New fermion discretizations and their applications

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Why New Fermions ?

(1) Further understanding on lattice field theory

(2) Improvement of lattice QCD simulations

Lattice fermion improvement

Wilson : O(a) errors & bad chiral properties

- Smeared-link clover (UV filtered, O(a) improved)

  DeGrand, Hasenfratz, Kovacs, MILC(98), BMW's intensive works
- Twisted-mass (unphysical zero-mode removed, O(a) improved)

  Frezzotti, et.al. ALPHA(00), ETM's intensive works

etc....

Staggered : taste breaking at O(a^2)

- HISQ (fat-link & O(a^2) Symanzik) Follana, et.al.(06), MILC's intensive works
- HYP Hasenfratz, Knechtli(01) ◆ Fat7 Orginos, Toussaint et.al.(99) ◆ Asqtad Lepage (98)

etc....

Domain-wall, Overlap : Numerical cost

- Fixed topology (kernel zero-mode removed, locality) Fukaya, et.al. (06), JLQCD intensive works
- Reweighting (enlarge 5th size, chiral properties) Hasenfratz et.al.(08), Ishikawa, et.al. (10)
- Hypercube overlap (perfect kernel→locality, scaling) Bietenholtz, et.al. (99)(12) etc....

New setups can contribute ?

1. Flavored mass

Adams (09) cf.) Golterman, Smit (84)

Staggered overlap → CPU time reduction (overlap)?

Taste symmetry improved (staggered)?

2. Central branch

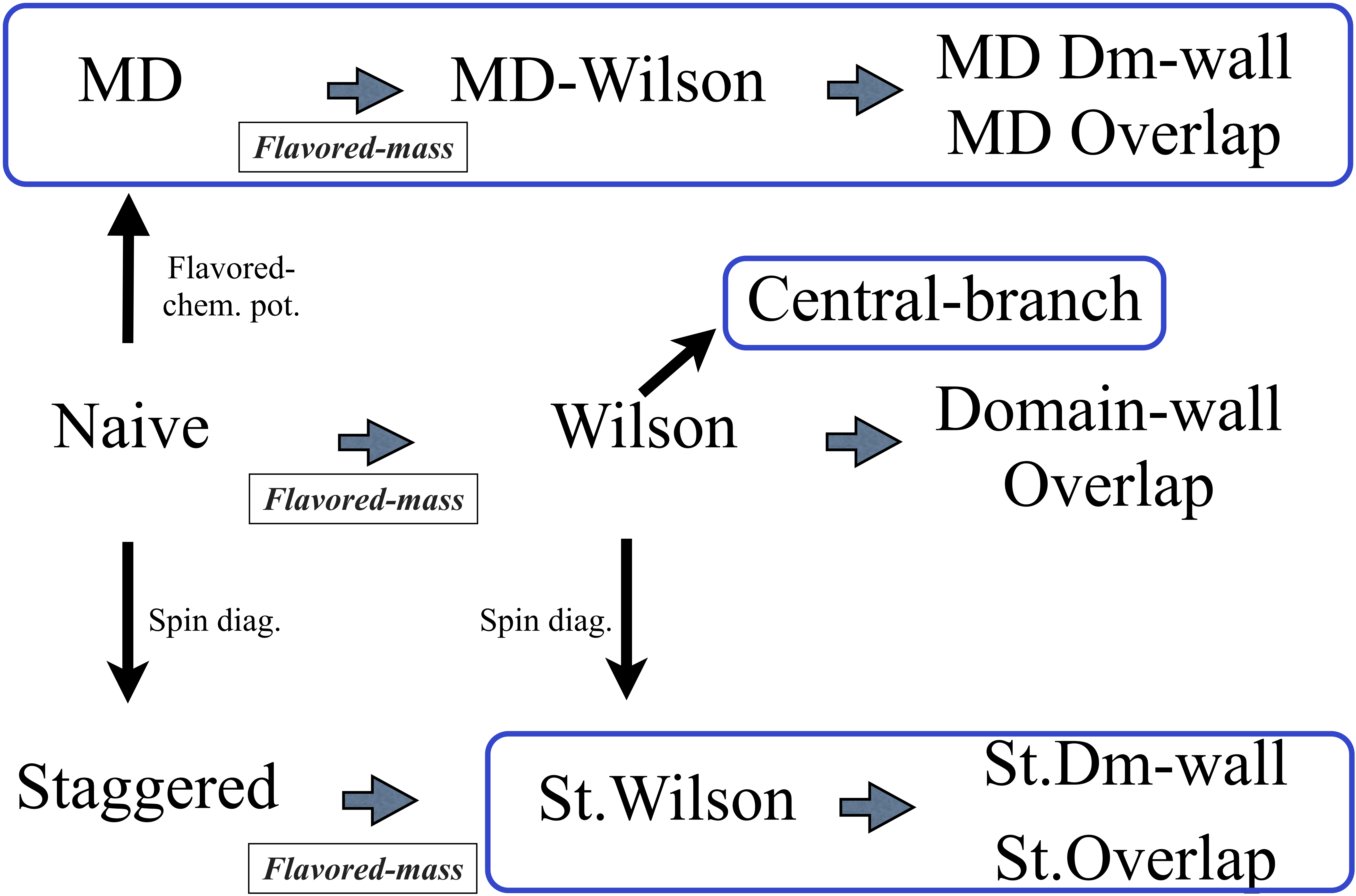
Kimura, Komatsu,TM, Noumi, Torii, Aoki (11)

Wilson w/o additive renorm. → Chiral symmetry (No fine-tuning?)
O(a) improved?

3. Minimal-doubling

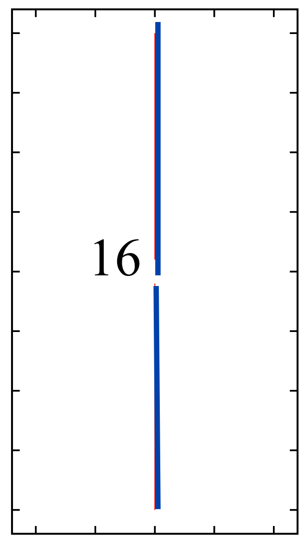
Karsten(81) Wilczek(87) Creutz(07) Borici(07)

Chiral two-flavor w/ ultra locality → Better chiral property?


MD
Flavored-mass
MD-Wilson
MD Dm-wall
MD Overlap
Flavored-
chem. pot.
Central-branch
Naive
Flavored-mass
Wilson
Domain-wall
Overlap
Spin diag.
Spin diag.
Staggered
Flavored-mass
St.Wilson
St.Dm-wall
St.Overlap

1. Flavored mass

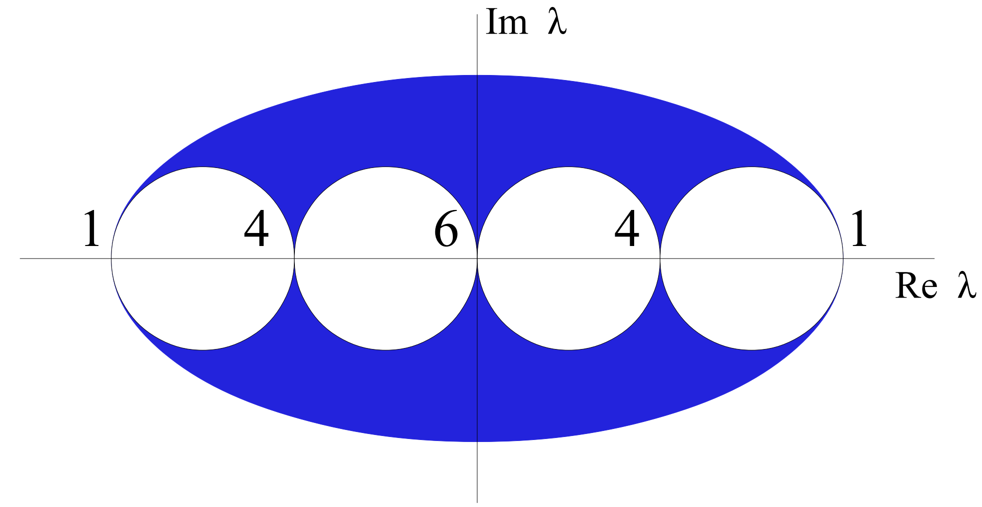
Naive



$$\sum_{\mu} C_{\mu}$$

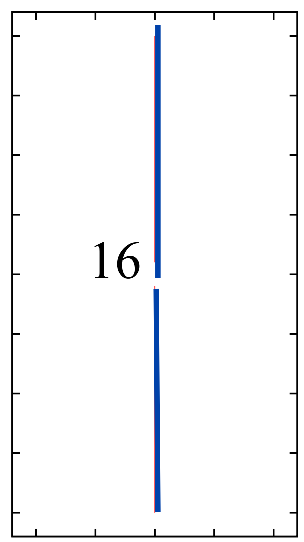
$$C_\mu = (T_{+\mu} + T_{-\mu})/2$$
$$T_{\pm\mu}\psi_n = U_{n,\pm\mu}\psi_{n\pm\mu}$$

Wilson



U(4)×U(4)

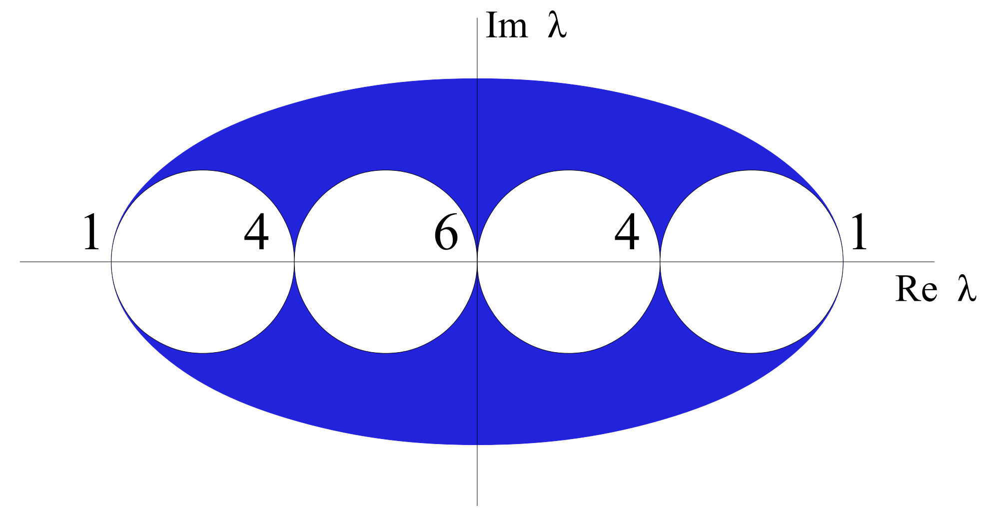
Naive



$$\sum_\mu C_\mu$$

$$C_\mu = (T_{+\mu} + T_{-\mu})/2$$
$$T_{\pm\mu}\psi_n = U_{n,\pm\mu}\psi_{n\pm\mu}$$

