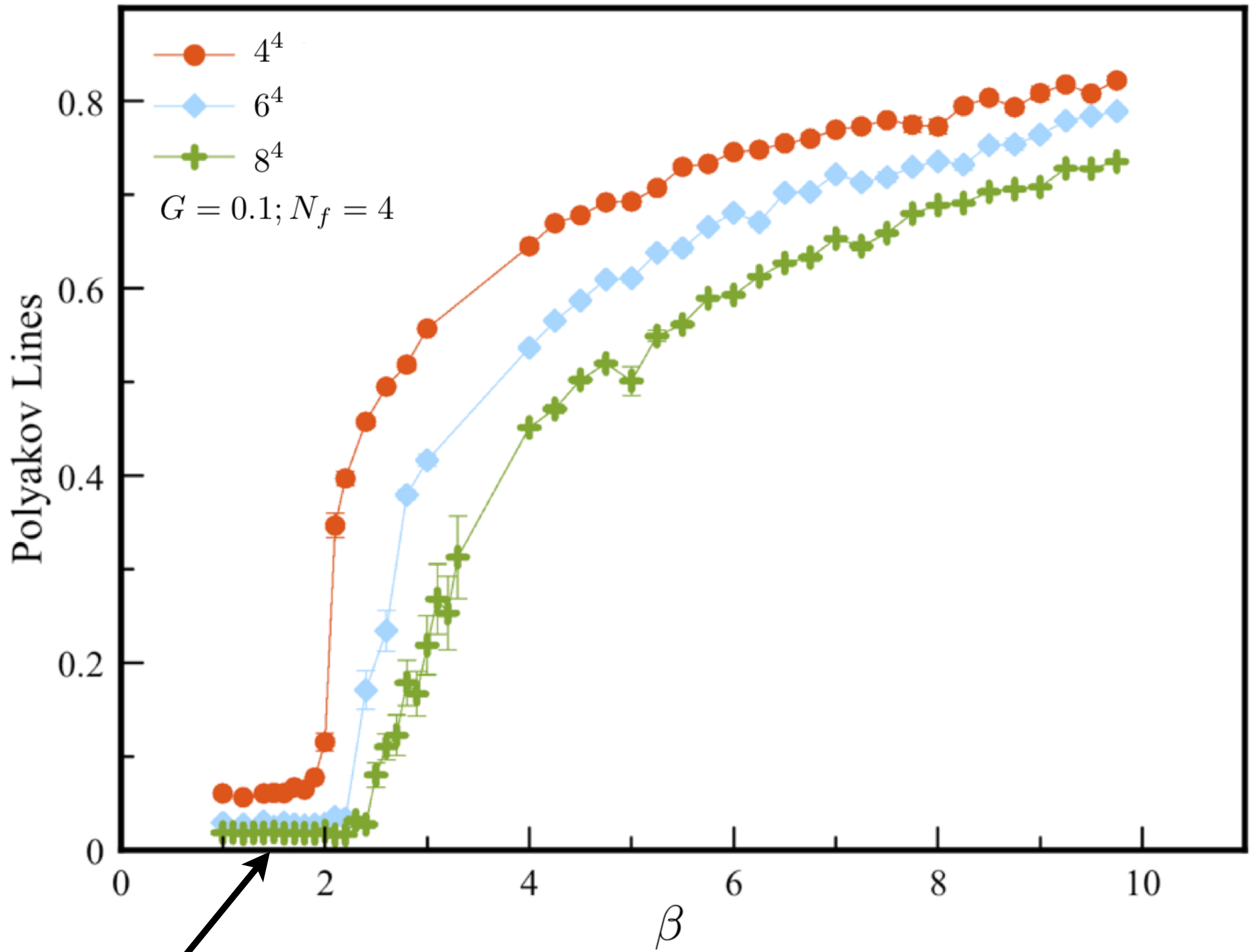
- Chiral condensate determined by 1-link operator

$$\langle \chi(x)\mathcal{U}_\mu(x)\chi(x+\mu)\xi_\mu(x)\epsilon(x)\rangle$$

Simulations

- Use gauge group SU(2) -- guarantees Pfaffian real. Positive if $N_F=4$
- Lattices: 4^4, 6^4, 8^4, 8^3x16.
- RHMC alg. GPU acceleration
- Range of couplings $\beta = 1.8 - 10.0, G = 0.1 - 2.6$
- Observables: Polyakov line, chiral condensate, plaquette, scalar two point function

