Conformal Finite Size Scaling of Twelve Fermion Flavors

Chik Him Wong

Lattice Higgs Collaboration (LHC):

Zoltán Fodor [$], *Kieran Holland* [*], *Julius Kuti* [†], *Dániel Nógrádi* [-], *Chik Him Wong* [†]

†: University of California, San Diego *: University of the Pacific $: University of Wuppertal -: Eötvös University

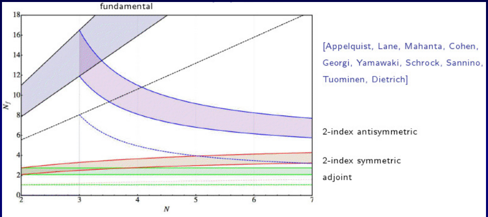
LATTICE 2012

Outline

- Background
  - Conformality Controversy of $N_f = 12$ $SU(3)$ Fundamental
  - Simulation Setup
- Spectroscopy Analysis
- Fitting under conformal hypothesis with Finite Size Scaling study
  - Fitting scaling function with physical model
  - Fitting general scaling function using B-form spline functions
- Conclusion
  - Outstanding Problems
  - Future Plans

Conformality Controversy of $N_f = 12$ $SU(3)$ Fundamental

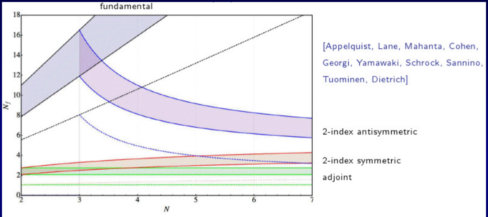
- Is $N_f = 12$ $SU(3)$ Fundamental inside Conformal Window?



- Previously...
  - Likely: T. Appelquist et al, A. Deuzeman et al,A Hasenfratz et al, Y. Aoki et al
  - Not Likely: X.-Y. Jin et al, Z. Fodor et al
- Latest findings of our group: **Finite Size Scaling Conformal scenario is Inconsistent and Suspect**

Conformality Controversy of $N_f = 12$ $SU(3)$ Fundamental

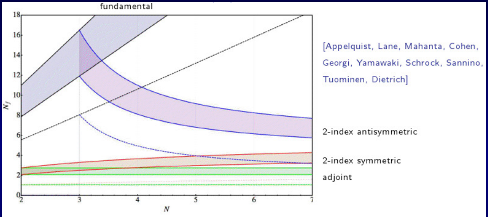
- Is $N_f = 12$ $SU(3)$ Fundamental inside Conformal Window?



- Previously...
  - Likely: T. Appelquist et al, A. Deuzeman et al, A Hasenfratz et al, Y. Aoki et al
  - Not Likely: X.-Y. Jin et al, Z. Fodor et al
- Latest findings of our group: **Finite Size Scaling Conformal scenario is Inconsistent and Suspect**

Conformality Controversy of $N_f = 12\ SU(3)$ Fundamental

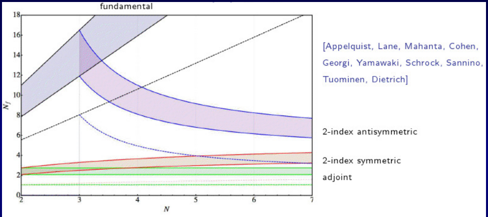
- Is $N_f = 12\ SU(3)$ Fundamental inside Conformal Window?



- Previously...
  - Likely: T. Appelquist et al, A. Deuzeman et al,A Hasenfratz et al, Y. Aoki et al
  - Not Likely: X.-Y. Jin et al, Z. Fodor et al
- Latest findings of our group:
  Finite Size Scaling Conformal scenario is Inconsistent and Suspect

Conformality Controversy of $N_f = 12\ SU(3)$ Fundamental

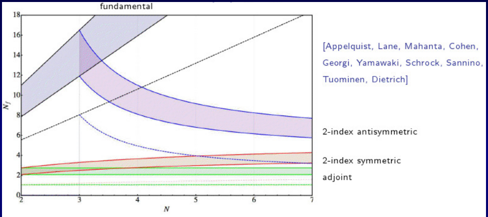
- Is $N_f = 12\ SU(3)$ Fundamental inside Conformal Window?