Wilson



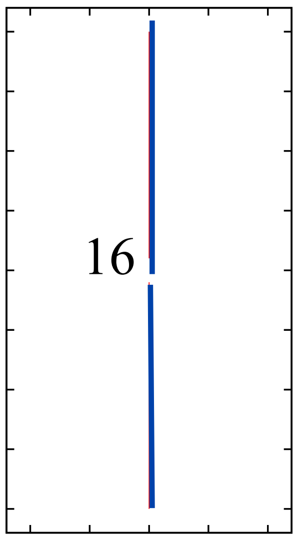
U(4)×U(4)

$$\Gamma_X^{(+)} \in \left\{ \mathbf{1}_4\,,\ (-1)^{n_1+\ldots+n_4}\gamma_5\,,\ (-1)^{\check{n}_\mu}\gamma_\mu\,,\ (-1)^{n_\mu} i\gamma_\mu\gamma_5\,,\ (-1)^{n_{\mu,\nu}}\frac{[\gamma_\mu\,,\gamma_\nu]}{2} \right\}$$

$$\Gamma_X^{(-)} \in \left\{ (-1)^{n_1+\ldots+n_4}\mathbf{1}_4\,,\ \gamma_5\,,\ (-1)^{n_\mu}\gamma_\mu\,,\ (-1)^{\check{n}_\mu}\gamma_\mu\gamma_5\,,\ (-1)^{\check{n}_{\mu,\nu}}\frac{[\gamma_\mu\,,\gamma_\nu]}{2} \right\}$$

$$\psi_n \to \psi'_n = \exp\left[\mathrm{i}\sum_X \left(\theta_X^{(+)}\Gamma_X^{(+)} + \theta_X^{(-)}\Gamma_X^{(-)}\right)\right]\psi_n\,, \qquad \bar\psi_n \to \bar\psi'_n = \bar\psi_n \exp\left[\mathrm{i}\sum_X \left(-\theta_X^{(+)}\Gamma_X^{(+)} + \theta_X^{(-)}\Gamma_X^{(-)}\right)\right]$$

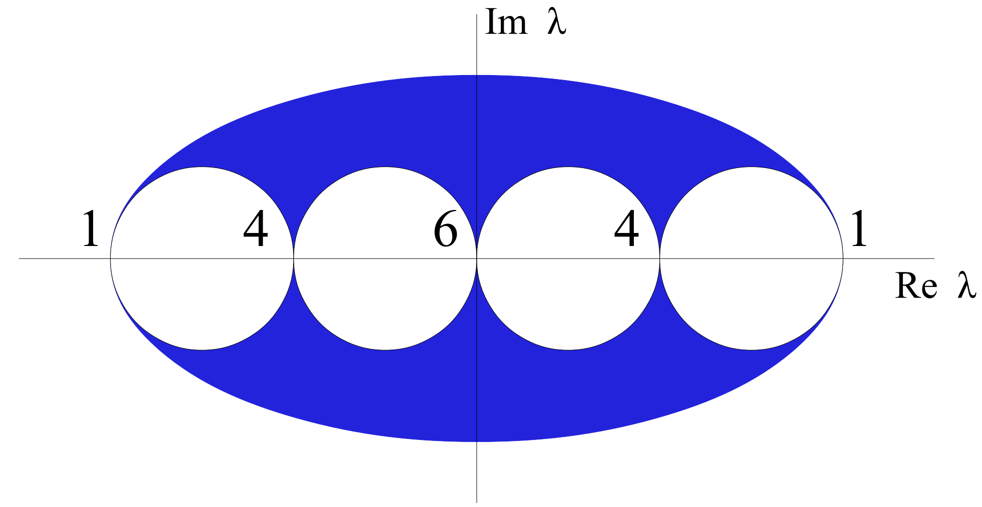
Naive



$$\sum_\mu C_\mu$$

$$C_\mu = (T_{+\mu} + T_{-\mu})/2$$
$$T_{\pm\mu}\psi_n = U_{n,\pm\mu}\psi_{n\pm\mu}$$

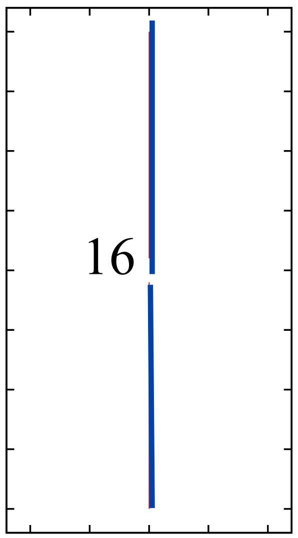
Wilson



U(4)×U(4) → U(1)

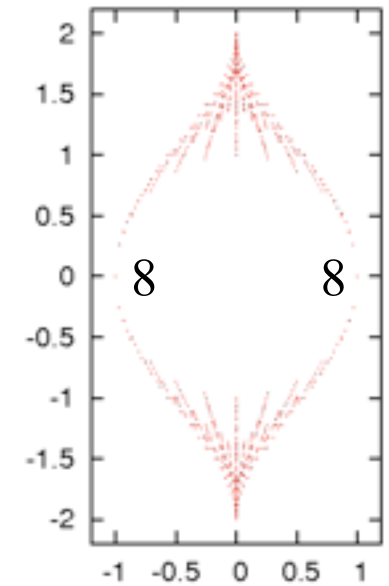
$$\Gamma_X^{(+)} \in \left\{ \mathbf{1}_4 \, , \qquad \qquad \right\}$$
$$\Gamma_X^{(-)} \in \left\{ \qquad \qquad \right\}$$

Naive		Wilson'



$$\sum_{sym.} C_1 C_2 C_3 C_4$$

$$C_\mu = (T_{+\mu} + T_{-\mu})/2$$
$$T_{\pm\mu}\psi_n = U_{n,\pm\mu}\psi_{n\pm\mu}$$



Kimura (10)

U(4)×U(4) → U(2)×U(2) TM, Lehner(12)

$$\Gamma_X^{(+)} \in \left\{ \mathbf{1}_4\,,\ (-1)^{n_1+\ldots+n_4}\gamma_5\,,\qquad (-1)^{n_{\mu,\nu}}\frac{[\gamma_\mu\,,\gamma_\nu]}{2} \right\}$$

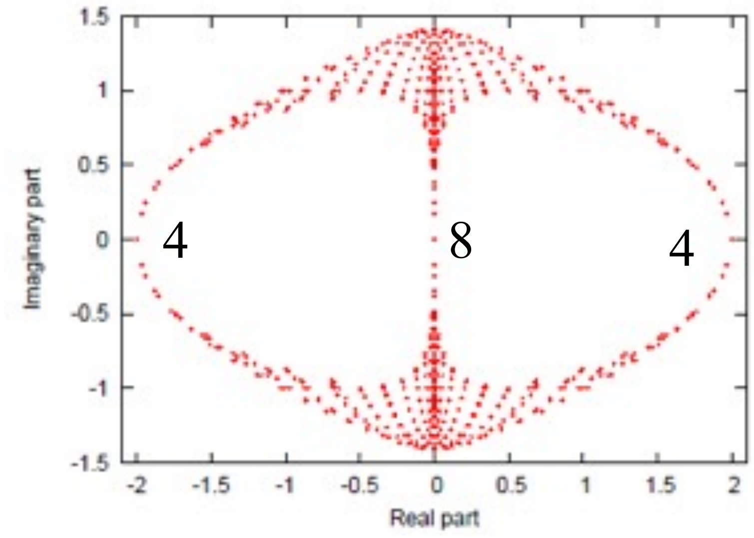
$$\Gamma_X^{(-)} \in \left\{ \qquad \right\}$$

◆Naive flavored mass

Creutz, Kimura, TM (10)

$$M_{\mathrm{V}} = \sum_{\mu} C_{\mu},$$ *Vector (1-link)*

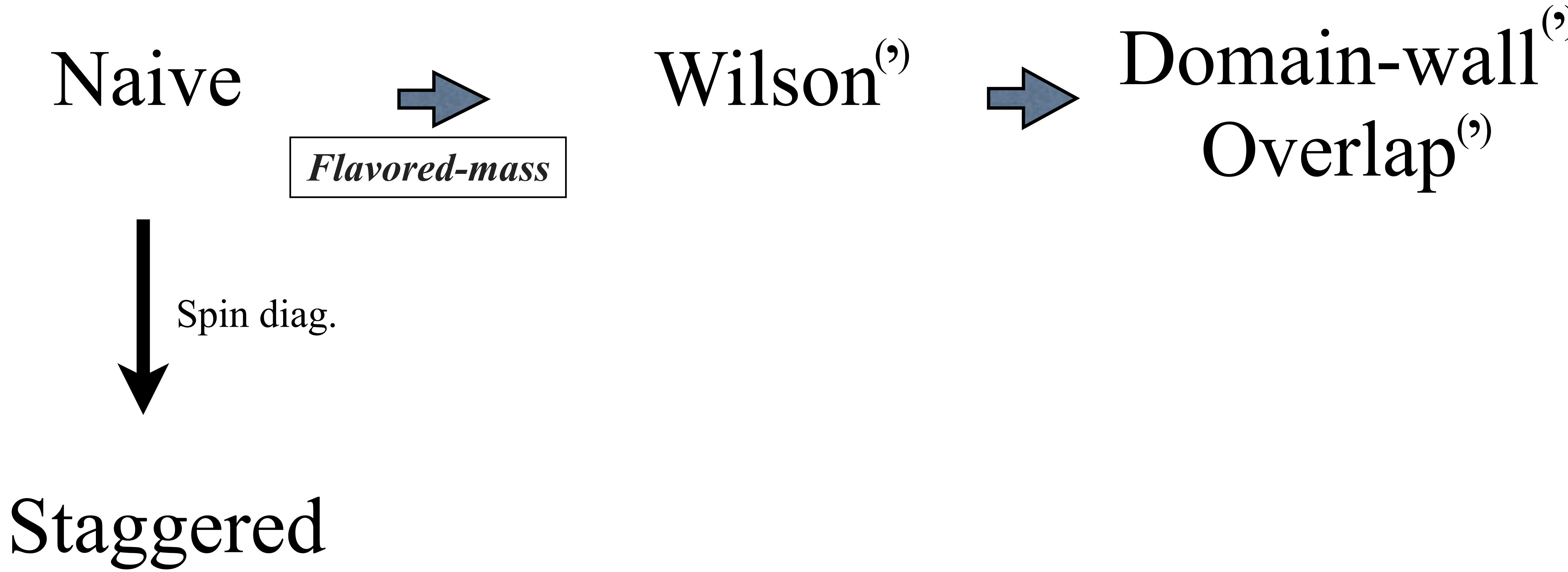
$$M_{\mathrm{T}} = \sum_{perm.} \sum_{sym.} C_{\mu} C_{\nu},$$ *Tensor (2-link)* →

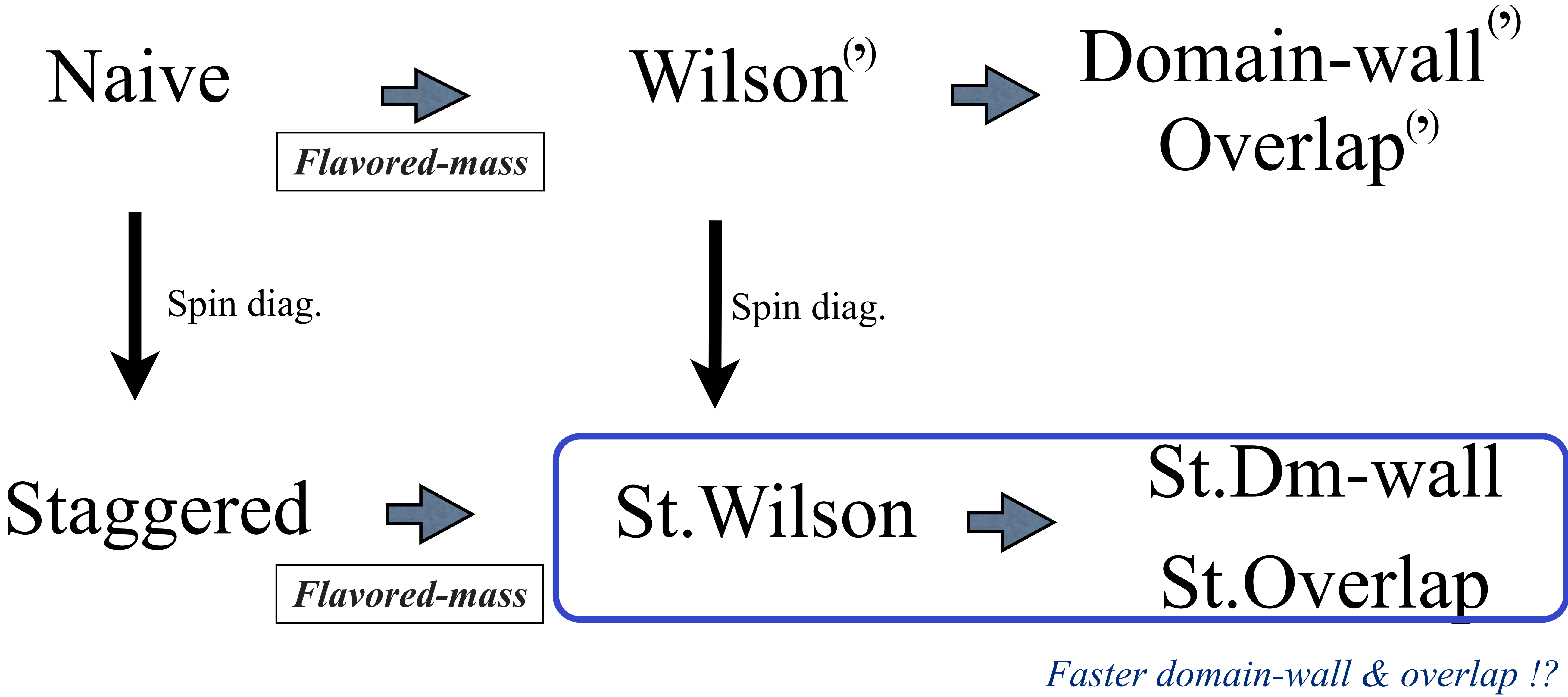


$$M_{\mathrm{A}} = \sum_{perm.} \sum_{sym.} \prod_{\nu} C_{\nu},$$ *Axial-V (3-link)*