Polyakov lines



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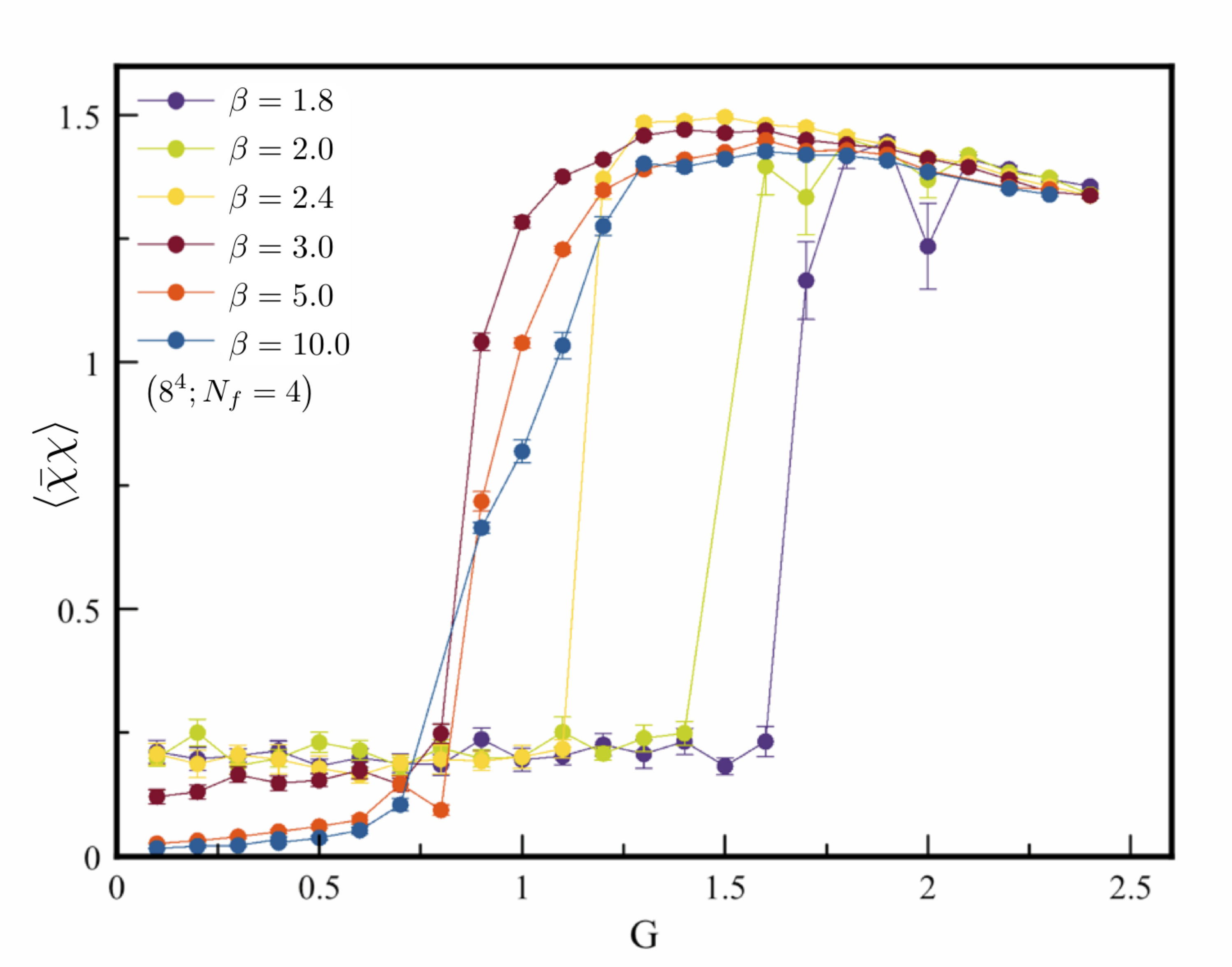
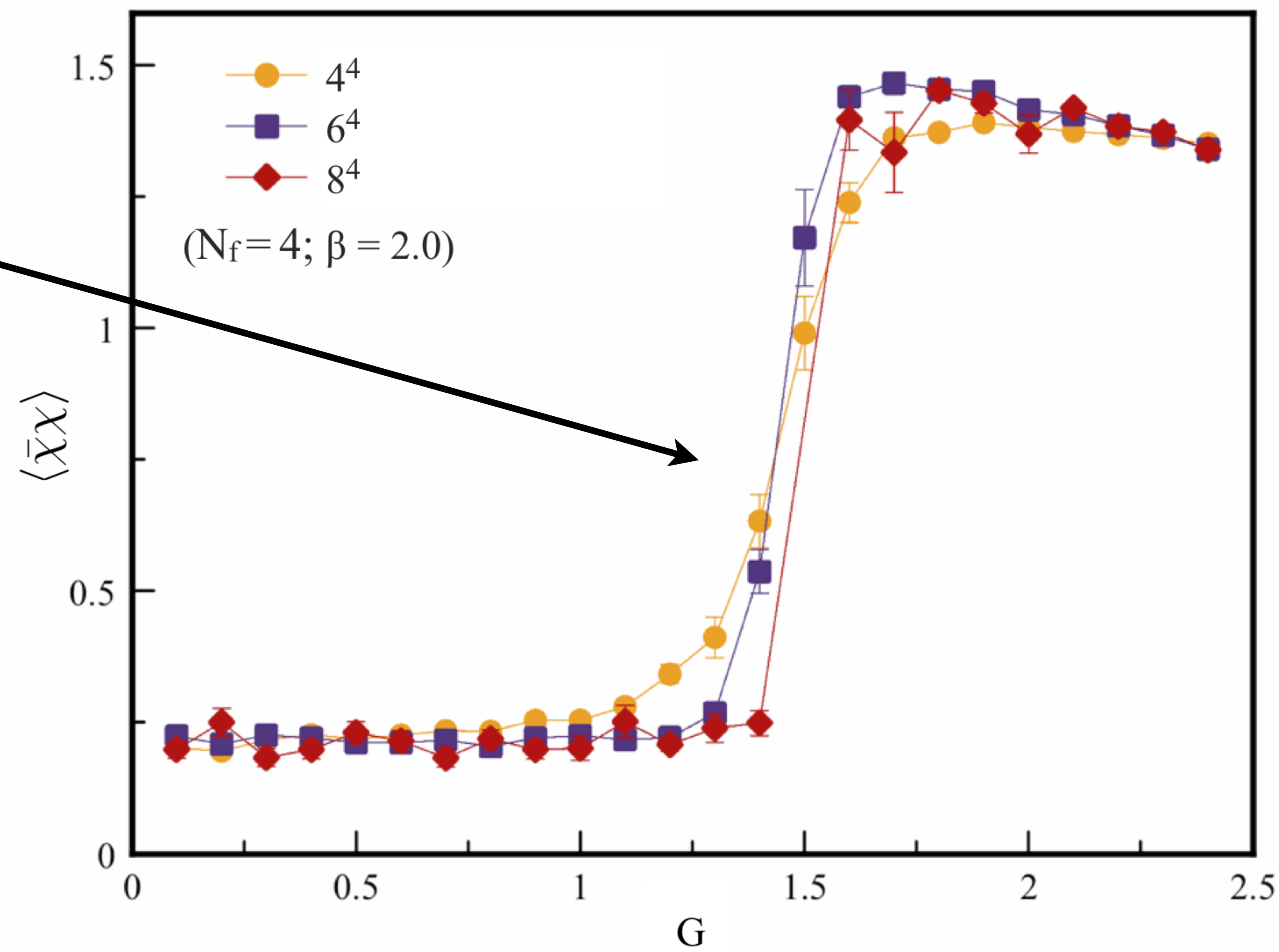
Condensate

