QCD with colour-sextet quarks

D. K. Sinclair and J. B. Kogut

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Introduction

We are interested in extensions of the standard model with a composite/strongly-interacting Higgs sector.

Technicolor theories – QCD-like theories with massless techni-quarks where the techni-pions play the rôle of the Higgs field, giving masses to the W and Z – show the most promise.

It is difficult to suppress flavour-changing neutral currents in extended Technicolor while giving large enough masses to the fermions. Technicolor theories which are simply scaled-up QCD fail the precision electroweak tests.

Walking Technicolor theories, where the fermion content of the gauge theory is such that the running coupling constant evolves very slowly over a considerable range of length/energy-momentum scales, can potentially avoid these problems.

QCD with $1\frac{28}{125} \leq N_f < 3\frac{3}{10}$ flavours of massless colour-sextet quarks is expected to be either a Walking or a Conformal field theory. (First term in the β function is negative, second positive.)

The $N_f = 3$ theory is presumably conformal.

The $N_f = 2$ theory could be either Walking or Conformal. In addition, if walking, it is minimal, having just the right number of Goldstone bosons (3) to give masses to the W and Z.

We simulate the $N_f = 2$ theory to see if it walks. We also study the $N_f = 3$ theory for comparison.

We simulate these theories at finite temperature, using the evolution of the lattice bare coupling at the chiral transition with N_t to determine if it is governed by asymptotic freedom – walking – or if it approaches a non-zero constant (bulk transition) – conformal.

The deconfinement transition occurs at appreciably smaller β (stronger coupling), and it is unlikely to give useful information on QCD-like versus conformal behaviour at the N_t values we use.

We simulate the $N_f = 2$ theory on lattices with $N_t = 4, 6, 8, 12$ and hope to extend this to larger N_t.

Preliminary results indicate that $\beta_\chi(N_t = 12)$ is significantly larger than $\beta_\chi(N_t = 8)$, but by less than what the 2-loop β-function would predict.

We are also performing preliminary runs with β fixed at a value above $\beta_\chi(N_t = 12)$ on lattices with fixed N_s varying N_t, keeping $N_t \leq N_s$ (we thank Julius Kuti for suggesting this). Here we look for the transition to the chirally restored state as N_t increases.

We simulate the $N_f = 3$ theory on lattices with $N_t = 4, 6, 8$ and hope to extend this to $N_t = 12$.

Preliminary results indicate that $\beta_\chi(N_t = 8)$ is probably significantly greater than $\beta_\chi(N_t = 6)$ which would indicate that we are not yet at weak enough coupling.

QCD with colour-sextet staggered quarks at finite T

We use the simplest (Wilson) gauge action:

$$S_g = \beta \sum_{\Box} \left[1 - \frac{1}{3}\mathrm{Re}(\mathrm{Tr}UUUU)\right]. \quad (1)$$

Formally, the unimproved staggered quark action action is:

$$S_f = \sum_{sites} \left[\sum_{f=1}^{N_f/4} \psi_f^\dagger [D\!\!\!/ + m]\psi_f\right], \quad (2)$$

where where $D\!\!\!/ = \sum_\mu \eta_\mu D_\mu$ with

$$D_\mu \psi(x) = \frac{1}{2}[U_\mu^{(6)}(x)\psi(x+\hat{\mu}) - U_\mu^{(6)\dagger}(x-\hat{\mu})\psi(x-\hat{\mu})]. \quad (3)$$

We use the RHMC algorithm to simulate values of $N_f/4$ which are not integers, in particular $N_f = 2, 3$.

Simulations near the chiral transition for $N_f = 2$

We are simulating lattice QCD with 2 colour-sextet quarks near the chiral transitions for lattices with $N_t = 8$ and $N_t = 12$, to accurately determine β_χ, the value of β at this transition.

Two loop perturbation theory predicts that

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.12$$

We look for evidence that $\beta_\chi(N_t)$ approaches zero as $N_t \to \infty$.

The peak in the (disconnected) chiral susceptibility

$$\chi_{\bar\psi\psi} = \frac{V}{T}\left[\langle(\bar\psi\psi)^2\rangle - (\langle\bar\psi\psi\rangle)^2\right]$$

extrapolated to $m = 0$, gives the estimate of the position of the chiral transition.

$N_t = 8$

We have completed our simulations on $16^3 \times 8$ lattices in the neighbourhood of the chiral transition. Our quark masses are $m = 0.0025$, $m = 0.005$, $m = 0.01$, $m = 0.02$. In the range $6.6 \leq \beta \leq 6.8$ we simulate at βs spaced by 0.02. At each β in this range we ran for 50,000 length-1 trajectories for each mass at $m = 0.005, 0.01, 0.02$ and for 100,000 trajectories at $m = 0.0025$.

Figure 1 shows the unrenormalized chiral condensates for these runs.

Figure 2 shows the unrenormalized chiral condensates and the subtracted chiral condensates (following Fodor $et\ al.$), for the lowest 2 masses. The subtracted chiral condensate

$$\langle\bar{\psi}\psi\rangle_{sub} = \langle\bar{\psi}\psi\rangle - \left(m_V \frac{\partial}{\partial m_V}\langle\bar{\psi}\psi\rangle\right)_{m_V=m}$$

removes (most of) the leading UV divergence from $\langle\bar{\psi}\psi\rangle$.

Figure 3 shows the chiral susceptibilities from these runs.