$$M_{\mathrm{P}} = \sum_{sym.} \prod_{\mu=1}^{4} C_{\mu},$$ *Pseudo-S (4-link)*

- *gamma-5 hermiticity*
- *2nd derivative terms* $\sum_{n} \bar{\psi}_n (M_P - 1) \psi_n \;\rightarrow\; -a \int d^4x \bar{\psi}(x) D_{\mu}^2 \psi(x) + O(a^2)$
- *Cousins of Wilson fermion*


Naive
Flavored-mass
Wilson(')
Domain-wall(')
Overlap(')
Spin diag.
Staggered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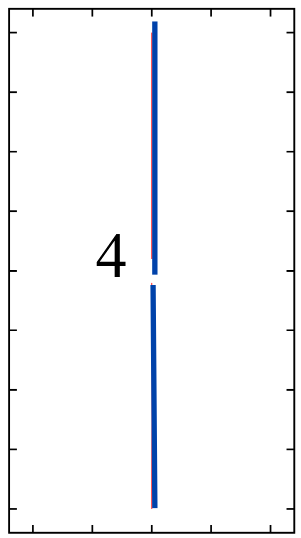
St.Dm-wall
St.Overlap
Faster domain-wall & overlap !?

◆Flavored mass Golterman, Smit (1984) Adams(2009)

Staggered

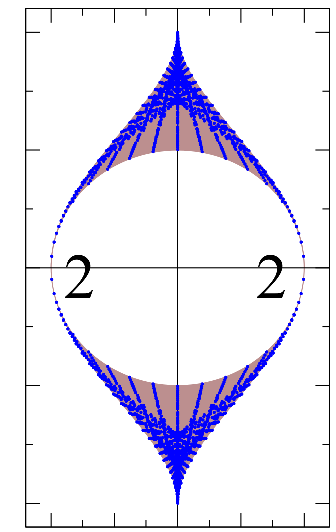


$$\epsilon_x \sum_{sym.} \eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4$$

$$\sim (\mathbf{1} \otimes \xi_5) + O(a)$$

St. Wilson Adams (09)

ξ_5=-1



ξ_5=+1

de Forcrand, Kurkela, Panero(2012)

$$\{C_0,\, \Xi_\mu,\, I_s,\, R_{\mu\nu}\} \;\times\; \{U^\epsilon(1)\}_{m=0} \;\Rightarrow\; \{C_0, \Xi'_\mu, R_{\mu\nu}\}$$

- Practical form

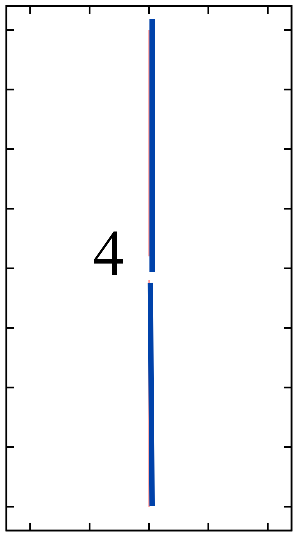
$$\eta_\mu D_\mu + \underline{r(1+M_\mathcal{A})} + \underline{m} \qquad M_\mathcal{A} = \epsilon_x \sum_{sym.} \eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4$$

Wilson-like term — mass parameter

With this mass shift →
ξ_5=-1 → physical sector : ℓ
ξ_5=+1 → decoupled sector : h

◆ Flavored mass Golterman, Smit (1984) Adams(2009)

Staggered

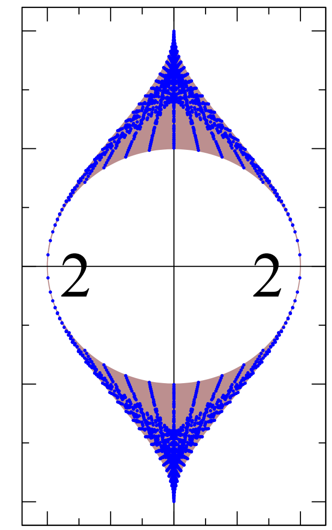


$$\boxed{\epsilon_x \sum_{sym.} \eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4}$$

$$\sim (\mathbf{1} \otimes \xi_5) + O(a)$$

St. Wilson Adams (09)

ξ_5=-1



ξ_5=+1

de Forcrand, Kurkela, Panero(2012)

$$\{C_0,\, \Xi_\mu,\, I_s,\, R_{\mu\nu}\} \quad \times \quad \{U^\epsilon(1)\}_{m=0}$$

$$\{C_0, \Xi'_\mu, R_{\mu\nu}\}$$

C P SW_4

No parameter tuning for Lorentz symmetry !

- Practical form

$$\eta_\mu D_\mu + \underline{r(1+M_{\mathcal{A}})} + \underline{m} \qquad M_{\mathcal{A}} = \epsilon_x \sum_{sym.} \eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4$$

Wilson-like term mass parameter

With this mass shift $\rightarrow$
ξ_5=-1 $\rightarrow$ physical sector : ℓ
ξ_5=+1 $\rightarrow$ decoupled sector : h

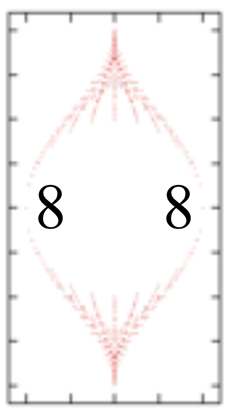
◆Staggered-Wilson (Domain-wall, Overlap)

How to apply : As Wilson → Mass parameter tuning required
As Domain-wall → 5th dimension introduced
As Overlap → Overlap formula with StWil kernel

- Spin diagonalization Creutz, Kimura, TM (10)

$$\bar{\psi}_x C_1 C_2 C_3 C_4 \psi_x \quad \rightarrow \quad \pm\bar{\chi}_x(\epsilon\eta_1\eta_2\eta_3\eta_4 C_1 C_2 C_3 C_4)\chi_x$$

$M_{\rm P}$



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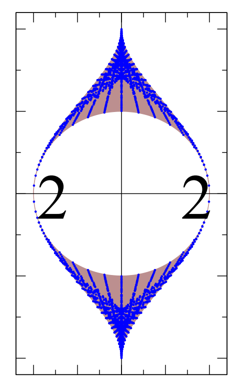


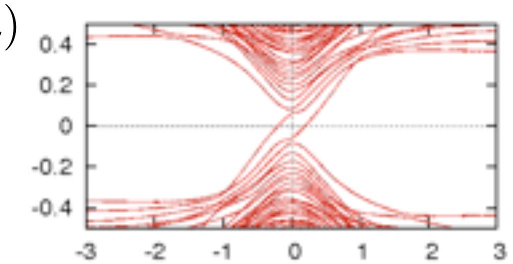
Staggered flavored-mass

- Index theorem Adams (09) Follana, Azcoiti, Di Carlo, Vaquero(11)

$$H_{SW}(m) = \epsilon(D_{SW} - m)$$

Index(D_{sw}) = - Spectral flow(H_{sw})

$\lambda(m)$

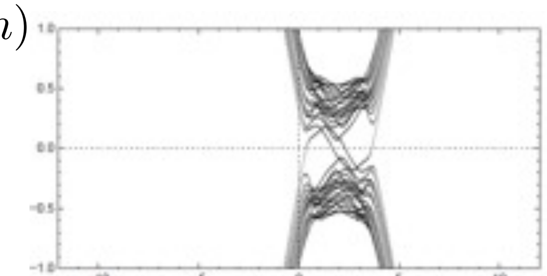


m

Spectral flow

cf.) Edwards, Heller, Narayanan (98)

$\lambda(m)$



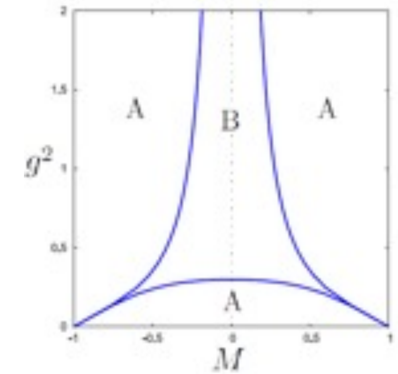
m

$$H_W(m) = \gamma_5(D_W - m)$$

- Aoki phase Creutz, Kimura, TM(11) TM, Nakano, Kimura, Ohnishi(12)

Strong-coupling lattice QCD & 2d model
→ Implies parity-flavor broken phase

ChPT analysis required → 1st or 2nd order ?



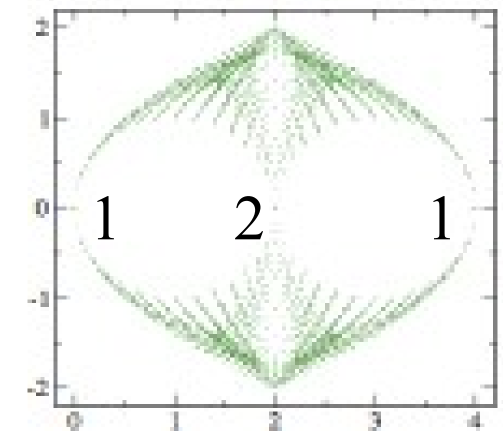
2d Aoki phase

cf.) Lee, Sharpe (99),
Aubin, Wang (04),
Cheng, et.al. (11)
"Possible Aoki phase"

- Another type (Hoelbling type) Hoelbling (10), de Forcrand, Kurkela, Panero (10)

$$\sum_{\mu\nu=12,34} i\eta_{\mu\nu}\eta_{\mu}\epsilon_{\mu\nu}(C_\mu C_\nu + C_\nu C_\mu)$$

$$\sim (\mathbf{1}\otimes i(\sigma_{12}+\sigma_{34})) + O(a)$$



Hoelbling (10)

$$\{C_0,\, \Xi_\mu,\, I_s,\, R_{\mu\nu}\} \times \{U^\epsilon(1)\}_{m=0} \Rightarrow \{C_T,\, \Xi'_\mu,\, R_{12}, R_{34}, R_{13}R_{42}\}$$

$C_T \to C$, $\Xi'_\mu \to P$

Rotation symmetry broken !

→ Requires fine-tuning of parameters for Lorentz sym. continuum Sharpe (12)

Let's focus only on Adams type.