- Previously…
  - Likely: T. Appelquist et al, A. Deuzeman et al,A Hasenfratz et al, Y. Aoki et al
  - Not Likely: X.-Y. Jin et al, Z. Fodor et al
- Latest findings of our group:
  Finite Size Scaling Conformal scenario is Inconsistent and Suspect

Conformality Controversy of $N_f = 12$ $SU(3)$ Fundamental

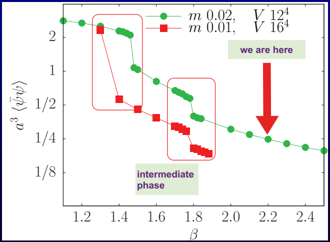
- Is $N_f = 12$ $SU(3)$ Fundamental inside Conformal Window?



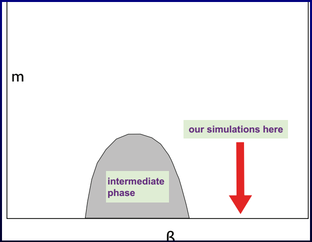
- Previously…
  - Likely: T. Appelquist et al, A. Deuzeman et al,A Hasenfratz et al, Y. Aoki et al
  - Not Likely: X.-Y. Jin et al, Z. Fodor et al
- Latest findings of our group:
  Finite Size Scaling Conformal scenario is Inconsistent and Suspect

Simulation Setup

- Action: Tree-level Symanzik-Improved gauge action with Staggered $N_f = 12$ Fundamental fermions
- HMC algorithm with multiple time scales and Omelyan integrator
- Autocorrelations monitored by time histories of Fermion condensate,plaquette and correlators
- $\beta = 6/g^2 = 2.20$, which is in the Weak Coupling regime



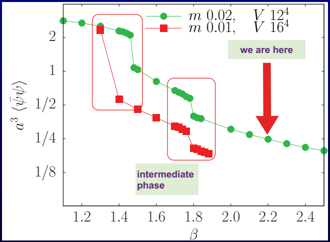
C. Schroeder et al



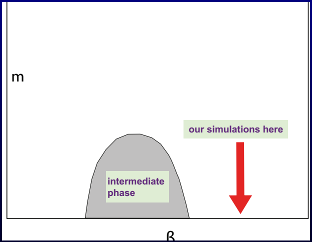
A. Hasenfratz et al

Simulation Setup

- Action: Tree-level Symanzik-Improved gauge action with Staggered $N_f = 12$ Fundamental fermions
- HMC algorithm with multiple time scales and Omelyan integrator
- Autocorrelations monitored by time histories of Fermion condensate,plaquette and correlators
- $\beta = 6/g^2 = 2.20$, which is in the Weak Coupling regime



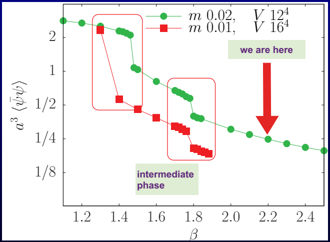
C. Schroeder et al



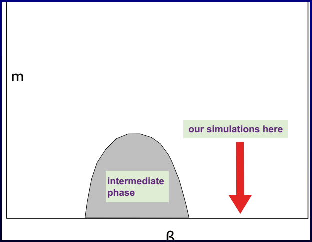
A. Hasenfratz et al

Simulation Setup

- Action: Tree-level Symanzik-Improved gauge action with Staggered $N_f = 12$ Fundamental fermions
- HMC algorithm with multiple time scales and Omelyan integrator
- Autocorrelations monitored by time histories of Fermion condensate,plaquette and correlators
- $\beta = 6/g^2 = 2.20$, which is in the Weak Coupling regime