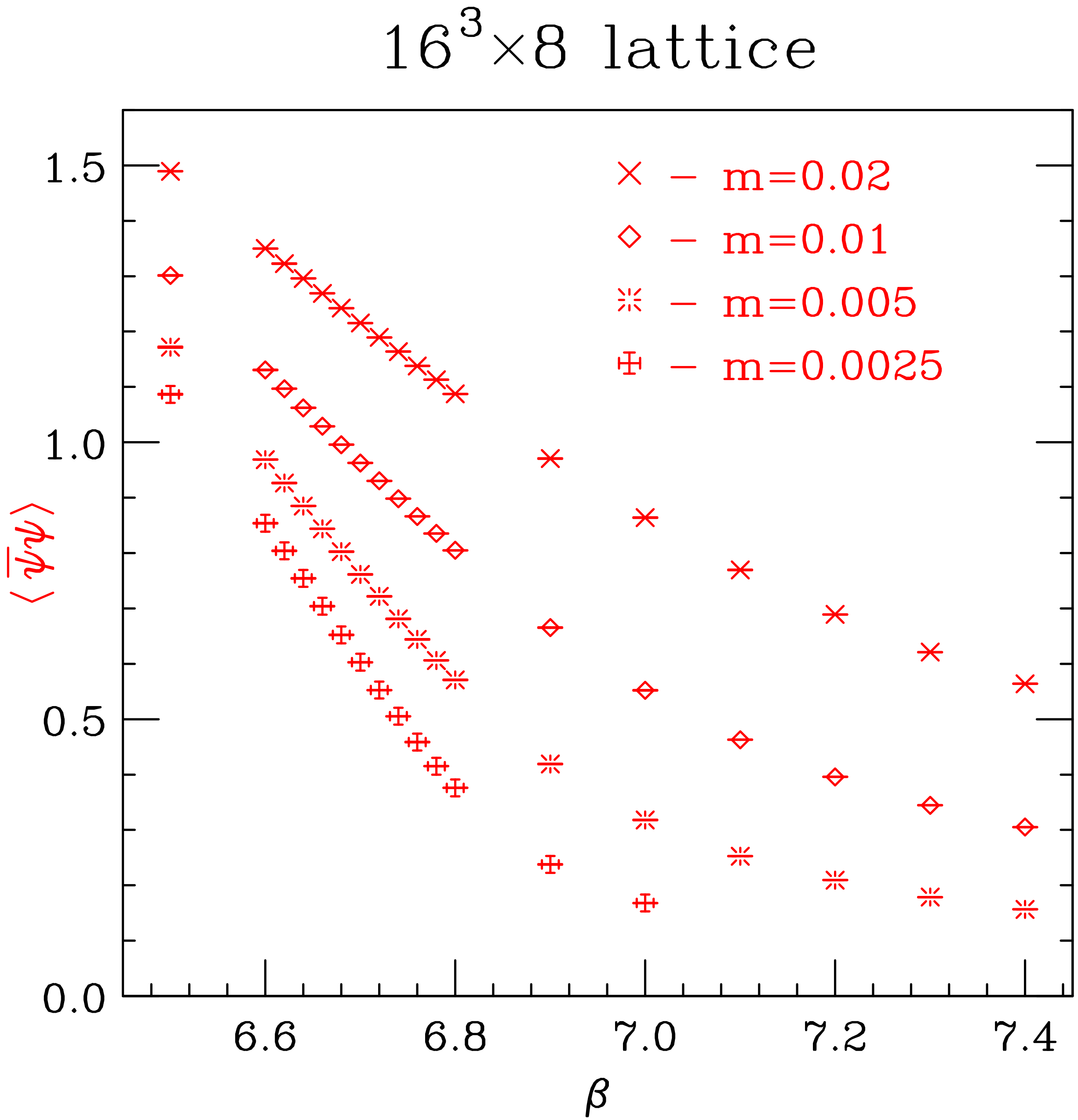

Figure 1: Chiral condensates on a $16^3 \times 8$ lattice.