- G_c decreases with increasing gauge coupling
- Smooth transition for weak gauge coupling
- Sharp transition when gauge theory confines

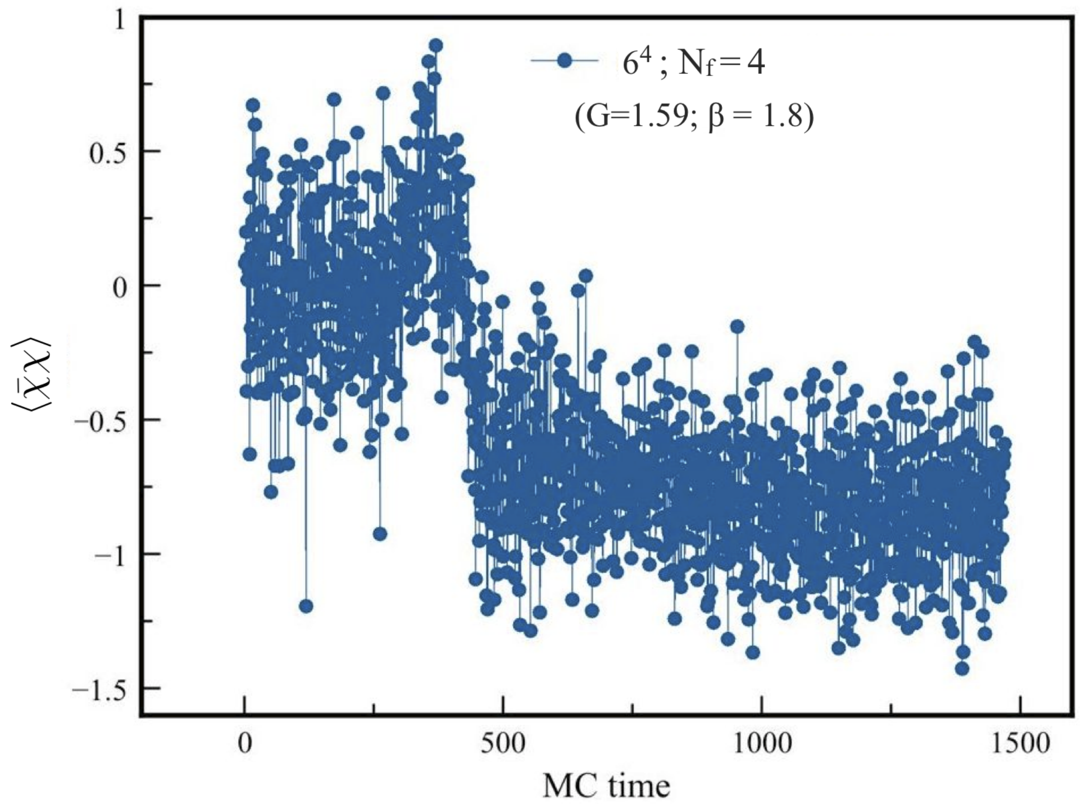


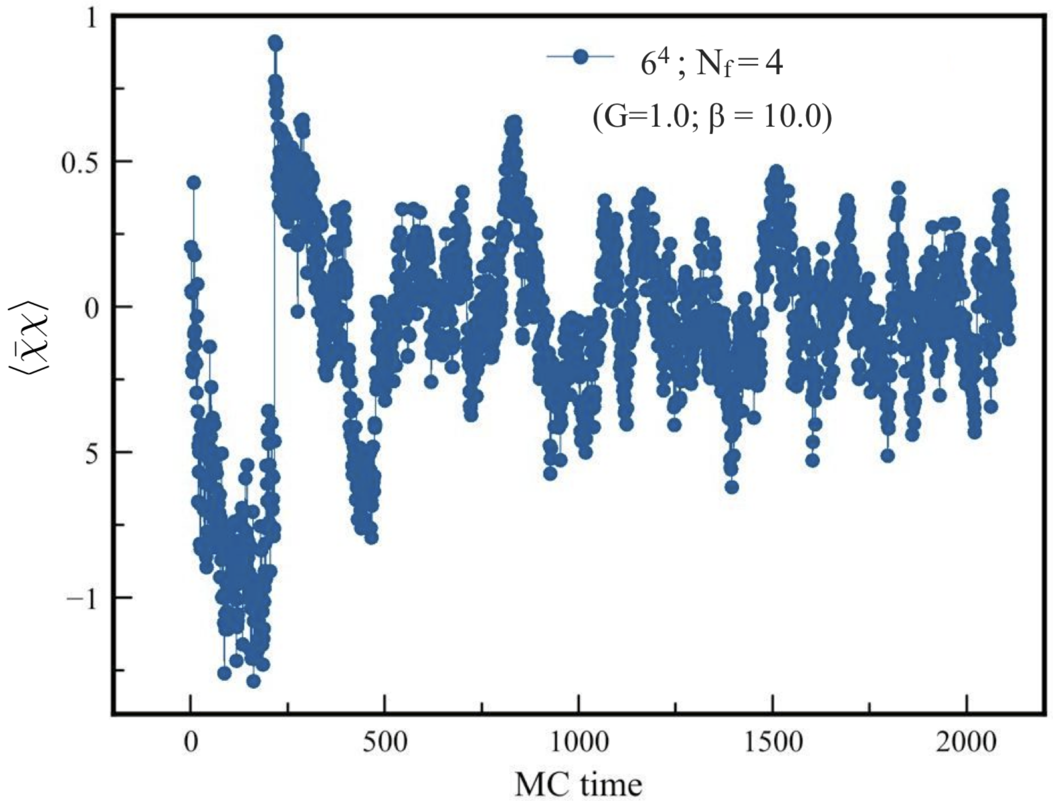
Finite volume effects

Transition sharpens as volume increases



Monte Carlo Time Series

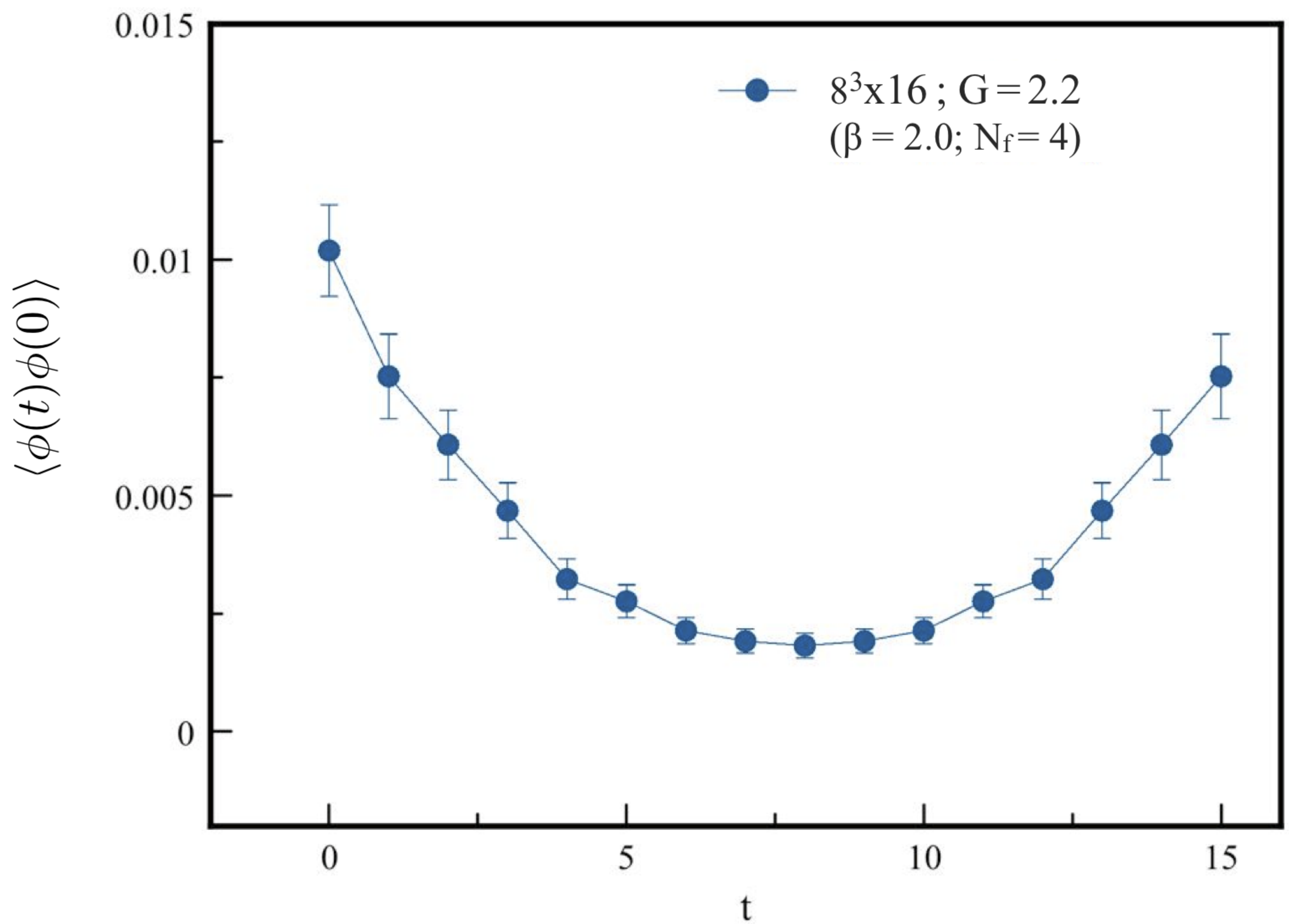




No sign of critical fluctuations for strong gauge coupling

Goldstones

Auxiliary field becomes dynamical pseudo GB



Summary

- First lattice study of gauged NJL model.
- Resembles pure NJL for weak gauge coupling - dynamical mass generation for strong G, spontaneous breaking chiral symmetry.
- Once theory confines see 1^{st} order transition. Chiral symmetry breaking in gauge theory generates non-zero mass even at G=0. No massless states to drive a continuous transition.
- Reduced staggered fermions allow just 2 flavors and minimal chiral invariant Yukawas. No single site mass term allowed.
- Positivity of measure guaranteed for real/pseudoreal gauge groups
- Future: what happens for theories in the conformal window ?