The sexet model and the composite Higgs mechanism

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Outline

- Sextet model: simplest composite Higgs mechanism

- RG flows of lattice actions and crossovers

- Two RG based scaling strategies

- Are we at weak enough coupling?

- New results on the Nf=2 sextet model with SU(3) gauge group

- What to expect from LHC phenomenology

- Summary and Outlook
composite Higgs? simplest example: Nf=2 SU(3) sextet representation
TC (ETC) language used
is it outside (but close to) conformal window?

- original Technicolor paradigm replaced with sextet SU(3) color rep:
- one massless fermion doublet chiral SB with three Goldstone pions
- longitudinal components of weak bosons
- composite Higgs mechanism scale of Higgs condensate $\sim F=250$ GeV
  $\Lambda_{TC} \sim TeV$
- conflicts with EW precision constraints?

running coupling
non-conformal QCD-like far from conformal window

near-conformal
$\Lambda_{ETC} = 100-1000 \Lambda_{TC}$
walking coupling separates two scales

Chiral symmetry breaking turns conformal FP into walking?
QCD-like lattice navigation map and massless fermion surface

- $m=0$ fermion surface
- Massless Goldstone pions
- $m=0$ RT continuum physics (baryon mass gap)
- $g^2$ gauge coupling
- Growing bare $m$
- $m$ (fermion mass)
QCD-like lattice navigation map and massless fermion surface

m = 0 fermion surface
massless Goldstone pions

m = 0 RT continuum physics (baryon mass gap)

UVFP

irrelevant couplings

g^2 gauge coupling

growing bare m

m (fermion mass)

light fermions: continuum RT (Goldstone dynamics)
QCD-like lattice navigation map and massless fermion surface

- \( m = 0 \) fermion surface
- massless Goldstone pions
- irrelevant couplings
- \( m = 0 \) RT continuum physics (baryon mass gap)
- 
- massive fermions: continuum RT (heavy non-Goldstone)
- \( g^2 \) gauge coupling
- growing bare \( m \)
- \( m \) (fermion mass)
- light fermions: continuum RT (Goldstone dynamics)
QCD-like lattice navigation map and massless fermion surface

- $m=0$ fermion surface
  - massless Goldstone pions
- $m=0$ RT continuum physics (baryon mass gap)
- Massive fermions: continuum RT (heavy non-Goldstone)
- Light fermions: continuum RT (Goldstone dynamics)

UVFP

$g^2$ gauge coupling

$m$ (fermion mass)

growing bare $m$
QCD-like lattice navigation map and massless fermion surface

- m = 0 fermion surface
- massless Goldstone pions
- irrelevant couplings
- m = 0 RT continuum physics (baryon mass gap)
- UVFP
- massive fermions: continuum RT (heavy non-Goldstone)
- light fermions: continuum RT (Goldstone dynamics)
- growing bare m
- g^2 gauge coupling
- m (fermion mass)
QCD-like lattice navigation map and massless fermion surface

- $m = 0$ fermion surface
- Massless Goldstone pions
- Irrelevant couplings
- $g^2$ gauge coupling
- $m$ (fermion mass)
- Growing bare $m$
- $m = 0$ RT continuum physics (baryon mass gap)
- UVFP
- Crossover
- Massive fermions: continuum RT (heavy non-Goldstone)
- Light fermions: continuum RT (Goldstone dynamics)
lattice navigation map and the critical (conformal) surface

Symanzik improved gauge action

staggered fermions with 2 stout smearing steps

massless fermions on critical surface

SU(2) x SU(2)
IRFP (conformal)

your favorite lattice action

RT mass deformed conformal continuum physics
lattice navigation map and the critical (conformal) surface

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staggered fermions with 2 stout smearing steps

massless fermions on critical surface

\[ g^2 = 0 \]

\[ g^2 = \infty \]

SU(2)\times SU(2)

IRFP (conformal)

UVFP

RT continuum physics (gapless)

irrelevant couplings

your favorite lattice action

RT mass deformed conformal continuum physics

m (fermion mass)

\[ g^2 \text{ gauge coupling} \]
lattice navigation map and the critical (conformal) surface

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$g^2 = 0$

$g^2 = \infty$

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$g^2$ gauge coupling

UVFP

SU(2)$\times$SU(2)

IRFP (conformal)

RT continuum physics (gapless)

irrelevant couplings

your favorite lattice action

phase transitions?