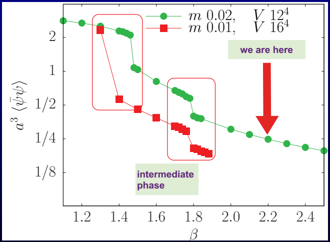
C. Schroeder et al



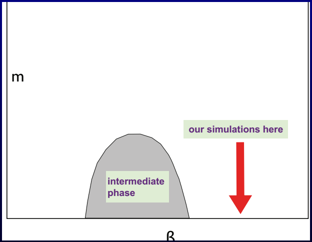
A. Hasenfratz et al

Simulation Setup

- Action: Tree-level Symanzik-Improved gauge action with Staggered $N_f = 12$ Fundamental fermions
- HMC algorithm with multiple time scales and Omelyan integrator
- Autocorrelations monitored by time histories of Fermion condensate,plaquette and correlators
- $\beta \equiv 6/g^2 = 2.20$, which is in the Weak Coupling regime



C. Schroeder et al



A. Hasenfratz et al

Simulation Setup

- Lattices used:($\sim 1000 - 2000$ Trajectories each)

L	T	m_q
48	96	0.0100, 0.0150
40	80	0.0100, 0.0150, 0.0200, 0.0250
32	64	0.0020, 0.0040, 0.0060, 0.0080, 0.0100, 0.0150, 0.0200, 0.0250
28	56	0.0150
24	48	0.0020, 0.0040, 0.0060, 0.0080, 0.0100, 0.0150, 0.0200, 0.0250, 0.0300, 0.0325, 0.0350
20	40	0.0020, 0.0040, 0.0060, 0.0080, 0.0100, 0.0150, 0.0200, 0.0250

Spectroscopy Analysis

- Double Jackknife to obtain good estimate on Covariance Matrix
  - Generate Jackknife ensembles of Correlators $\{C(t)\}_i$
  - Fit individual ensemble to obtain m_0^i or F_π^i using Covariance Matrix

$$C^i_{x_4 x'_4} = \sum_{j=1}^{N-1} \{m_{eff}(x_4)^i_j - \langle m_{eff}(x_4)\rangle_i\}\{m_{eff}(x'_4)^i_j - \langle m_{eff}(x'_4)\rangle_i\}$$

With χ^2 defined as:

$$(\chi^2)^i = \sum_{x_4=t_0}^{t_1} \sum_{x'_4=t_0}^{t_1} \{m_0^i - \langle m_{eff}(x_4)\rangle_i\}[(C^i)^{-1}]_{x_4 x'_4}\{m_0^i - \langle m_{eff}(x'_4)\rangle_i\}$$

(above expressions also apply to F_π^i)

Spectroscopy Analysis

- Double Jackknife to obtain good estimate on Covariance Matrix
  - Generate Jackknife ensembles of Correlators $\{C(t)\}_i$
  - Fit individual ensemble to obtain m_0^i or F_π^i using Covariance Matrix

$$C^i_{x_4x'_4} = \sum_{j=1}^{N-1} \{m_{eff}(x_4)^i_j - \langle m_{eff}(x_4)\rangle_i\}\{m_{eff}(x'_4)^i_j - \langle m_{eff}(x'_4)\rangle_i\}$$

With χ^2 defined as:

$$(\chi^2)^i = \sum_{x_4=t_0}^{t_1} \sum_{x'_4=t_0}^{t_1} \{m_0^i - \langle m_{eff}(x_4)\rangle_i\}[(C^i)^{-1}]_{x_4x'_4}\{m_0^i - \langle m_{eff}(x'_4)\rangle_i\}$$

(above expressions also apply to F_π^i)