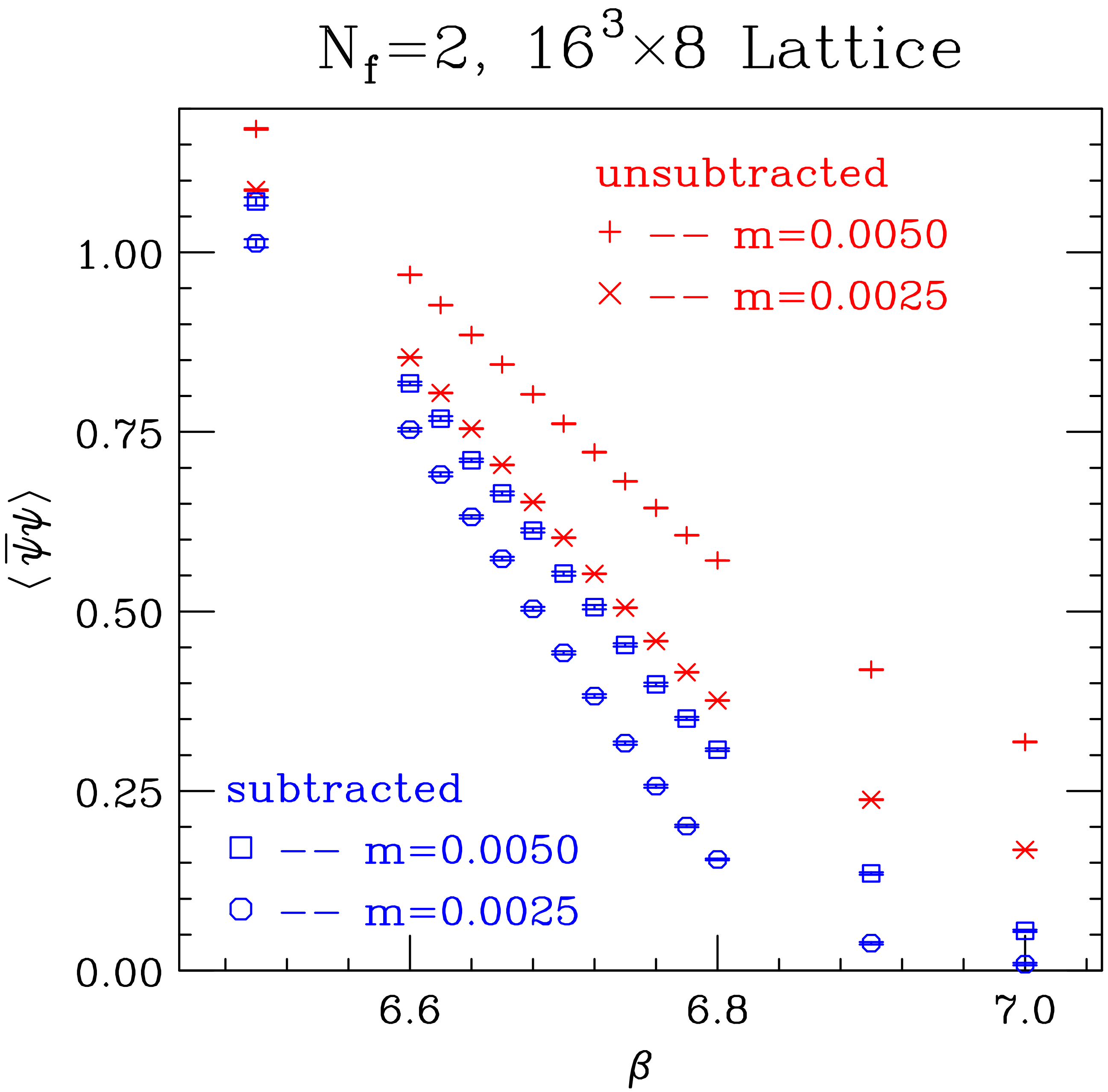

Figure 2: Chiral condensates and subtracted chiral condensates on a $16^3 \times 8$ lattice.

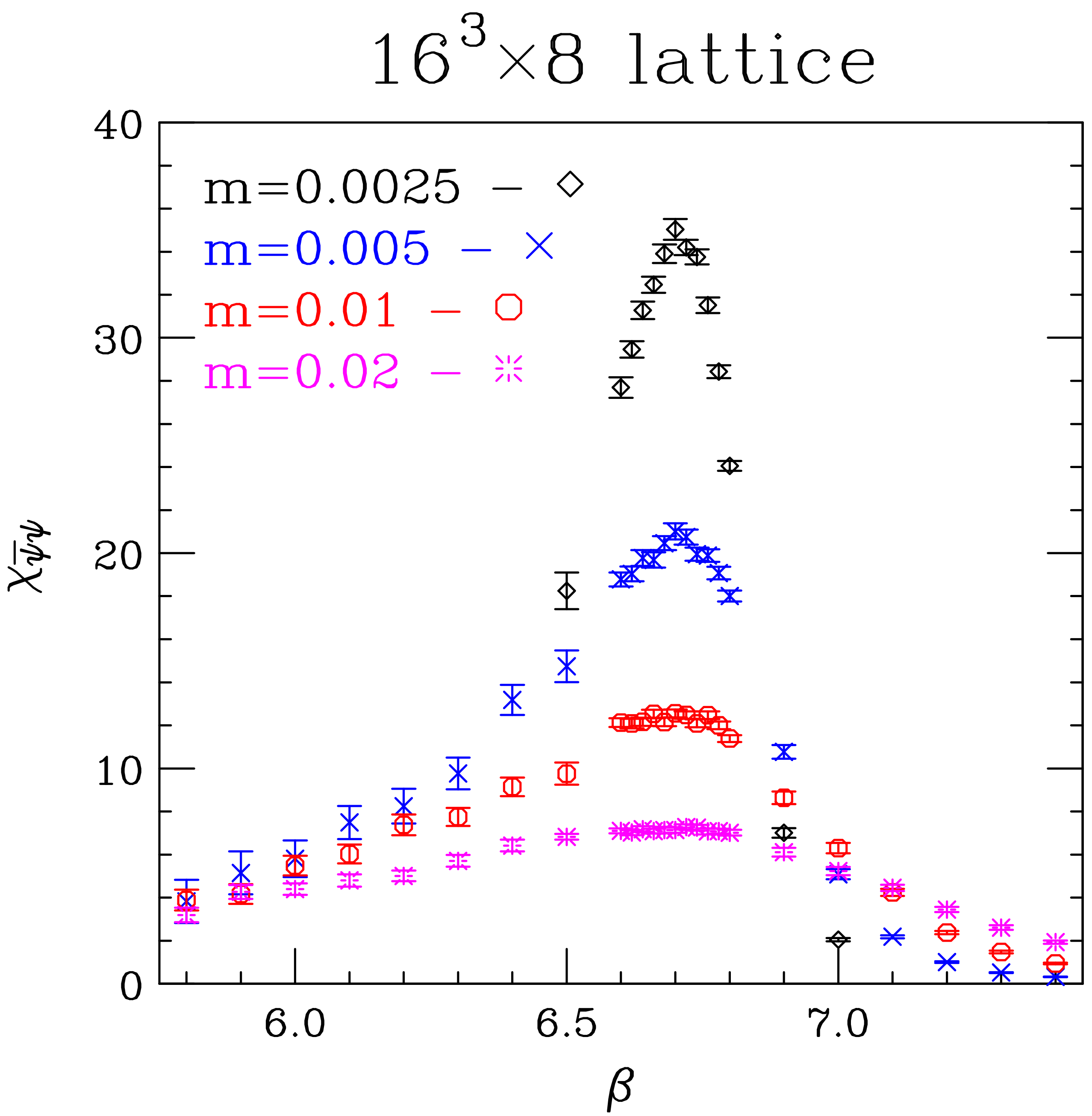

Figure 3: Chiral susceptibilities on a $16^3 \times 8$ lattice.

While the behaviour of the chiral condensate (figure 1) strongly suggests that it vanishes in the chiral limit for β sufficiently large, it would be difficult, if not impossible, to make a precise estimate of β_χ from these graphs.

The subtracted chiral condensate (figure 2) enhances the vanishing of the chiral condensate in the chiral limit, but is still inadequate for estimating β_χ.

The chiral susceptibility, which should diverge at β_χ, in the chiral limit shows clear peaks for $m = 0.005$ and $m = 0.0025$. The height of the peaks grows as m is decreased. In addition, we note that the position of the peaks does not change as m is decreased from 0.005 to 0.0025. Hence the position of the peak at $m = 0.0025$ should be a good estimate of β_χ.