RT mass deformed conformal continuum physics
lattice navigation map and the critical (conformal) surface

Symanzik improved gauge action

staggered fermions with 2 stout smearing steps

massless fermions on critical surface

$m = 0$ bulk phase at weak coupling? chiral SB?
will determine the physics of the $m=0$ surface and the (non)existence of IRFP
Nf=2 sextet bulk phase structure? are we sitting in the weak coupling phase when $\beta=3.2$? (most of the results)

Goldstone pion and non–Goldstone spectra

$\beta=3.20$

$M_\pi^2 = c_1 m + c_2 m^2$

$M_{i5}^2 = c_0 + c_1 m$

taste breaking is comparable to HISQ performance when $M_{\rho}$ is matched at $\beta=3.25$ we see considerably reduced splitting

small taste breaking further reduced at $\beta=3.25$
Nf=2 sextet bulk phase structure? are we sitting in the weak coupling phase when $\beta=3.2$? (most of the results)

new data: $\beta=3.2 \rightarrow \beta=3.25$

(non)Goldstone splittings and spectroscopy like in weak coupling QCD full scan of bulk phase in progress to re-confirm chiSB phase
our group: mass-deformed theory close to m=0 critical surface and m \to 0 limit:

- two strategies complement: (1) inf volume mass deformed chiral or conformal
  (2) finite volume mass-deformed FSS

- direct access to effective anomalous dimension \( \gamma \) if conformal

- similar to tests of RG scaling laws of moments of current correlator functions
  \( (m=0 \text{ and } m\neq 0 \text{ RG with running coupling and running; } \gamma(\mu) \text{ exponent in progress}) \)

- we will work with chiSB hypothesis which is consistent with all data

- conformal tests badly failed so far – additional conformal FSS tests in the works will be illustrated for Nf=12 \( (\text{Ricky Wong's talk}) \)
mass deformed chiral regime in finite volume below conformal window:

Goldstone dynamics is different in each regime
We study \( \delta \) and \( \epsilon \)-regimes (RMT)
and p-regime (probing chiral loops)
complement each other
interpretation of rotator levels in \( m_q \to 0 \) limit:

Veff: chiral condensate in flavor space
arbitrary orientation of condensate

tilted condensate

Not to misidentify rotator gaps
as evidence of chirally symmetric phase!

Our sextet simulations are in the p-regime \( \beta=3.2 \) (and \( \beta=3.25 \))
strategy 1: \( L \to \infty \) extrapolation first
mass-deformed theory
close to \( m=0 \) critical surface
\( L \to \infty \) extrapolated chiral and conformal
scaling tests in sextet model

due to \( L \cdot M_\pi > 5 \)
less than one percent \( L \) correction
\( C_M \) and \( C_F \) signs correct, values are off
by factor of 3–4 (similar in QCD)
**Strategy I: $L=\infty$ extrapolation first and scaling mass-deformed test**

Chiral hypothesis

(in)complete analysis on both sides

Conformal hypothesis

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*K* sextet model is close to chiral log regime

\[
(M_\pi^2)_{NLO} = (M_\pi^2)_{LO} + (\delta M_\pi^2)_{1-loop} + (\delta M_\pi^2)_{m^2} + (\delta M_\pi^2)_{a^2 m} + (\delta M_\pi^2)_{a^4}
\]

\[
\sim m^2 \quad \sim a^2 m \quad \sim a^4
\]

\[
(M_\pi^2)_{LO} = 2B \cdot m + a^2 \Delta_B
\]

kept cutoff term in $B$ see LO $a^2$ term

would require more data

\[
(\delta M_\pi^2)_{1-loop} = [(M_\pi^2)_{LO} + a^2] \ln(M_\pi^2)_{LO}
\]

\[
M_\pi^2 = c_1 m + c_2 m^2 + \text{logs}
\]

fitted function for Goldstone

\[
M_{\text{nuc}} = c_0 + c_1 m + \text{logs}
\]

nucleon states, rho, a1, higgs, ...