Spectroscopy Analysis

- Double Jackknife to obtain good estimate on Covariance Matrix
  - Generate Jackknife ensembles of Correlators $\{C(t)\}_i$
  - Fit individual ensemble to obtain m_0^i or F_π^i using Covariance Matrix

$$C^i_{x_4x'_4} = \sum_{j=1}^{N-1} \{m_{eff}(x_4)^i_j - \langle m_{eff}(x_4)\rangle_i\}\{m_{eff}(x'_4)^i_j - \langle m_{eff}(x'_4)\rangle_i\}$$

With χ^2 defined as:

$$(\chi^2)^i = \sum_{x_4=t_0}^{t_1} \sum_{x'_4=t_0}^{t_1} \{m_0^i - \langle m_{eff}(x_4)\rangle_i\}[(C^i)^{-1}]_{x_4x'_4}\{m_0^i - \langle m_{eff}(x'_4)\rangle_i\}$$

(above expressions also apply to F_π^i)

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ, c_1, $c_{\exp}$, α, x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ, c_1, $c_{\exp}$, α, x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\text{exp}} (c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\text{exp}}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ, c_1, c_{exp}, α, x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv L m^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ, c_1, $c_{\exp}$, α, x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\mathrm{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\mathrm{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ, c_1, $c_{\exp}$, α, x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ , c_1 , $c_{\exp}$, α , x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ , c_1 , $c_{\exp}$, α , x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\mathrm{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\mathrm{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ , c_1 , $c_{\exp}$, α , x_{cut}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 1: Fitting scaling function with physical model

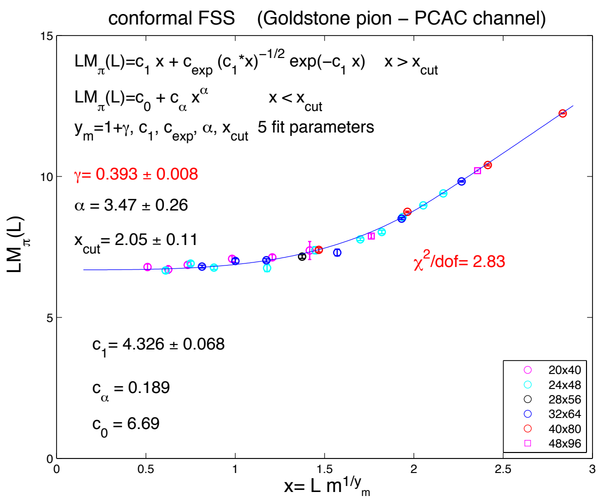
$$LM = c_1 x + c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}, \qquad x > x_{\text{cut}}$$
$$LM = c_0 + c_\alpha x^\alpha, \qquad x < x_{\text{cut}}$$

where $x \equiv Lm^{1/1+\gamma}$ and $M = M_\pi, M_\rho, M_N$ or F_π

- Conformal behavior:
  - $M \sim m^{1/1+\gamma}$ as $L \to \infty$ at fixed m
  - $M \sim L^{-1}$ as $m \to 0$
  - Other terms are finite size corrections
    - $c_{\exp}(c_\pi x)^{-1/2} e^{-c_\pi x}$: leading exponential correction from wrap-around effect of the lightest Goldstone pion state ($c_\pi = c_1$ for pions and fitted value becomes c_π of other channels)
    - $c_\alpha x^\alpha$: simplest ansatz, could be polynomials
  - Continuity and first derivative continuity imposed at x_{cut}
  - 5 fit parameters: γ , c_1 , $c_{\exp}$, α , x_{cut}