Ferrenberg-Swendsen reweighting from the chiral susceptibilities at $\beta = 6.68$ and $\beta = 6.70$ for mass $m = 0.0025$ yields $\beta_{peak} = 6.690(5)$. We therefore estimate that $\beta_\chi = 6.69(1)$.

$N_t = 12$

We are performing simulations on $24^3 \times 12$ lattices at $m = 0.0025$, $m = 0.005$, $m = 0.01$. For $6.6 \le \beta \le 6.9$ we are performing simulations at βs spaced by 0.02. Outside this range our βs are spaced by 0.1. The preliminary results presented here represent 10,000–25,000 trajectories for each (β, m), except at $m = 0.005$, $\beta = 6.6$ where we have 42,750 trajectories.

Figure 4 shows the unrenormalized chiral condensates for these runs.

Figure 5 shows the unrenormalized chiral condensates and the subtracted chiral condensates for the lowest mass.

Figure 6 shows the chiral susceptibilities from these runs.

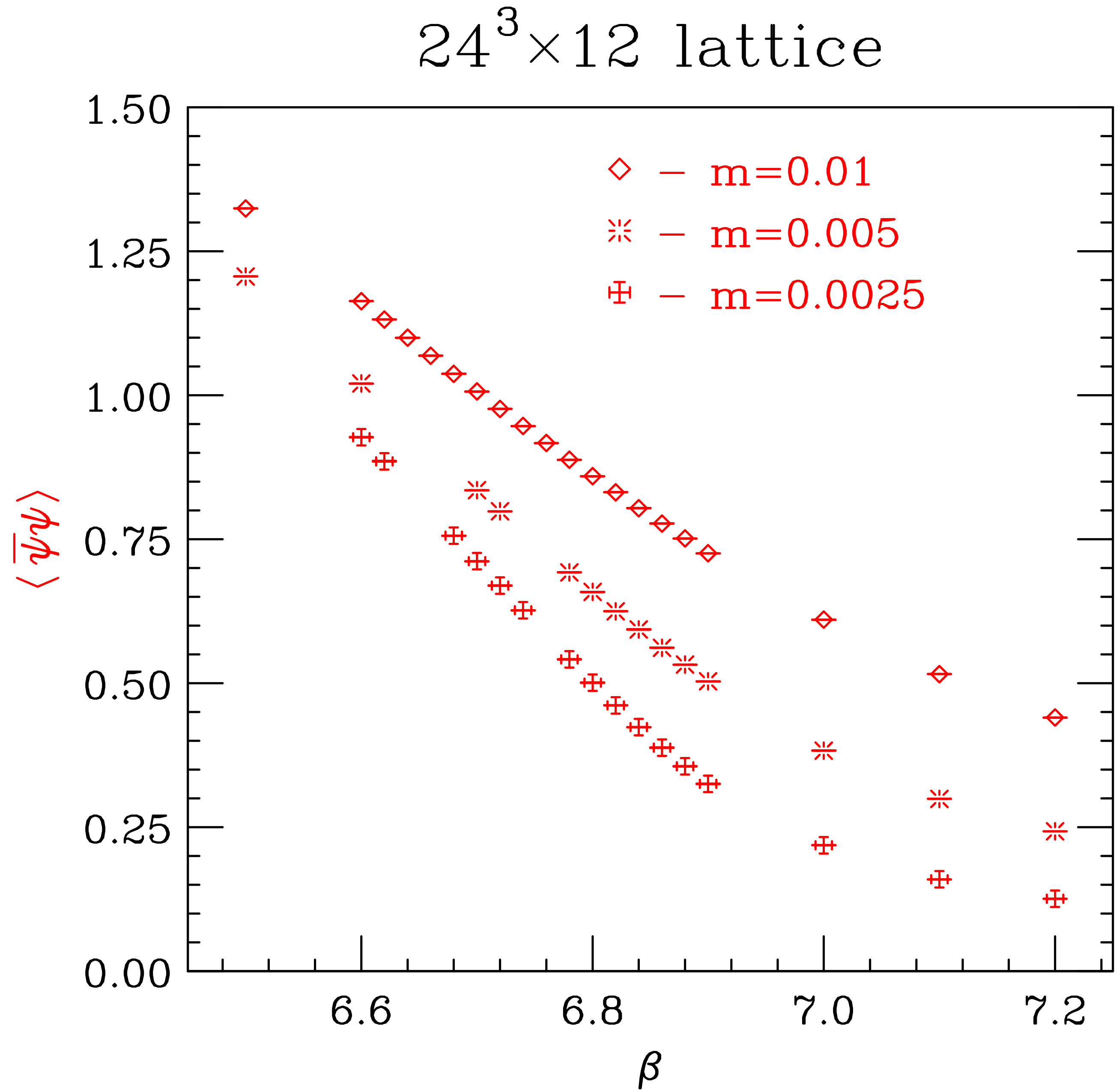

Figure 4: Chiral condensates on a $24^3 \times 12$ lattice.