\[
(F_\pi)_{LO} = F, \quad (\delta F_\pi)_{1-loop} = [(M_\pi^2)_{LO} + a^2] \ln(M_\pi^2)_{LO}
\]

chiral log regime was not reached in fermion mass range

\[
(\delta F_\pi)_{m^2} \sim m, \quad (\delta F_\pi)_{a^2 m} = a^2
\]

kept cutoff term in $F$

\[
F_\pi = F + c_1 m + \text{logs}
\]

fitted function

\[
\langle \bar{\psi} \psi \rangle = \langle \bar{\psi} \psi \rangle_0 + c_1 m + c_2 m^2 + \text{logs}
\]

chiral condensate

---

\[
M_\pi = c_\pi \cdot m^{1/y_m}, \quad y_m = 1 + \gamma
\]

leading conformal scaling functional form for all hadron masses

\[
F_\pi = c_F \cdot m^{1/y_m}, \quad y_m = 1 + \gamma
\]

universal critical exponent

\[
\langle \bar{\psi} \psi \rangle = c_\gamma \cdot m^{(3-\gamma)/y_m} + c_1 m
\]

Del Debbio et al.

recent improvements (Patella) from Dirac spectrum based on Giusti and Luscher

conformal scaling violation analysis?

fails in sextet model
- two independent determinations of the chiral condensate
- consistently non-vanishing in chiral limit
- all sextet results are treated as inf volume (m=0.003 is extrapolated in Ls)
Nf=2 SU(3) sextet chiral fits of $M_\pi$ and $F_\pi$

- sextet model Goldstone pion in PCAC channel with quadratic chiral fit
  
  \[ M_\pi^2 = c_1 m + c_2 m^2 \quad \beta=3.2 \]

  - \( c_1 = 6.43 \pm 0.095 \)
  - \( c_2 = -48.2 \pm 15 \)
  - \( \chi^2/\text{dof} = 1.6 \)

- \( \chi^B \) Goldstone boson

- m fit range: 0.003 – 0.008

- inputs from: \( 24^3 \times 48, 32^3 \times 64, 48^3 \times 96 \)

$\beta=3.2$

- \( F_\pi = F_0 + c_1 m \quad \beta=3.2 \)

  - \( F_0 = 0.02805 \pm 0.00052 \)
  - \( c_1 = 3.08 \pm 0.12 \)

  - \( \chi^2/\text{dof} = 0.87 \)

- inputs from: \( 24^3 \times 48, 32^3 \times 64, 48^3 \times 96 \)

  - m fit range: 0.003 – 0.006

m=0.003–0.006 range close to chiral log regime

Nf=2 helps: SU(2) flavor analysis QCD-like

log detection will require more precise data (pqSChiPT works but not well-tested)
Nf=2 SU(3) sextet chiral fits $M_\rho$ and $M_H$

sextet model Rho meson in cRho4 channel with linear chiral fit

$$M_\rho = M_0 + c_1 \, m$$ \[ \beta = 3.2 \]

$M_0 = 0.1892 \pm 0.0061$
$c_1 = 24.7 \pm 0.92$

$M_\rho/F_0 \sim 7$

$\chi^2$/dof = 0.56

fitted volumes: $24^3 \times 48, 32^3 \times 64, 48^3 \times 96$
m fit range: 0.003 – 0.01

sextet model $M_{Higgs}$ in scPion channel with linear chiral fit

$$M_{Higgs} = M_0 + c_1 \, m$$ \[ \beta = 3.2 \]