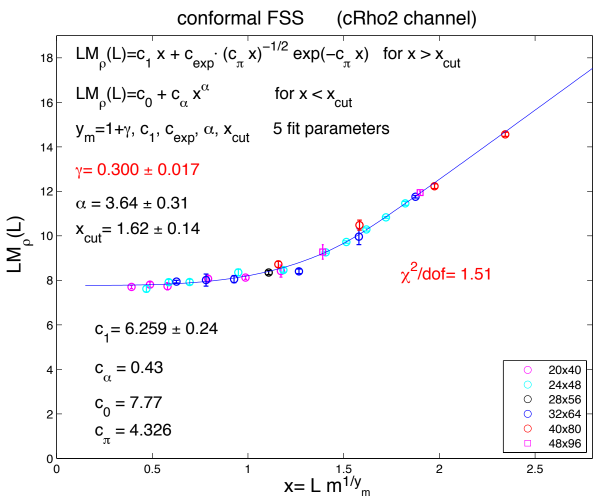
Fitting scaling function with physical model

- M_π



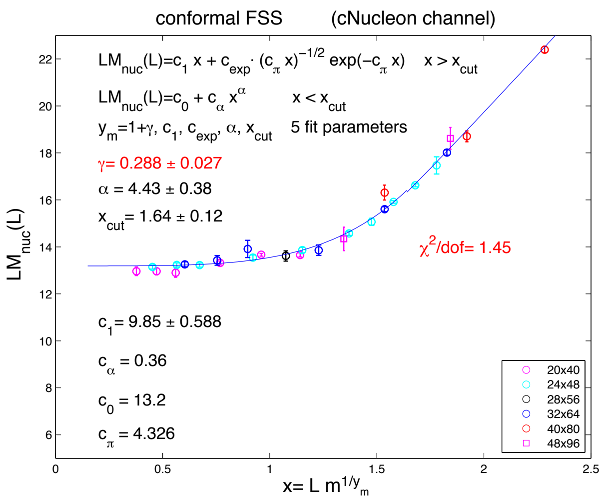
Fitting scaling function with physical model

- M_ρ



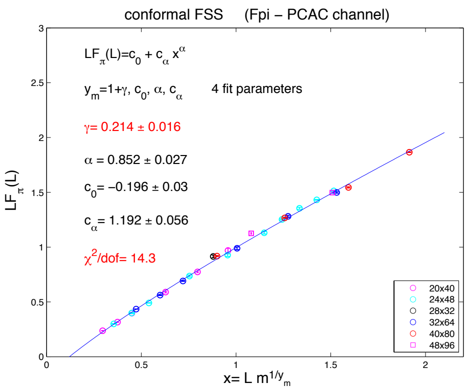
Fitting scaling function with physical model

- M_N



Fitting scaling function with physical model

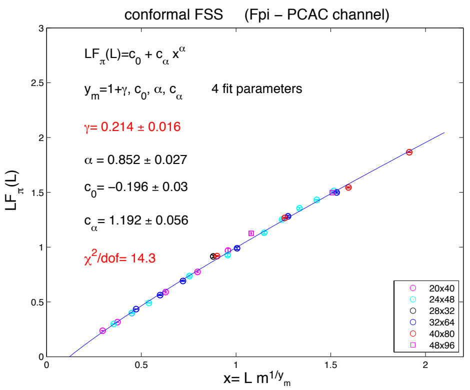
- F_π



- Only 4 parameters are kept because 5 parameter fit is not stable
- The unexpectedly curious behavior of the data set against conformal FSS remains unresolved.
- Question: What to be expected if conformal, e.g. $SU(2)$ Adjoint?

Fitting scaling function with physical model

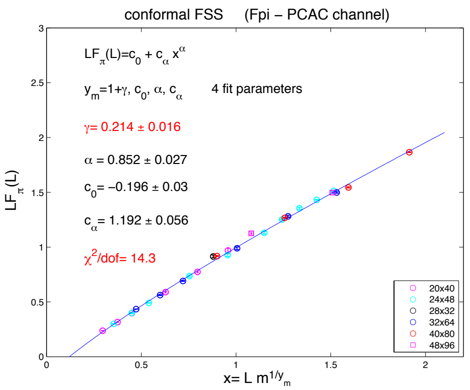
- F_π



- Only 4 parameters are kept because 5 parameter fit is not stable
- The unexpectedly curious behavior of the data set against conformal FSS remains unresolved.
- Question: What to be expected if conformal, e.g. $SU(2)$ Adjoint?