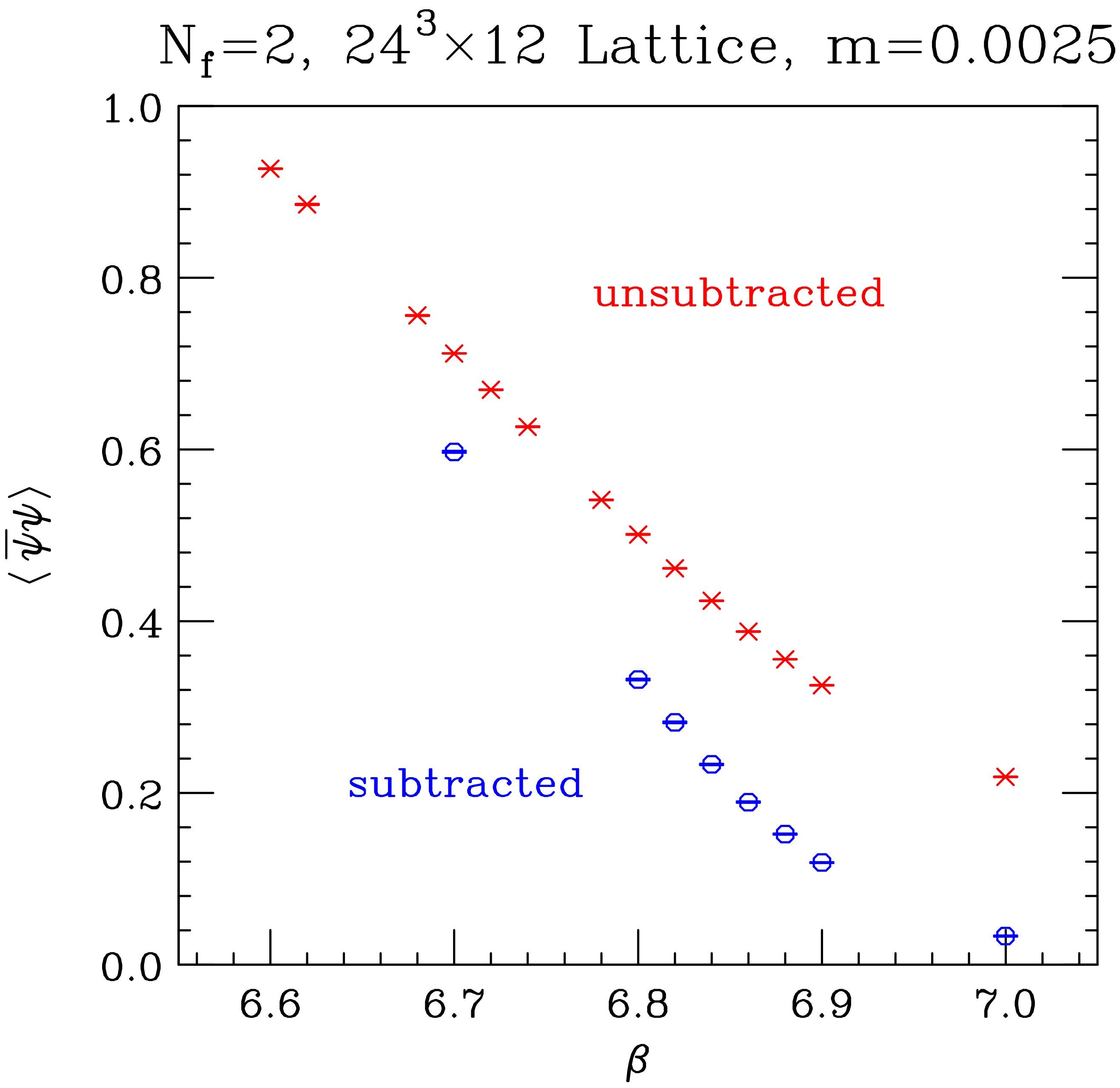

Figure 5: Chiral condensates and subtracted chiral condensates on a $24^3 \times 12$ lattice at $m = 0.0025$.

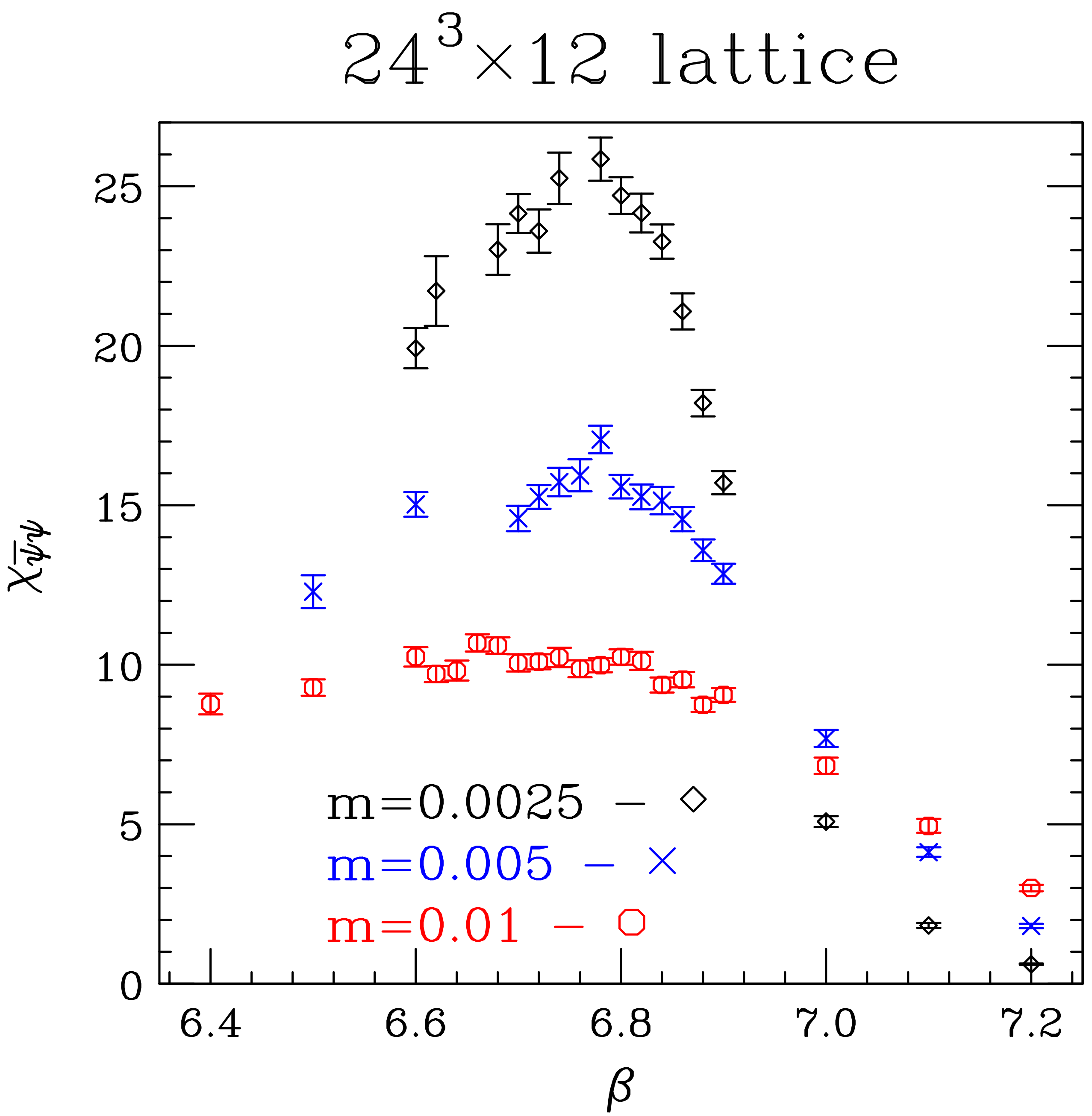

Figure 6: Chiral susceptibilities on a $24^3 \times 12$ lattice.