§ Potential problems of $\eta_\mu D_\mu + r(1 + M_{\mathcal{A}})$

1. Lorentz symmetry restored ?

Euclidian Lorentz symmetry, C, P, T

$\rightarrow$ likely to be restored from $\{C_0, \Xi'_\mu, R_{\mu\nu}\}$ *No parameter tuning!*

2. Multi-link terms require numerical costs ?

(i) 24 terms for symmetric sum, (ii) 4 transporters

VS

One component fermion (small matrix size)

§ Potential advantages of $\eta_\mu D_\mu + r(1 + M_{\mathcal{A}})$

1. could reduce numerical costs in 2-flavor overlap

One-component action $\rightarrow$ *Small matrix size of propagator*

2. could reduce influence of taste-breaking for 2-flavor

Staggered sym. vs *4 tastes*

Halved staggered sym. vs *2 tastes*

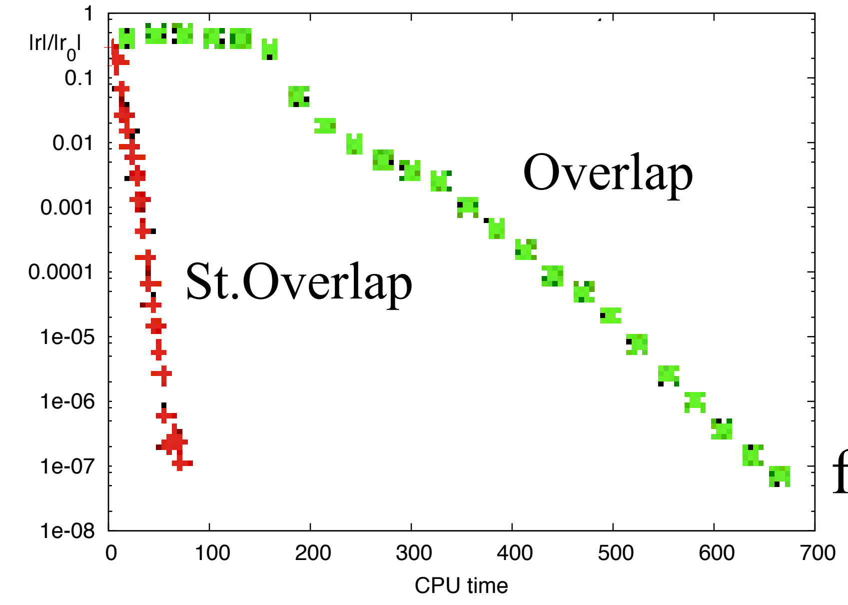
The situation should be different, but better or worse ?

1. Numerical costs reduced ?

de Forcrand, Kurkela, Panero(2011)

Staggered-Overlap Dirac propagator

CG solver with MxV (12^4, m=0.1)



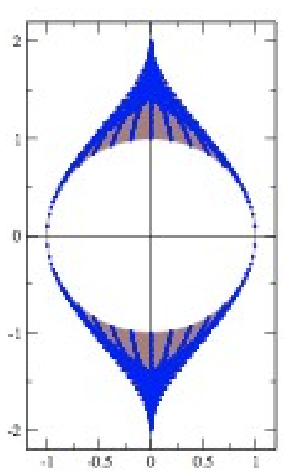
free

◆Small matrix size

Requires fewer Matrix-Vector multiplications for sign function !

◆4-link hopping terms

Gauge fluctuation is raised to 4th power !
→ splitting of two branches reduced



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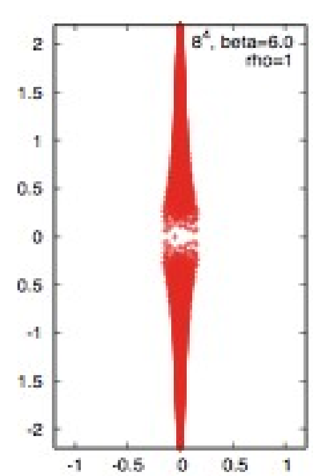
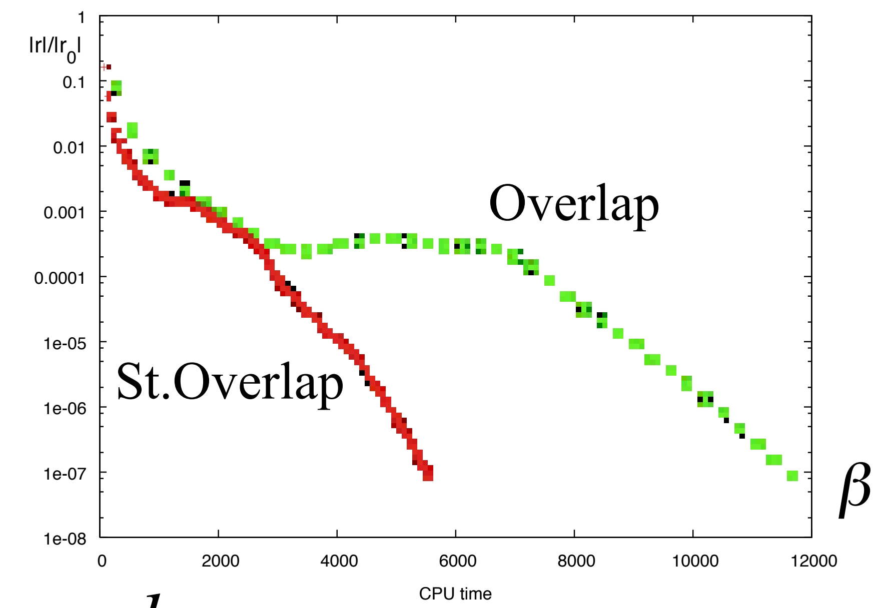


$\beta = 6$



$\beta = 6$

CPU

Staggered-Wilson is better as an overlap kernel, but not much better.

2. How about taste breaking ? Sharpe (12)

Pion spectrum

§ Staggered $\{C_0, \Xi_\mu, I_s, R_{\mu\nu}\} \times \{U^\epsilon(1)\}_{m=0}$ ➡ $\{C_0, \Xi_j, I_s, R_{ij}\}$
Transfer-matrix sym.

➡ classify 15 pseudoscalar operators Golterman (1986)

$\mathbf{1}$: $\xi_4,\ \xi_{45},\ \xi_5,$

$\mathbf{3}$: $\xi_i,\ \xi_{i5},\ \xi_{ij}\ \xi_{i4}$

7 irreps

cf.) ChPT by Lee, Sharpe (1999)

4 irreps of SO(4) upto $O(a^4),\ O(a^2m)\ \ O(a^2p^2)$

$\mathbf{1}$: $\xi_5,$

$\mathbf{4}$: $\xi_\mu,\ \xi_{\mu5},$

$\mathbf{6}$: $\xi_{\mu\nu}.$

§ Staggered-Wilson $\boxed{\{C_0, \Xi'_\mu, R_{\mu\nu}\}}$ ➡ $\{C_0, \Xi'_j, R_{ij}\}$

Irreps mix in ξ_5 pairs

Physical sector

$\mathbf{1}$ & ξ_5 $\longrightarrow$ $\bar{\ell}\ell\ ,\ \bar{h}h$ $\longrightarrow$ $\boxed{\bar{\ell}(\gamma_5\otimes\mathbf{1})\ell}$ η'

➡ ξ_4 & ξ_{45} $\longrightarrow$ $\bar{\ell}h\ ,\ \bar{h}\ell$

ξ_{i4} & ξ_{i45} $\longrightarrow$ $\bar{\ell}\sigma_j\ell\ ,\ \bar{h}\sigma_j h$ $\longrightarrow$ $\boxed{\bar{\ell}(\gamma_5\otimes\sigma_i)\ell}$ $\pi_0,\ \pi_\pm$ States in 3d irrep

ξ_i & ξ_{i5} $\longrightarrow$ $\bar{\ell}\sigma_j h\ ,\ \bar{h}\sigma_j\ell$

cf.) SU(2) in ChPT
potential upto $O(a^4),\ O(a^2m)$

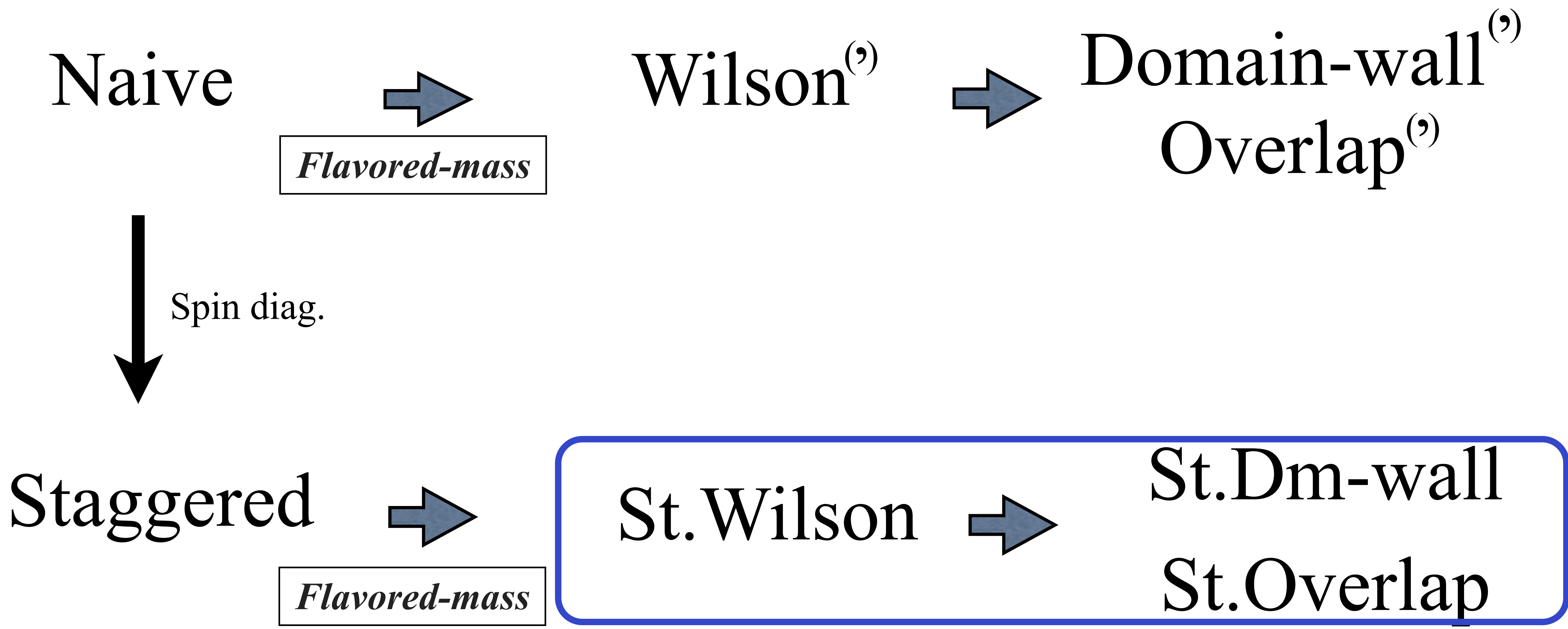
Discrete symmetries are sufficient for degenerate pion triplet!

◆ Short summary

- Adams fermion will work as 2-flavor Wilson.

- Taste-breaking exists, but small enough to have degenerate pion triplet. How about other mesons and baryons?