Fitting scaling function with physical model

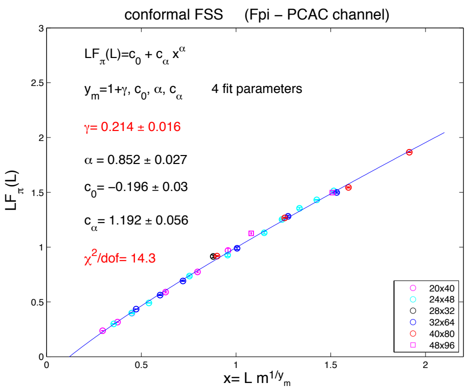
- F_π



- Only 4 parameters are kept because 5 parameter fit is not stable
- The unexpectedly curious behavior of the data set against conformal FSS remains unresolved.
- Question: What to be expected if conformal, e.g. $SU(2)$ Adjoint?

Fitting scaling function with physical model

- F_π



- Only 4 parameters are kept because 5 parameter fit is not stable
- The unexpectedly curious behavior of the data set against conformal FSS remains unresolved.
- Question: What to be expected if conformal, e.g. $SU(2)$ Adjoint?

Fitting scaling function with physical model

- Summary:

Quantity	γ	χ^2/dof
M_π	0.393(8)	2.83
M_ρ	0.300(17)	1.51
M_N	0.288(27)	1.45
F_π	0.214(16)	14.3

- The composite particle masses in several quantum number channels can be reasonably fitted with conformal scaling functions
- BUT γ's are incompatible across different channels.
  $\Rightarrow$ **Global conformal FSS fit with identical γ will fail**
- Conformal fit of F_π unexpectedly failed

Fitting scaling function with physical model

- Summary:

Quantity	γ	χ^2/dof
M_π	0.393(8)	2.83
M_ρ	0.300(17)	1.51
M_N	0.288(27)	1.45
F_π	0.214(16)	14.3

  - The composite particle masses in several quantum number channels can be reasonably fitted with conformal scaling functions
  - BUT γ's are incompatible across different channels.
    $\Rightarrow$ **Global conformal FSS fit with identical γ will fail**
  - Conformal fit of F_π unexpectedly failed

Fitting scaling function with physical model

- Summary:

Quantity	γ	χ^2/dof
M_π	0.393(8)	2.83
M_ρ	0.300(17)	1.51
M_N	0.288(27)	1.45
F_π	0.214(16)	14.3

  - The composite particle masses in several quantum number channels can be reasonably fitted with conformal scaling functions
  - BUT γ's are incompatible across different channels.
    $\Rightarrow$ **Global conformal FSS fit with identical γ will fail**
  - Conformal fit of F_π unexpectedly failed

Fitting scaling function with physical model

- Summary:

Quantity	γ	χ^2/dof
M_π	0.393(8)	2.83
M_ρ	0.300(17)	1.51
M_N	0.288(27)	1.45
F_π	0.214(16)	14.3

  - The composite particle masses in several quantum number channels can be reasonably fitted with conformal scaling functions
  - BUT γ's are incompatible across different channels.
    $\Rightarrow$ **Global conformal FSS fit with identical γ will fail**
  - Conformal fit of F_π unexpectedly failed

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain γ_{min} that minimizes $\chi^2(\gamma)/\text{dof}$
- The range of γ that reduce/increase χ^2/dof by 1 is regarded as the error estimate of γ_{min}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain $\gamma_{\rm min}$ that minimizes $\chi^2(\gamma)/{\rm dof}$
- The range of γ that reduce/increase $\chi^2/{\rm dof}$ by 1 is regarded as the error estimate of $\gamma_{\rm min}$

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain $\gamma_{\rm min}$ that minimizes $\chi^2(\gamma)/{\rm dof}$
- The range of γ that reduce/increase $\chi^2/{\rm dof}$ by 1 is regarded as the error estimate of $\gamma_{\rm min}$