As for $N_t = 8$, the chiral condensates both unsubtracted and subtracted, strongly suggest that there is some $\beta = \beta_\chi$ above which the chiral condensate vanishes in the chiral limit. However, at the masses we use, it would be difficult if not impossible to extract β_χ from either of these condensates.

The peaking of the chiral susceptibility (figure 6) promises to allow us to estimate β_χ with some precision. However, more statistics are needed.

A rough estimate of β_χ from the present 'data' yields $\beta_\chi \approx 6.78$. This means that

$$\beta_\chi(N_t = 12) - \beta_\chi(N_t = 8) \approx 0.09$$

compared with the 0.12 from 2-loop perturbation theory.

$24^3 \times N_t$

We simulate this 2-flavour theory on $24^3 \times N_t$ lattices with $N_t \leq 24$ at fixed β, to search for evidence of a transition back to the chirally broken phase as N_t is increased.

We choose $\beta = 6.9$. If the evolution of β_χ is governed by the 2-loop β-function, $\beta_\chi \approx 6.9$ for $N_t = 18$. Hence we should see evidence for the chiral transition as N_t is increased.

At present, we have fixed the quark mass at $m = 0.0025$, and are running on $24^3 \times 8$, $24^3 \times 12$, $24^3 \times 18$ and 24^4 lattices.

Figure 7 shows the unsubtracted and subtracted chiral condensates as functions of N_t for these runs.

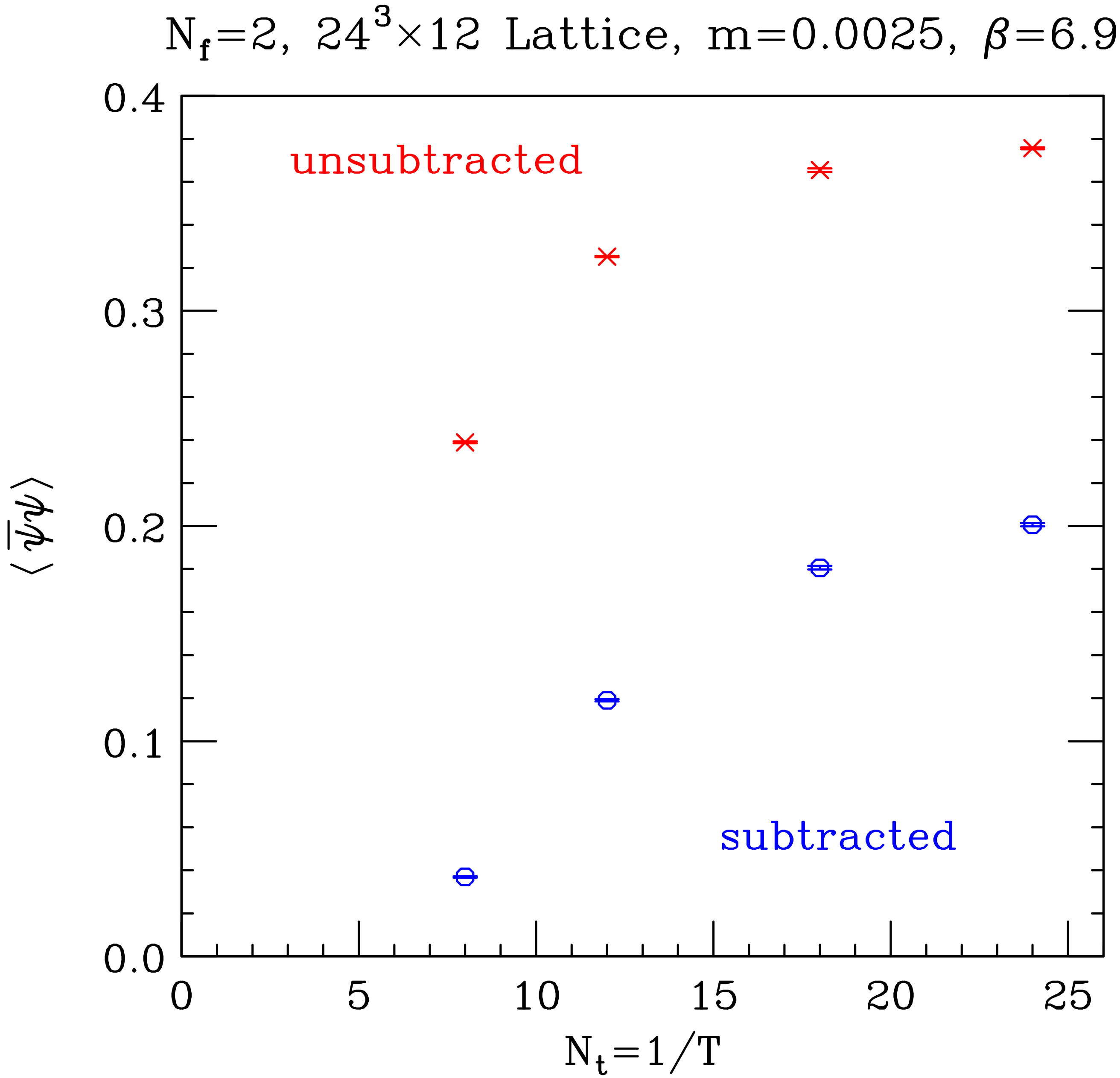

Figure 7: Chiral condensates on a $24^3 \times N_t$ lattices at $\beta = 6.9$ and $m = 0.0025$.