$M_0 = 0.1699 \pm 0.0077$
$c_1 = 22.4 \pm 1.45$

$M_{Higgs}/F_0 \sim 6$

without disconnected diagram

$\chi^2$/dof = 0.48

plotted volumes: $24^3 \times 48, 32^3 \times 64, 48^3 \times 96$
m fit range: 0.003 – 0.008

m=0.003–0.006 range close to chiral log regime

Nf=2 helps: SU(2) flavor analysis QCD-like

log detection will require more precise data (pqSChPT works but not tested)
**a1-rho split in m=0 limit requires more precise data:**

\[
M_{a_1} = M_0 + c_1 \, m \quad \beta = 3.2
\]

\[
M_0 = 0.281 \pm 0.044
\]

\[
c_1 = 21.5 \pm 7.99
\]

\[
\chi^2/\text{dof} = 1
\]

**sextet \(a_1\) meson (linear fit) split from \(M_\rho\)**

- **Fit range:** 0.004 – 0.009
- **\(M_{a_1}\) fit values:**
  - \(M_0 = 0.281 \pm 0.044\)
  - \(c_1 = 21.5 \pm 7.99\)
  - \(\chi^2/\text{dof} = 1\)
further $N_f=2$ $SU(3)$ sextet model tests?

- $L=\infty$ conformal scaling tests ✓

- conformal FSS tests will be shown in $N_f=12$ model

- confining force in chiral limit? Kieran Holland's talk
conformal hypothesis breaks down in global fits:

sextet model Goldstone pion in PCAC channel with conformal fit

\[ M_\pi = c_m m^{1/1+\gamma} \]  

\[ c_m = 2.21 \pm 0.09 \]  
\[ \gamma = 1.091 \pm 0.034 \]  
\[ \chi^2/dof = 2.04 \]

inputs from: 24\(^3\) x 48, 32\(^3\) x 64, 48\(^3\) x 96
m fit range: 0.003 – 0.008

\[ \beta = 3.2 \]

large anomalous dimension!

inconsistent large critical exponents \( \gamma \) forced by chiral behavior in far infrared

it is not the running \( \gamma(\mu) \) at some scale \( \mu \)!
conformal hypothesis breaks down in global fits:

large and inconsistent critical exponents $\gamma$

are we close enough to the critical surface?
1. conformal scaling test with FSS heavy use of RG theory

\[ LM = f(x) + L^{-\omega} g(x) \]
\[ x = m^{1/1+\gamma} L \]
\[ \omega = \beta'(g^*) \]

2. confining force and its critical m=0 limit  Holland’s talk

Time for LHC phenomenology
We are doing LHC phenomenology based on MadGraph and FeynCalc (Sannino et al).

Drell-Yen production of composite vector bosons on the TeV scale

Feynman diagram of TeV scale new vector meson production
Nf=2 SU(3) sextet model summary:
- No inconsistency with $\chi_{SB}$ in Nf=2 SU(3) sextet model

- We find inconsistency with conformal symmetry in all $L=\infty$ like tests

- Effective large anomalous dimension inconsistent and forced ($\gamma$ is in 1-2 range)

- Kogut and Sinclair: looking for finite temperature $\chi_{SB}$ phase transition
- No inconsistency with $\chi_{SB}$ in Nf=2 SU(3) sextet model

- We find inconsistency with conformal symmetry in all L=∞ like tests

- Effective large anomalous dimension inconsistent and forced (γ is in 1-2 range)

- Kogut and Sinclair: looking for finite temperature $\chi_{SB}$ phase transition

- DeGrand et al. find: Nf=2 sextet beta function might have an IRFP zero?
- model has small anomalous dimension ?
  $\gamma(\mu) < 0.45$ controversy, if conformal; if $\chi_{SB}$ what is $\gamma(\mu)$ ?
- No inconsistency with $\chi_{SB}$ in Nf=2 SU(3) sextet model

- We find inconsistency with conformal symmetry in all L=∞ like tests

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- Kogut and Sinclair: looking for finite temperature $\chi_{SB}$ phase transition

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  $\gamma(\mu) < 0.45$ controversy, if conformal; if $\chi_{SB}$ what is $\gamma(\mu)$ ?

- The Nf=2 sextet model with SU(3) color is an interesting candidate for the composite Higgs mechanism

- Viability requires: confirmation, our own running couplings (see Nogradi’s talk from Wilson flow and Holland’s talk on the F(R) force), the S-parameter, and composite Higgs physics with LHC phenomenology. What will happen on July 4th?