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain γ_{min} that minimizes $\chi^2(\gamma)/\text{dof}$
- The range of γ that reduce/increase χ^2/dof by 1 is regarded as the error estimate of γ_{min}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain γ_{min} that minimizes $\chi^2(\gamma)/\text{dof}$
- The range of γ that reduce/increase χ^2/dof by 1 is regarded as the error estimate of γ_{min}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain γ_{min} that minimizes $\chi^2(\gamma)/\mathrm{dof}$
- The range of γ that reduce/increase χ^2/dof by 1 is regarded as the error estimate of γ_{min}

Fitting under conformal hypothesis with Finite Size Scaling study

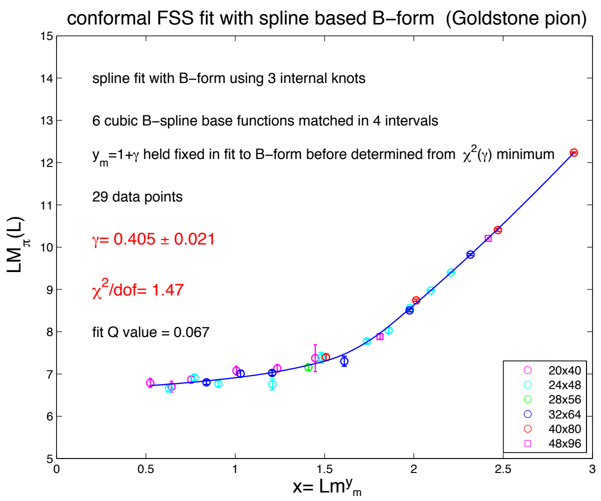
- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain γ_{min} that minimizes $\chi^2(\gamma)/\text{dof}$
- The range of γ that reduce/increase χ^2/dof by 1 is regarded as the error estimate of γ_{min}

Fitting under conformal hypothesis with Finite Size Scaling study

- Approach 2: Fitting general scaling function using B-form spline functions
- Vary γ and perform Spline Fits at fixed γ's:
  - Piecewise Polynomial fitting
  - Non-uniform B-form used
  - 3 knots used
  - 6 cubic spline functions
- Obtain $\gamma_{\rm min}$ that minimizes $\chi^2(\gamma)/{\rm dof}$
- The range of γ that reduce/increase $\chi^2/{\rm dof}$ by 1 is regarded as the error estimate of $\gamma_{\rm min}$

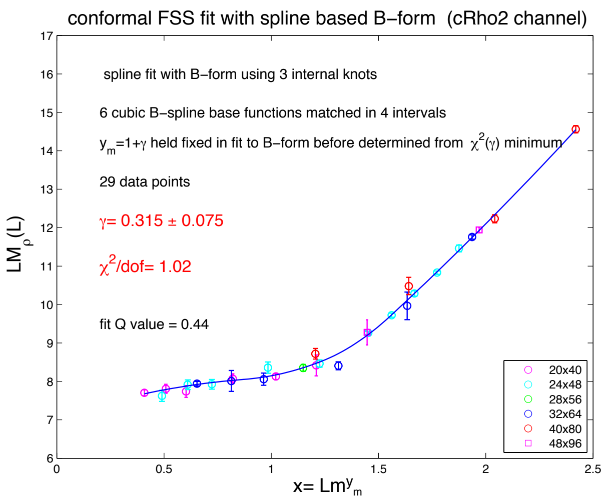
Fitting general scaling function using B-form spline functions