While this appears promising as evidence for there being a transition back to a chirally broken phase in the chiral limit, we clearly need more masses to make it convincing.

Simulations near the chiral transition for $N_f = 3$

We are extending our simulations with 3 flavours of sextet quarks to $N_t = 8$ lattices. Our simulations are performed on $16^3 \times 8$ lattices with $m = 0.005$ and $m = 0.01$.

2-loop evolution of the lattice(bare) coupling would predict

$$\beta_\chi(N_t = 8) - \beta_\chi(N_t = 6) \approx 0.0025,$$

which would be indistinguishable from zero at our expected resolution. Hence we will be unable to distinguish a finite temperature from a bulk transition.

However, we should be able to tell if we are yet in the asymptotic regime, by observing if there is any appreciable shift in β_χ between $N_t = 6$ and $N_t = 8$.

Figure 8 shows preliminary results for the chiral susceptibilities for $N_f = 3$ on a $16^3 \times 8$ lattice. Each point represents 10,000–20,000 trajectories.

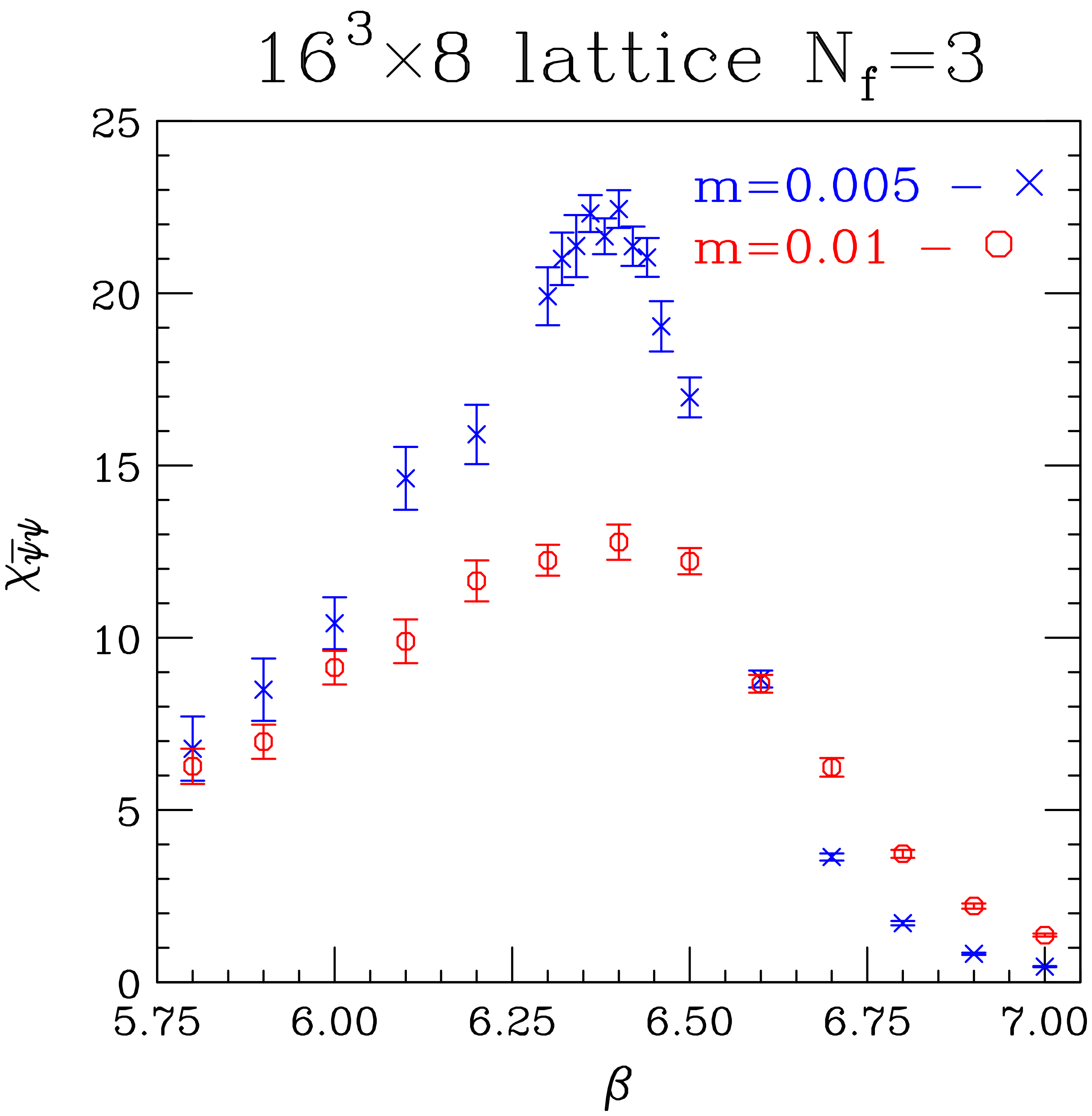

Figure 8: Chiral susceptibilities on a $16^3 \times 8$ lattice for $N_f = 3$.

Clearly we need more statistics to determine the position of the peak with any accuracy.

A rough estimate of the position of the peaks yields $\beta_\chi \approx 6.38$.

Both the chiral susceptibility and the condensates themselves indicate that there is an appreciable shift in β_χ between $N_t = 6$ and $N_t = 8$. This indicates that we have yet to enter the weak-coupling regime where there should be no discernible shift.

Discussion and Conclusions

- We simulate the thermodynamics of QCD with 2 colour-sextet quarks on lattices with $N_t = 4, 6, 8, 12$ in the neighbourhood of the chiral transition. If chiral-symmetry restoration is a finite-temperature phase transition, measuring β_χ as a function of N_t yields the running of the bare lattice coupling β with a at $a = 1/(N_t T_\chi)$. Asymptotic freedom would imply that $\beta_\chi \longrightarrow \infty$ ($g^2 \longrightarrow 0$) as $N_t \longrightarrow \infty$.