- Further study is needed to reveal numerical merit or demerit. Usual improvement works ? (Fixed topology, smearing)



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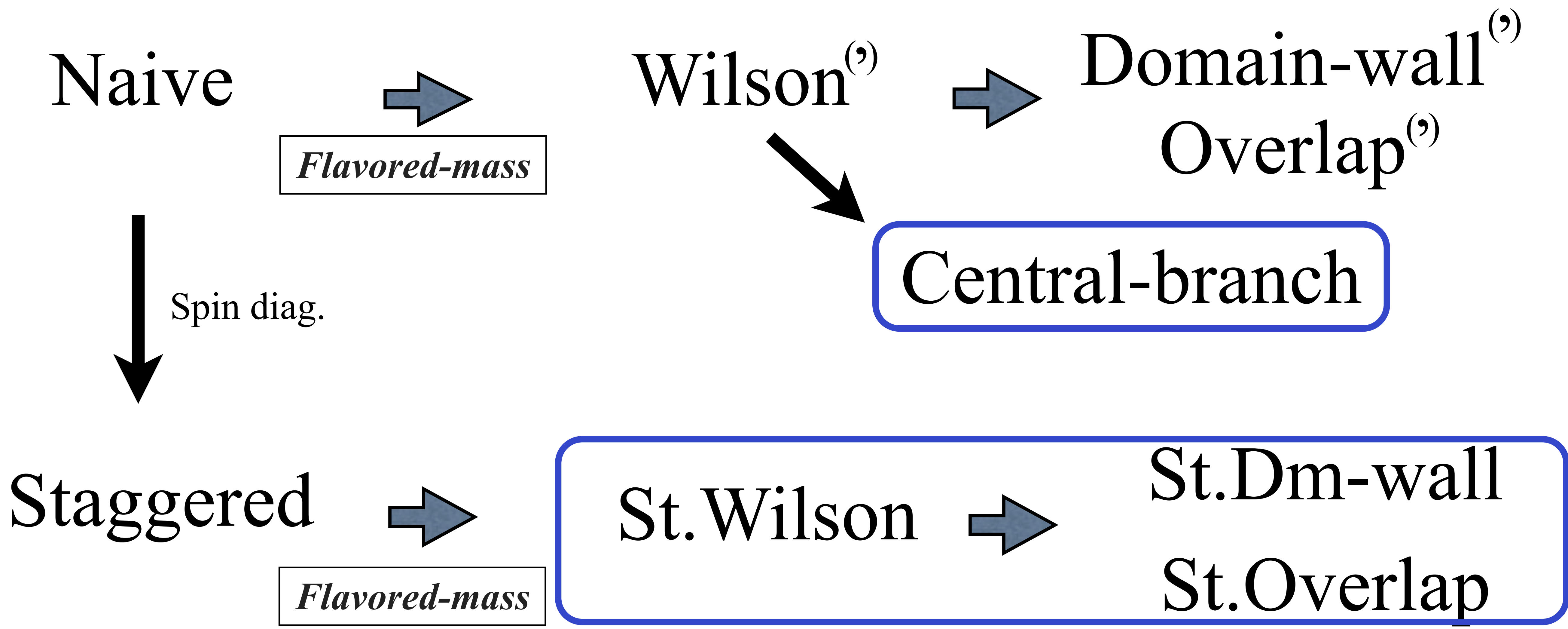
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2. Central-branch

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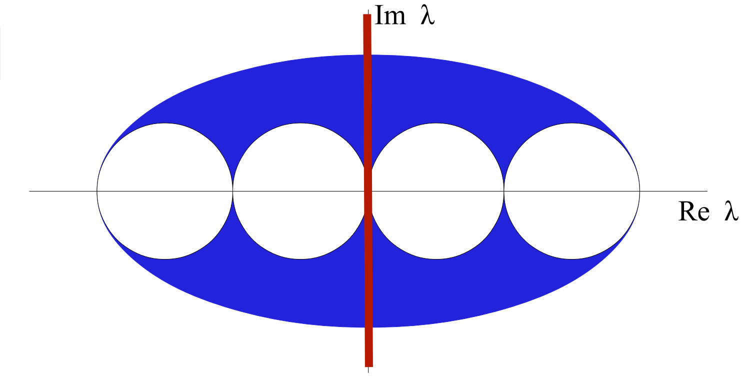
Creutz, Kimura, TM (11)
Kimura, Komatsu,TM, Noumi, Torii, Aoki (11)

- Wilson w/o onsite term $M_W \equiv m + 4r = 0$

$$S = \frac{1}{2}\sum_{x,\mu}\bar{\psi}_x[\gamma_\mu(U_{x,\mu}\psi_{x+\mu} - U_{x,-\mu}\psi_{x-\mu}) - (U_{x,\mu}\psi_{x+\mu} + U_{x,-\mu}\psi_{x-\mu})]$$



Another U(1) !

$$\psi_x \to e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}\psi_x,\;\; \bar{\psi}_x \to \bar{\psi}_x e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}$$

$$\Gamma_X^{(+)} \in \left\{\mathbf{1}_4\,,\;(-1)^{n_1+\ldots+n_4}\gamma_5\,,\;(-1)^{\check{n}_\mu}\gamma_\mu\,,\;(-1)^{n_\mu}i\gamma_\mu\gamma_5\,,\;(-1)^{n_{\mu,\nu}}\frac{[\gamma_\mu\,,\gamma_\nu]}{2}\right\}$$

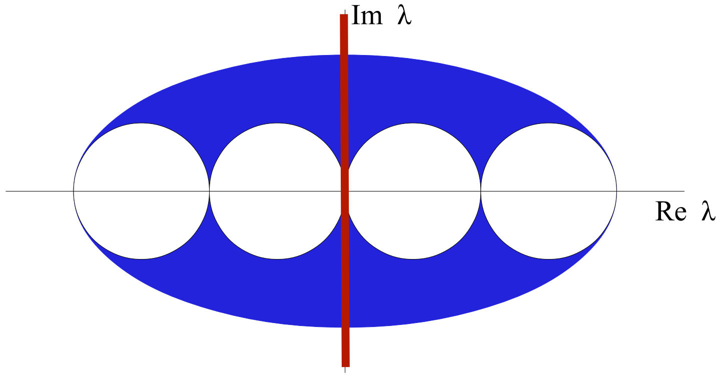
$$\Gamma_X^{(-)} \in \left\{(-1)^{n_1+\ldots+n_4}\mathbf{1}_4\,,\;\gamma_5\,,\;(-1)^{n_\mu}\gamma_\mu\,,\;(-1)^{\check{n}_\mu}\gamma_\mu\gamma_5\,,\;(-1)^{\check{n}_{\mu,\nu}}\frac{[\gamma_\mu\,,\gamma_\nu]}{2}\right\}$$

2. Central-branch

Creutz, Kimura, TM (11)
Kimura, Komatsu,TM, Noumi, Torii, Aoki (11)

- Wilson w/o onsite term $M_W \equiv m + 4r = 0$

$$S = \frac{1}{2}\sum_{x,\mu} \bar{\psi}_x[\gamma_\mu(U_{x,\mu}\psi_{x+\mu} - U_{x,-\mu}\psi_{x-\mu}) - (U_{x,\mu}\psi_{x+\mu} + U_{x,-\mu}\psi_{x-\mu})]$$



⇨ Another U(1) !

$$\psi_x \rightarrow e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}\psi_x,\ \ \bar{\psi}_x \rightarrow \bar{\psi}_x e^{i\theta(-1)^{x_1+x_2+x_3+x_4}}$$

$$\Gamma_X^{(+)} \in \left\{\mathbf{1}_4\,, \right\}$$

$$\Gamma_X^{(-)} \in \left\{\underline{(-1)^{n_1+\ldots+n_4}\mathbf{1}_4}\,, \right\}$$

→ $\gamma_5 \otimes \xi_5$

Prohibits additive mass renormalization !
SSB gives NG boson !

◆ Strong-coupling QCD

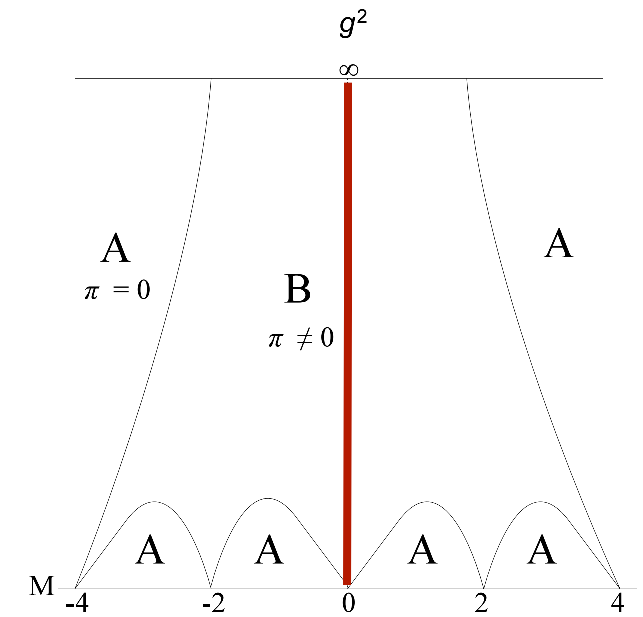
Kimura, Komatsu, TM, Noumi, Torii, Aoki (11)

$$\cosh(m_{SPA}) = 1 + \frac{2M_W^2(16 + M_W^2)}{16 - 15M_W^2}$$

NG boson associated with SSB of U(1)

- Pion (eta) condensate $\langle\bar{\psi}\gamma_5\psi\rangle \neq 0$
- No chiral condensate $\langle\bar{\psi}\psi\rangle = 0$

$$\bar{\psi}\psi \overset{?}{\longleftrightarrow} \bar{\psi}\gamma_5\psi$$



Aoki phase

§ Advantages

- No additive mass renormalization (no fine-tuning)
- SSB of U(1) and massless NG boson
- No O(a) errors

$\bar{\psi}\psi \leftrightarrow \bar{\psi}\gamma_5\psi$ *change of mass base*

6-flavor massless QCD

6th-rooting works ?

§ Potential drawbacks

- sign problem
- U(1) problem
- Quark mass

Twisted-mass works ?