- M_π



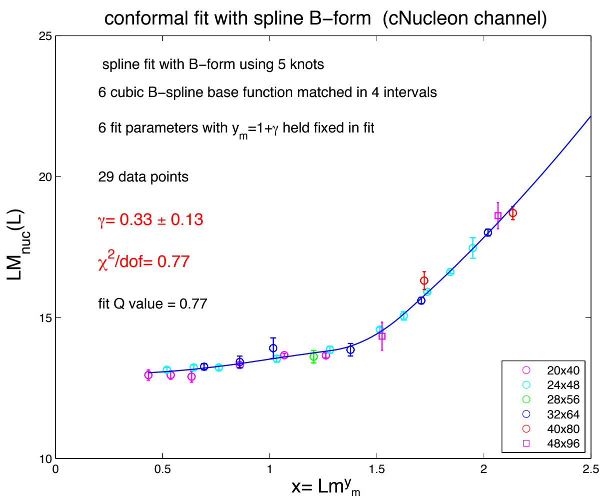
Fitting general scaling function using B-form spline functions

- M_ρ



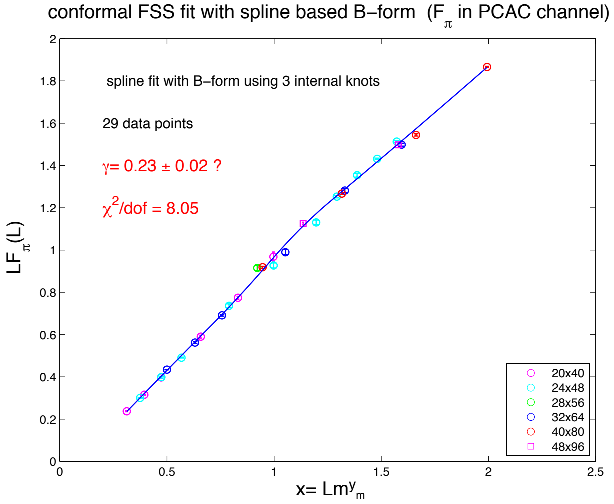
Fitting general scaling function using B-form spline functions

- M_N



Fitting general scaling function using B-form spline functions

- F_π



Fitting general scaling function using B-form spline functions

- Summary

Quantity	γ	χ^2/dof
M_π	0.405(21)	1.47
M_ρ	0.315(75)	1.02
M_N	0.33(13)	0.77
F_π	0.23(2)	8.05

- M_π improved as expected
- Tension of γ across channels decreased, but still problematic
- Fit to F_π remains unacceptable

Fitting general scaling function using B-form spline functions

- Summary

Quantity	γ	χ^2/dof
M_π	0.405(21)	1.47
M_ρ	0.315(75)	1.02
M_N	0.33(13)	0.77
F_π	0.23(2)	8.05

- M_π improved as expected
- Tension of γ across channels decreased, but still problematic
- Fit to F_π remains unacceptable

Fitting general scaling function using B-form spline functions

- Summary

Quantity	γ	χ^2/dof
M_π	0.405(21)	1.47
M_ρ	0.315(75)	1.02
M_N	0.33(13)	0.77
F_π	0.23(2)	8.05

- M_π improved as expected
- Tension of γ across channels decreased, but still problematic
- Fit to F_π remains unacceptable

Fitting general scaling function using B-form spline functions

- Summary

Quantity	γ	χ^2/dof
M_π	0.405(21)	1.47
M_ρ	0.315(75)	1.02
M_N	0.33(13)	0.77
F_π	0.23(2)	8.05

- M_π improved as expected
- Tension of γ across channels decreased, but still problematic
- Fit to F_π remains unacceptable

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing

Conclusion

- The extended study of the spectroscopy of $N_f = 12$ Fundamental shows problems in the conformal scenario, consistent with our previous findings
- Outstanding Problem:
  - Scaling violation effect may fake scaling with inconsistent gamma values, but unlikely to be the only effect $\Rightarrow$ Leading scaling violation effects has to be analyzed
  - Can the strongly squeezed wave function effects in chirally broken scenario explain the good scaling form in separate quantum number channels?
- Bottom line:
  - Conformal scenario remains inconsistent and suspect in our analysis
  - The issues are not settled yet. More definitive analyses are needed and ongoing