- We present preliminary results indicating that β_χ increases with N_t over the range of N_ts considered. This suggests that the theory does walk. However, the change in β_χ between $N_t = 8$ and $N_t = 12$ appears to be about 25% smaller than would be predicted from the 2-loop β-function. This is of concern, since this lies in the assumed weak-coupling domain ($\beta \gtrsim \beta_\chi(N_t = 6)$). If the theory is QCD-like, it suggests that the 2-loop β-function is inadequate to describe the running of the bare coupling for unimproved staggered lattice QCD at these couplings.

- A series of runs performed on $24^3 \times N_t$ lattices for several N_ts ($N_t \leq 24$) at a fixed (small) quark mass and a fixed coupling intermediate between $\beta_\chi(N_t = 12)$ and the expected value of $\beta_\chi(N_t = 24)$, does show an increase both the unsubtracted an subtracted chiral condensates consistent with a transition to a chirally broken theory as N_t increases.
- We need more statistics to accurately determine $\beta_\chi(N_t = 12)$.
- A more accurate determination of $\beta_\chi(N_t = 6)$ is also needed.
- Our $24^3 \times N_t$ runs need to be performed at more masses (and more N_ts).
- The zero temperature properties of this theory need to be studied and the results compared with Fodor $et\ al$ and DeGrand $et\ al$.
- We are extending our $N_f = 3$ runs to $N_t = 8$. Preliminary results indicate that there is a substantial increase in β_χ between $N_t = 6$ and $N_t = 8$. This indicates that this range of N_ts does not lie completely in the weak-coupling domain.

- Other theories we plan to study include $SU(2)_{colour}$ with 3/2 and with 1 colour-adjoint (symmetric) quarks, and $SU(4)_{colour}$ with colour-antisymmetric quarks.
- These simulations were performed on Hopper, Carver and Franklin at NERSC, on Kraken at NICS and on Fusion at the Argonne LCRC.

Appendix

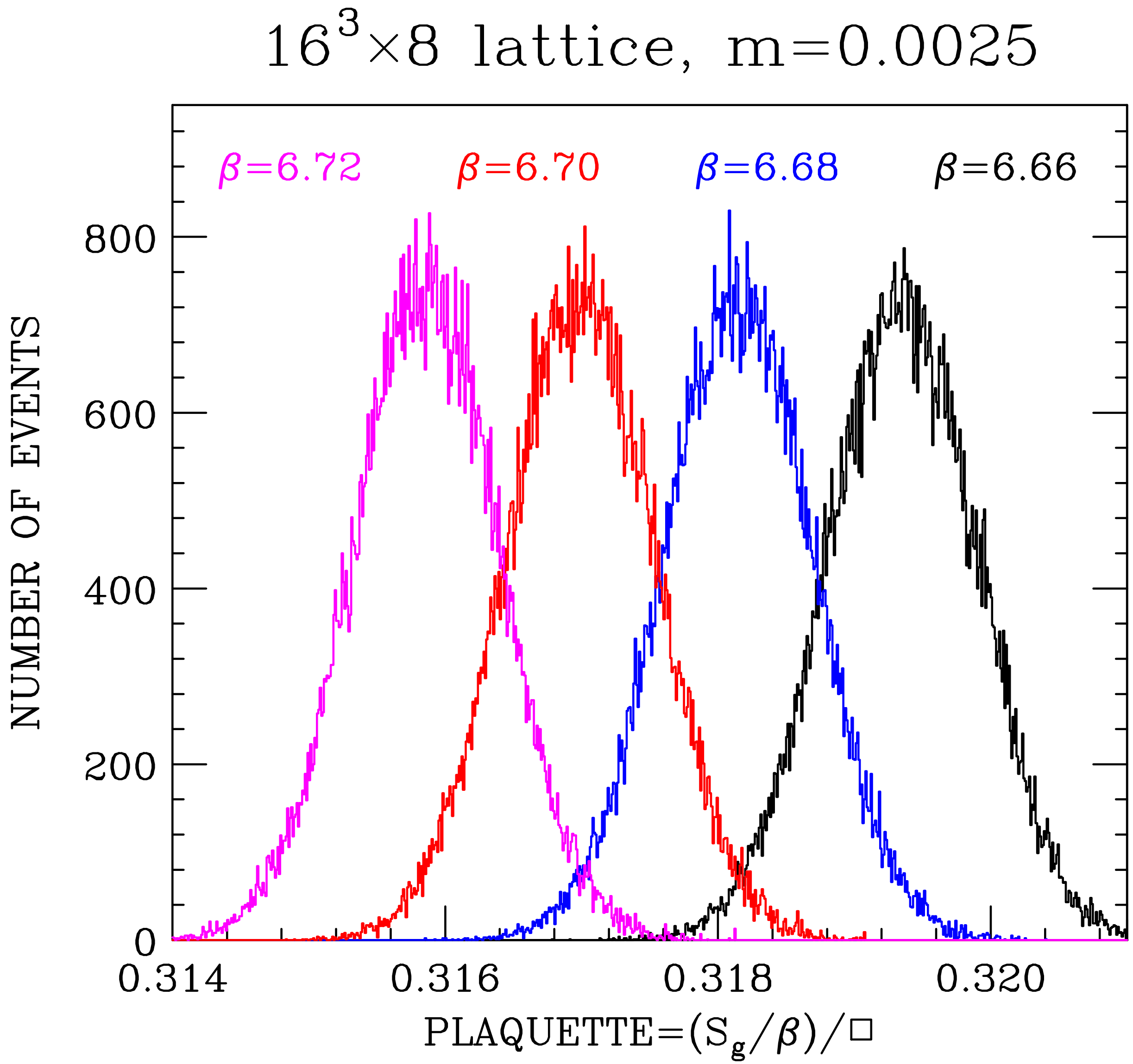

Figure 9: Plaquette distributions near the chiral transition for $N_f = 2$, $m = 0.0025$ on a $16^3 \times 8$ lattice.