→ 12-flavor QCD

Could be a new possibility of 12-flavor lattice QCD

◆ Central points for other flavored masses

- For other naive flavored mass terms

  M_{A} : *U(1)* restored

  M_{T} : *U(2)* restored

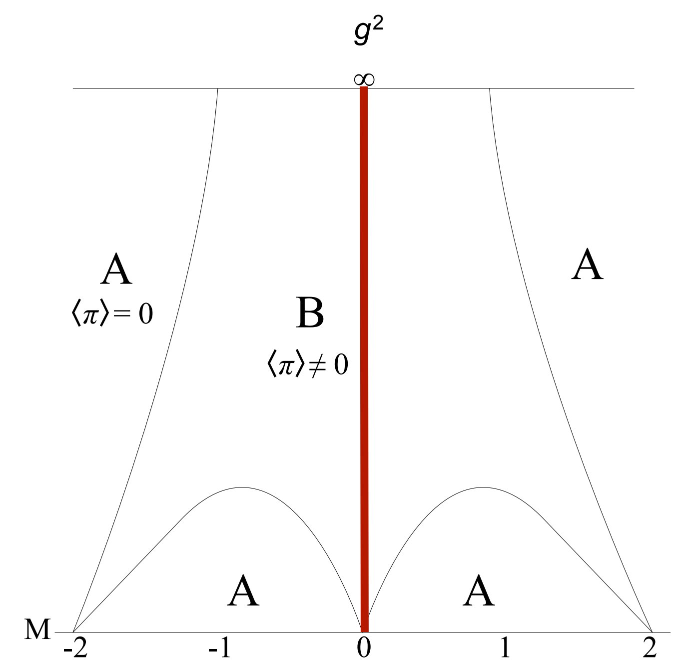
  M_{P} : *U(4)* restored

- For staggered flavored mass terms

  $M_{\mathcal{A}}$: $C_T'\Xi$, $C_T'I$ restored

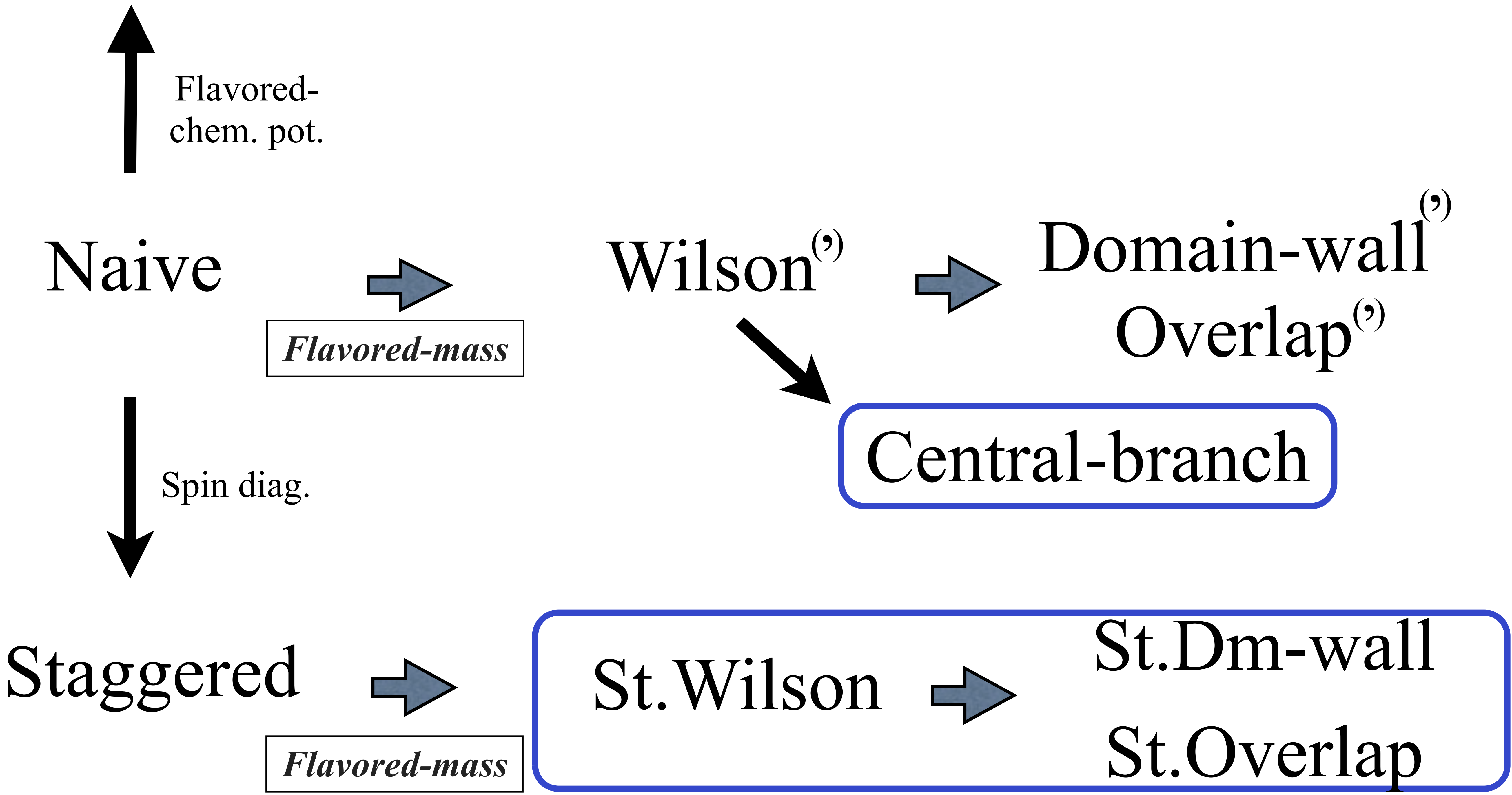
  $M_{\mathcal{H}}$: C_T' restored

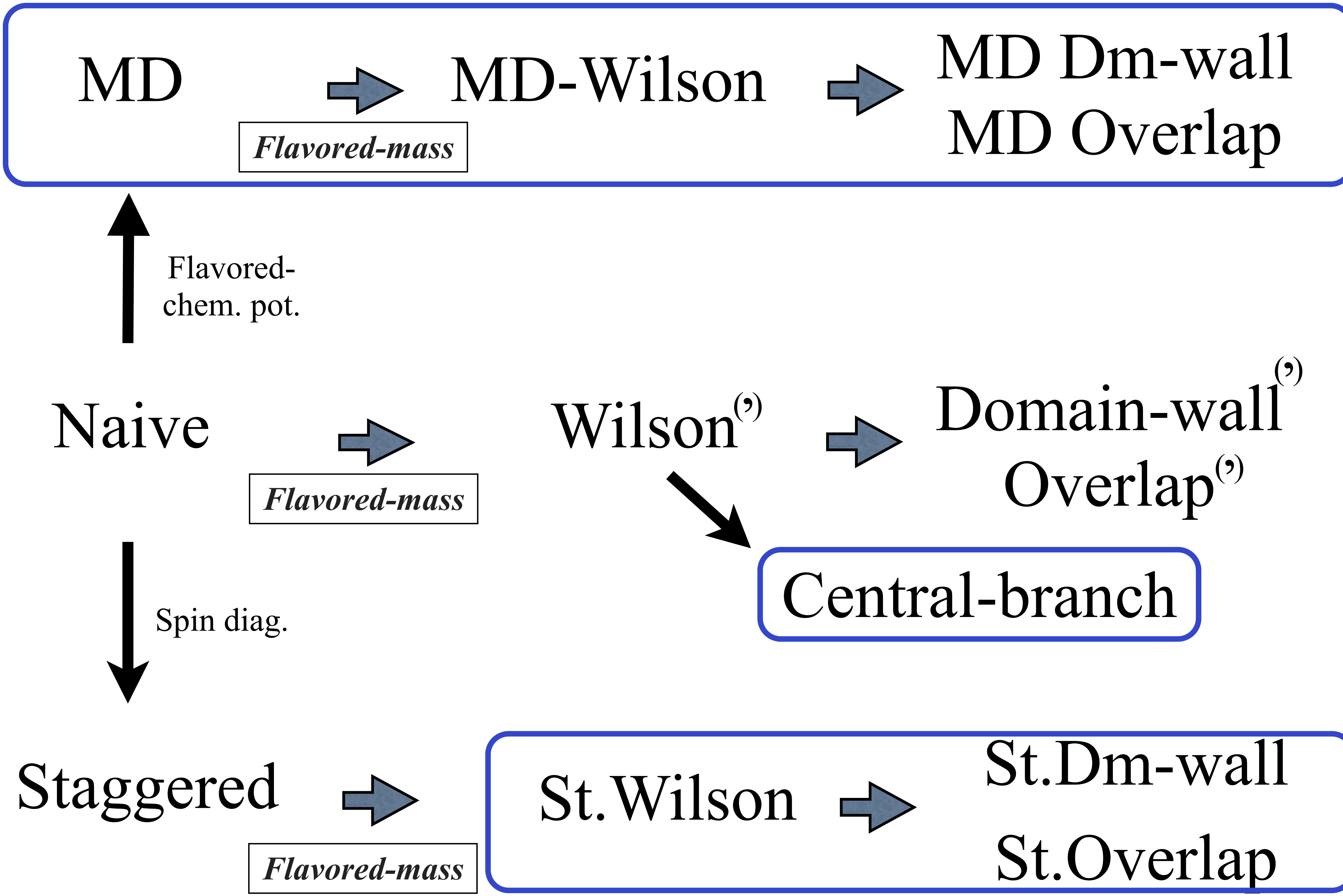
$$C_T' \,:\, \chi_x \rightarrow \bar{\chi}_x^T, \quad \bar{\chi}_x \rightarrow \chi_x^T, \quad U_{x,\mu} \rightarrow U_{x,\mu}^*$$



Aoki phase in Hoelbling fermion

	C_T'	Ξ_μ	I_μ	$C_T'\Xi_\mu$	$C_T'I_\mu$	$\Xi_\mu I_\mu$
S_{st}	○	○	○	○	○	○
$S_{\mathcal{A}}$	×	×	×	○	○	○
$S_{\mathcal{H}}$	○	×	×	×	×	○
S_m	×	○	○	×	×	○


Flavored-
chem. pot.
Naive
Flavored-mass
Wilson(')
Domain-wall(')
Overlap(')
Central-branch
Spin diag.
Staggered
Flavored-mass
St.Wilson
St.Dm-wall
St.Overlap


MD
MD-Wilson
MD Dm-wall
MD Overlap
Flavored-mass
Flavored-
chem. pot.
Naive
Wilson(')
Domain-wall(')
Overlap(')
Flavored-mass
Central-branch
Spin diag.
Staggered
St.Wilson
St.Dm-wall
St.Overlap
Flavored-mass

3. Minimal-doubling

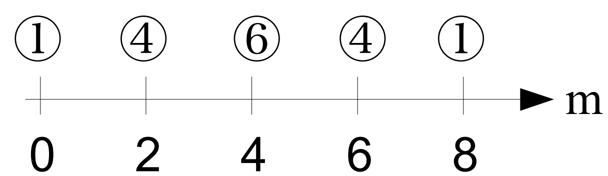
3. Minimal-doubling

Karsten(81) Wilczek(87)
Creutz(07) Borici(87) Creutz,TM(10)

Flavored imaginary chemical potential term lifts species degeneracy.

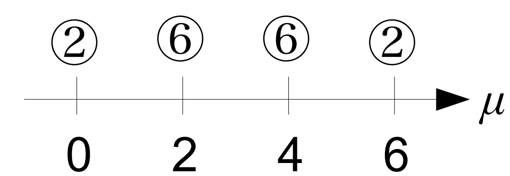
cf.) Flavored mass in Wilson

Wilson



$$\sum_{\mu}(1-\cos p_\mu)$$

Flavored chemical-pot.



$$\text{(i)}\ \gamma_4 \sum_{j=1}^{3}(1-\cos p_j) \quad \rightarrow \quad \textit{keeping one chiral sym.}$$

Finite-mass system(Wil) ⇆ Finite-density system(FCP)

◆ Advantage

- U(1) chiral symmetry
- Ultra-local
- 2 flavor possible

◆ Drawbacks

- Hypercubic symmetry breaking
- Tuning parameters for a correct continuum limit

Bedaque, Buchoff, Tiburzi, Walker-Loud(08)

Capitani, Creutz, Weber, Wittig (09)(10)

◆Symmetries

Bedaque, Buchoff, Tiburzi, Walker-Loud(08)

(1) $U(1)$ chiral symmetry

(2) P

(3) CT

(4) Cubic symmetry.

→ symmetries of finite-density systems

◆Counterterms

Capitani, Creutz, Weber, Wittig (09)(10)

dim3 $\bar{\psi}_n i\gamma_4\psi_n$ *dim4* $\bar{\psi}_n\gamma_4 D\psi_n$ $F_{i4}F_{i4}$

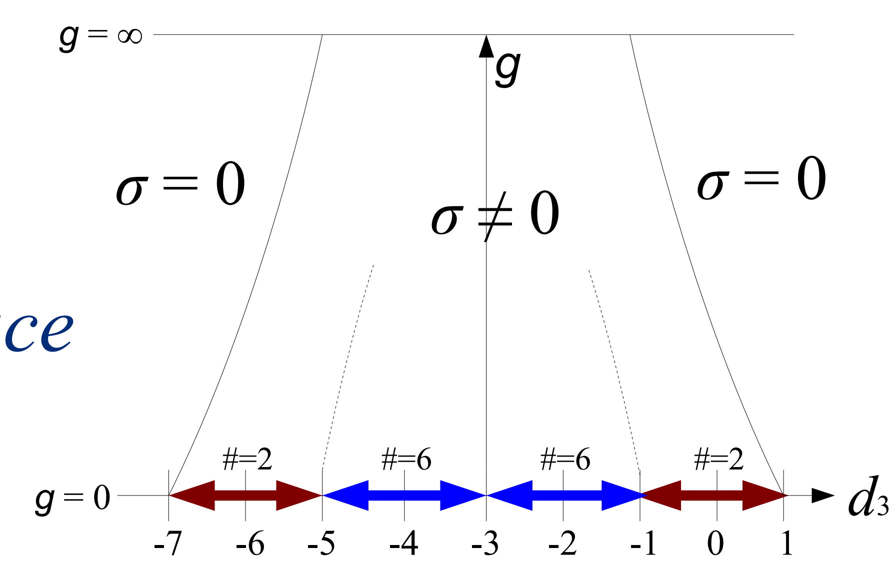
Fine-tuning of three parameters are required for Lorentz sym.

◆Chiral phase structure

TM (12)

Nontrivial phase diagram in the parameter space

Parameter phase structure



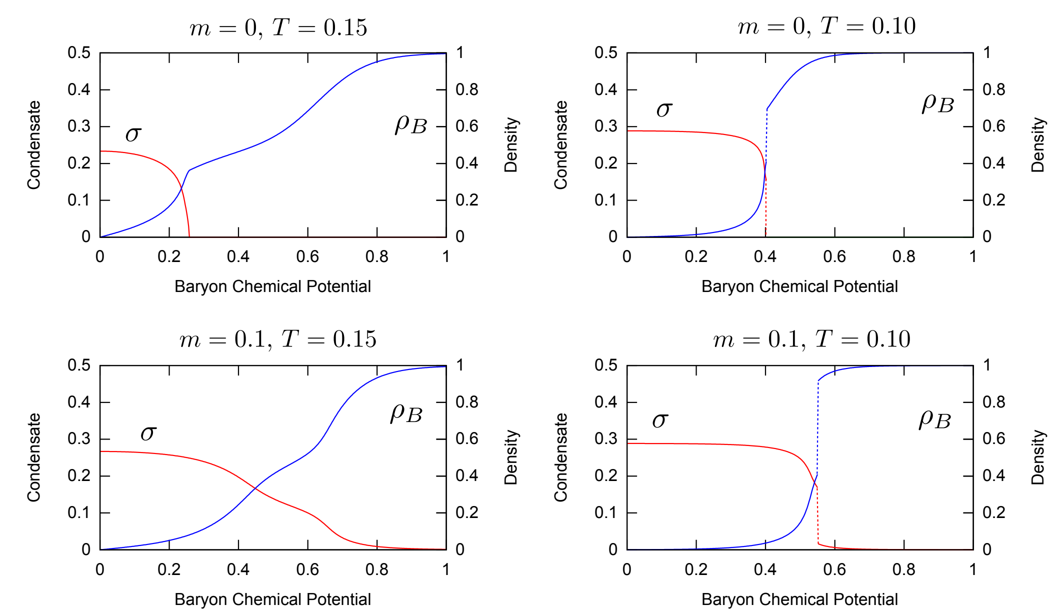
TM(2012)

◆ Finite (T, μ) QCD with FCP   Misumi, Kimura, Ohnishi (2012)

Still fine-tuning for O($1/a$) chemical potential renorm.....   cf.)additive mass in Wilson

But the discrete symmetries suit this case.   P.Hasenfratz, Karsch (83)

$$S_{\rm md} = \sum_x \left[\frac{1}{2} \sum_{j=1}^{3} \bar{\psi}_x \gamma_j \left(U_{x,x+j}\psi_{x+j} - U_{x,x-j}\psi_{x-j} \right) + \frac{1}{2} \bar{\psi}_x \gamma_4 \left(e^{\mu} U_{x,x+4}\psi_{x+4} - e^{-\mu} U_{x,x-4}\psi_{x-4} \right) \right.$$
$$\left. + \frac{i}{2} \sum_{j=1}^{3} \bar{\psi}_x \gamma_4 \underline{\left(2\psi_x - U_{x,x+j}\psi_{x+j} - U_{x,x-j}\psi_{x-j} \right)} + i d_3 \bar{\psi}_x \gamma_4 \psi_x \right]$$

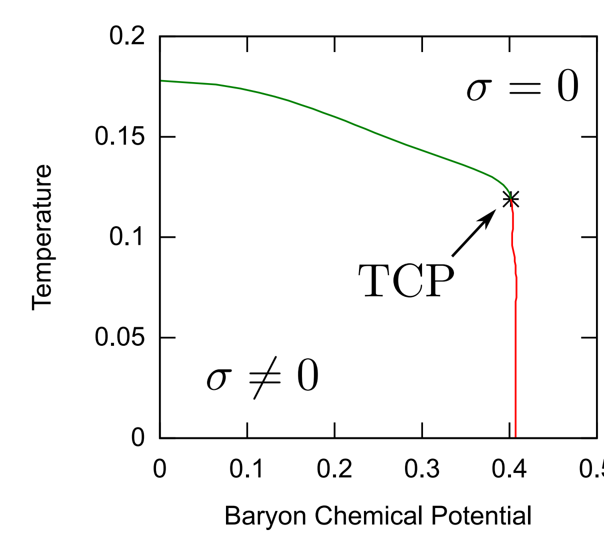


§ Strong-coupling study

Effective potential of σ as a function of T, μ and d3

Chiral phase structure

- 1st and 2nd phase transition (m=0)
- 1st, critical point and crossover (m≠0)



New possibility of (T,μ) lattice QCD !

4. Summary

1. Flavored-mass terms give us new types of Wilson and overlap fermions.

2. Staggered-Wilson can be an alternative Wilson and overlap for 2-flavor QCD (3 degenerate pion spectrum)

3. Central-branch fermion is a new possibility of use of Wilson for many-flavor QCD without fine-tuning of parameters.

4. Flavored-chemical-potential fermion would be useful for finite-temperature & density lattice QCD.

Related talks

Tuesday 15:30 Room 8 Taro KIMURA
"QCD Phase diagram with 2-flavor discretization"

Wednesday 9:30 Room 5 Takashi NAKANO
"Strong coupling analysis of Aoki phase in St-Wil fermions"

Back-up slides

◆ Spectral flow

(i) Hermitian operator

$$H(m) = \gamma_5(D - m) \quad (H^2 = D^\dagger D + m^2 \geq 0)$$

(ii) Eigenvalue flow $\lambda_i(m)$

$\lambda_0(m) = \mp m$ only for zero modes

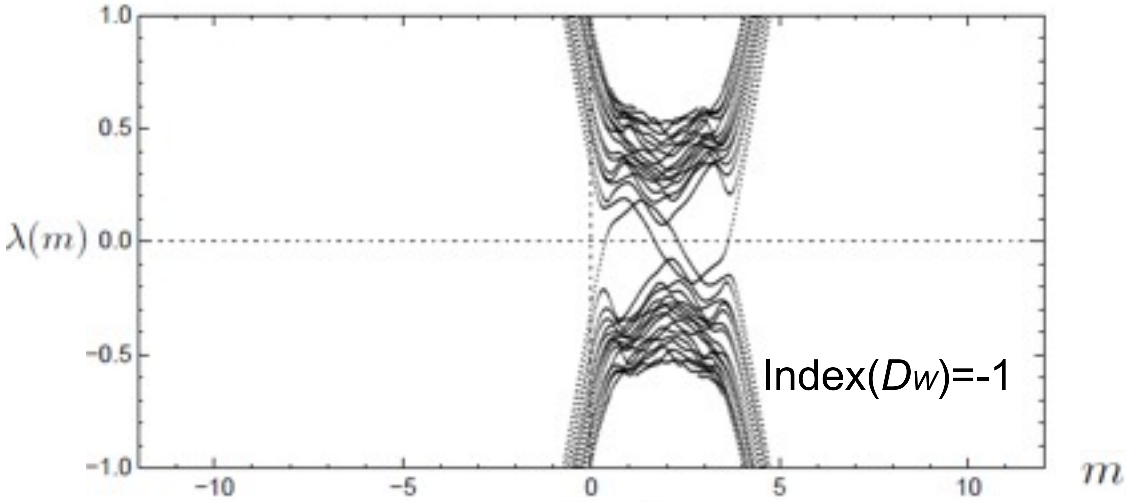
zero mode : low-lying crossing

chirality : minus the sign of slope

⇒ Index theorem

2d Wilson fermion

$H_W(m) = \gamma_5(D_W - m)$



$Q=1$

R.Edwards, U.Heller, R.Narayanan (1998)

- lattice theory (Wilson fermion)

(i) Hermitian operator

$$H_W(m) = \gamma_5(D_W - m)$$

(ii) Eigenvalue flow

would-be zero modes : low-lying real crossing

approximate chirality : $\lambda'(m) = -\psi(m)^\dagger\gamma_5\psi(m)$

Index(D_W) = - Spectral flow(H_W)

⇒ $\text{Index}(D_W) = (-1)^{d/2}Q$

※ Spectral flow :
Crossings counted with ± slopes

◆For *generalized Wilson fermions*

M. Creutz, T. Kimura, TM, *JHEP*1012:041 (2010)

$$H_{gw} = \gamma_5(D_{nf} - M_P)$$

Index(D_{gw}) = - Spectral flow(H_{gw})

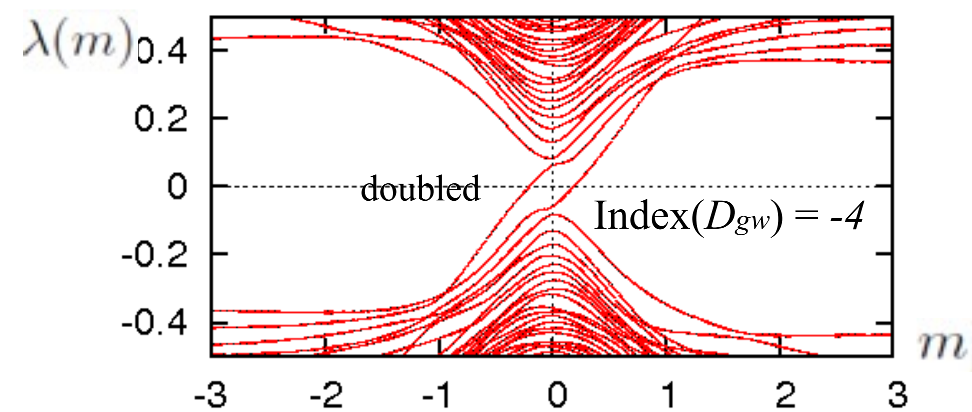
⇩

$$\text{Index}(D_{gw}) = 2^d(-1)^{d/2}Q$$

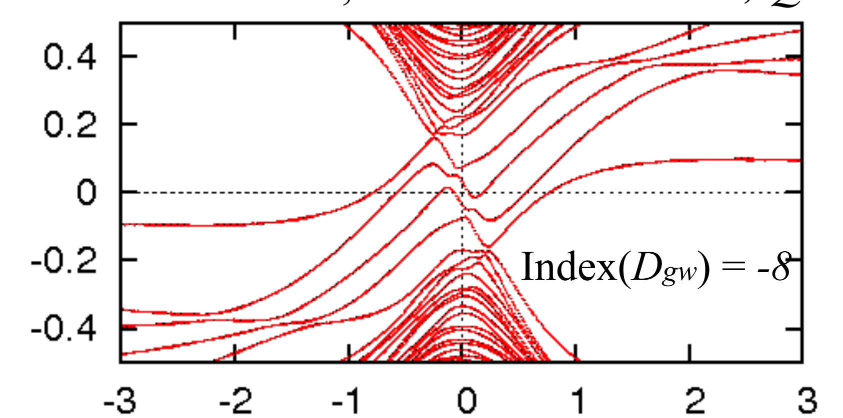
※ gauge configuration :

$$U_{x,\hat{1}} = e^{i\omega x_2}, \qquad U_{x,\hat{2}} = \begin{cases} 1 & (x_2 = 1, 2, \cdots, L-1) \\ e^{i\omega L x_1} & (x_2 = L) \end{cases} \qquad \omega = 2\pi Q$$

36×36 lattice, randomness *δ=0.25, Q=1*



36×36 lattice, randomness *δ=0.25, Q=2*



◆For *staggered-Wilson fermions*

D.H.Adams (2010)

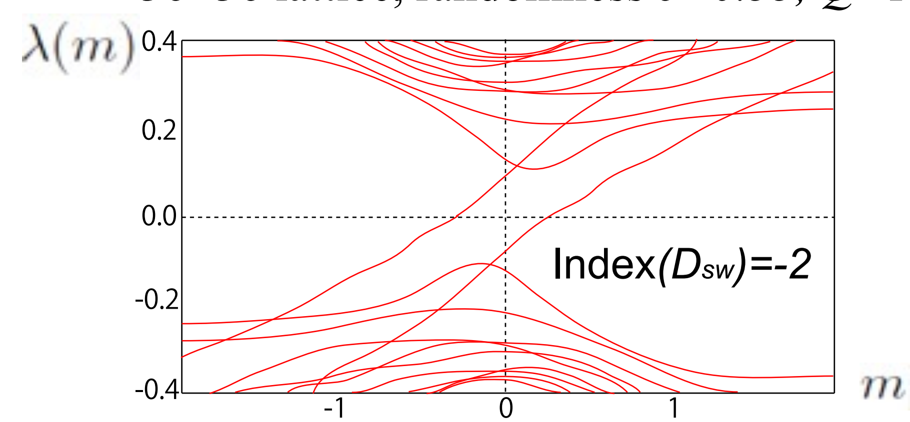
$$H_{sw} = \epsilon(D_{st} - M_f^{(A)}) = \Gamma_{55}(D_{st} - M_f^{(A)})$$

Index(D_{sw}) = - Spectral flow(H_{sw})

⇩

$$\text{Index}(D_{sw}) = 2^{d/2}(-1)^{d/2}Q$$

36×36 lattice, randomness *δ=0.33, Q=1*

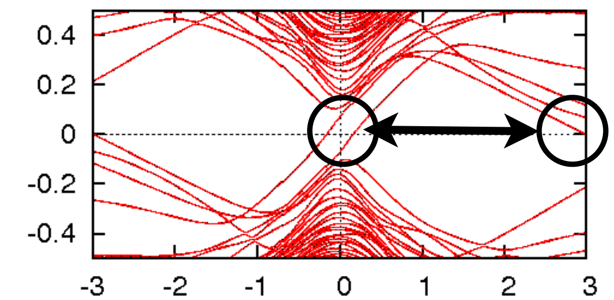


Index theorem holds for them.

◆Overlap formulation

negative-mass mode in D_W → massless mode in D_{ov}

Low-lying crossings are far from high-lying ones



- <u>Generalized overlap</u>

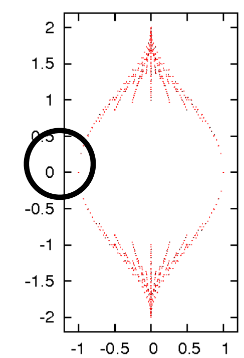
$$D_{go} = 1 + \gamma_5 \frac{H_{gw}(m)}{\sqrt{H_{gw}^2(m)}}$$

Any-flavor (1~15) overlap is possible!

cf.) 2 or 3-flavor overlap → lattice QCD

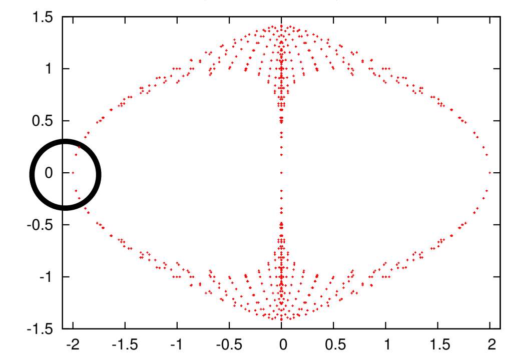
12-flavor overlap → conformal window

(8,8)



8-flavor overlp

(4,8,4)



4-flavor overlap

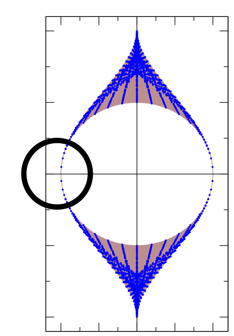
- <u>Staggered-overlap</u>

$$D_{so} = 1 + \Gamma_{55} \frac{H_{sw}(m)}{\sqrt{H_{sw}^2(m)}}$$

Less expensive overlap!

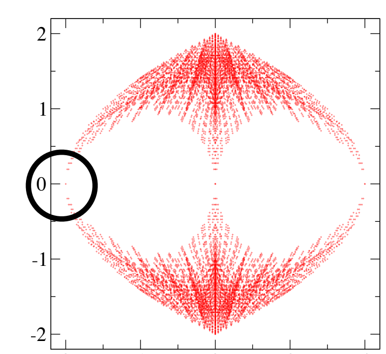
cf.) 1/4 matrix size → less CPU cost for Lanczos process

(2,2)



2-flavor overlap

(1,2,1)



1-flavor overlap

- ***Shift symmetry*** → broken to *2-link shift for S_A*
  broken to *4-link shift for S_H*

$$\mathcal{S}_\rho: \ \chi_x \to \zeta_\rho(x)\chi_{x+\hat\rho}, \quad \bar\chi_x \to \zeta_\rho(x)\bar\chi_{x+\hat\rho}, \quad U_{\mu,x} \to U_{\mu,x+\hat\rho}$$

$$\mathcal{S}_\mu: \ \phi(p) \ \to \ \exp(ip_\mu)\Xi_\mu\,\phi(p)$$

- ***Axis reversal*** → broken to *shifted axis reversal*

$$\mathcal{I}_\rho: \ \chi_x \to (-1)^{x_\rho}\chi_{Ix}, \quad \bar\chi_x \to (-1)^{x_\rho}\bar\chi_{Ix}, \quad U_{\mu,x} \to U_{\mu,Ix}$$

$$\mathcal{I}_\rho: \phi(p) \ \to \ \Gamma_\rho\Gamma_5\Xi_\rho\Xi_5\,\phi(Ip)$$

- ***Rotation*** → remain *in S_A*
  broken to *subgroup in S_H*

$$\mathcal{R}_{\rho\sigma}: \ \chi_x \to S_R(R^{-1}x)\chi_{R^{-1}x}, \quad \bar\chi_x \to S_R(R^{-1}x)\bar\chi_{R^{-1}x}, \quad U_{\mu,x} \to U_{\mu,Rx}$$

$$\mathcal{R}_{\rho\sigma}: \ \phi(p) \ \to \ \exp(\frac{\pi}{4}\Gamma_\rho\Gamma_\sigma)\exp(\frac{\pi}{4}\Xi_\rho\Xi_\sigma)\,\phi(R^{-1}p)$$

- ***Conjugation*** → remain *in S_A*
  broken *in S_H*

$$\mathcal{C}: \ \chi_x \to \epsilon_x\bar\chi_x^T, \ \bar\chi_x \to -\epsilon_x\bar\chi_x^T, \ U_{\mu,x} \to U^*_{\mu,x}$$

$$\mathcal{C}: \ \phi(p) \ \to \ \bar\phi(-p)^T$$

Axis and Rotation $\rightarrow$ $(\Gamma_4 \rtimes SW_{4,\mathrm{diag}})$

Details of StWil symmetries

$$\{\Xi_\mu, I_s, R_{\mu\nu}\} \quad \longrightarrow \quad \Gamma_4 \rtimes SW_4$$

$$\{\Xi'_\mu, R_{\mu\nu}\} \quad \longrightarrow \quad \Gamma_3 \rtimes SW_4$$

Physical-sector symmetry

$$\Xi'_j \Xi'_4 R^2_{j4} = \Xi_j \Xi_4 \sim (1 \otimes \sigma_j)$$
$$\Xi'_4 R^2_{34} R^2_{12} = \Xi_4 I_s \sim (\gamma_4 \otimes \mathbf{1})$$
$$C_0 \Xi'_2 \Xi'_4 R^2_{24} \sim C$$

Details of timeslice symmetries

Enlarged staggered sym : $\{C_0, \Xi_\mu, I_s, R_{\mu\nu}, T_\mu^{1/2}\}$ $\Xi_\mu^2 = 1$

$$\longrightarrow T_\mu^{1/2} \rtimes [\{C_0, \Xi_\mu\} \rtimes \{R_{\mu\nu}, I_s\}] = (\otimes_j Z_{N_\mu}) \rtimes [\Gamma_{4,1} \rtimes W_4]$$

Timeslice sym : $T_\mu^{1/2} \rtimes [\{C_0, \Xi_\mu\} \rtimes \{R_{ij}, I_s\}] = (\otimes_j Z_{N_j}) \rtimes [\Gamma_{4,1} \rtimes W_3]$

Relevant group at rest

$$\Gamma_{4,1} \rtimes W_3 \sim [\{R_{ij}, \Xi_{ij}\} \times \{C_0, \Xi_4, \Xi_{123}, I_s\}]/Z_2$$
$$= [\{R_{ij}, \tilde{R}_{4i} \equiv \epsilon_{ijk} R_{jk} \Xi_{kj}\} \times \{C_0, \Xi_4, \Xi_{123}, C_0 \Xi_4 I_s\}]/Z_2$$
$$= [SW_4 \times \Gamma_{2,2}]/Z_2$$

Staggered-Wilson

$\{C_0, \Xi'_\mu, R_{\mu\nu}, T'^{1/2}_\mu\}$

$$\sim [\{R_{ij}, \Xi'_{ij}\} \times \{C_0, \Xi'_4, \Xi'_{123}, I_s\}]/Z_2$$
$$= [\{R_{ij}, \tilde{R}_{4i} \equiv \epsilon_{ijk} R_{jk} \Xi'_{kj}\} \times \{C_0, \Xi'_4, \Xi'_{123}]/Z_2$$
$$= [SW_4 \times \Gamma_{1,2}]/Z_2$$

Dim3, 4 : $\bar{Q}(1 \otimes \xi_F)Q \quad \bar{Q}(\gamma_\mu \otimes \xi_F)D_\mu Q$ for $\xi_F = 1$ or ξ_5 ⇨ $\bar{\ell}\gamma_\mu D_\mu \ell, \quad \bar{\ell}\ell$

Dim5 O(a): $\bar{Q}(i\sigma_{\mu\nu}F_{\mu\nu} \otimes \xi_F)Q$ for $\xi_F = 1$ or ξ_5 ⇨ $\bar{\ell}i\sigma_{\mu\nu}F_{\mu\nu}\ell$

No unphysical term nor taste-breaking term up to O(a)

Dim6 O(a^2) : 2 types of four-fermi operators $\mathcal{L}_6^{FF(A)}$ and $\mathcal{L}_6^{FF(B)}$

In $\mathcal{L}_6^{FF(A)}$ the spin and flavor independently forms scalar

⇨ 25 operators with ξ_5 pair $\rightarrow$ 50 operators

SA, SV, AS, VS, PV, PA, VP, AP, TV, TA, VT, AT, AA, PP , SP, PS, ST, PT, TS, TP, VV, AA, VA, AV, TT

$\rightarrow$ No taste-breaking. No derivative terms. Contributes to potential $\mathcal{V}_6^{FF(A)}$

In $\mathcal{L}_6^{FF(B)}$ the spin and flavor are not independent

⇨ 10 operators with ξ_5 pair $\rightarrow$ 20 operators

TV, TA, VT, AT, VV, AA, VA, AV, TT+, TT-

$\rightarrow$ Taste-breaking. Derivative terms. No contribution to potential $\mathcal{V}_6^{FF(B)}$

No taste-breaking in ChPT potential upto O(a^2): SU(2)