# Review of lattice studies of resonances

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# Outline

# Hadron resonances - experiment

- Most hadrons are resonances under the strong interaction

| hadron | Γ [MeV] |
|---|---|
| $b_1(1235)$ | $142 \pm 9$ |
| $a_1(1260)$ | $250 - 600$ |

| hadron | Γ [MeV] |
|---|---|
| $K^\star(1410)$ | $232 \pm 31$ |
| $K_0^\star(1430)$ | $270 \pm 80$ |

| hadron | Γ [MeV] |
|---|---|
| $D_0^\star(2400)$ | $267 \pm 40$ |
| $D_1(2430)$ | $384 \pm^{130}_{110}$ |

- Widths and branching fractions often known poorly
- Experiment data is analyzed with a partial wave analysis
- Elastic scattering: Scattering amplitudes $a_l$ and related phases $\delta_l$:

$$a_l = \sin\delta_l \mathrm{e}^{\delta_l} = \frac{\mathrm{e}^{2i\delta_l} - 1}{2i}$$

- Near a single relativistic Breit-Wigner shaped resonance

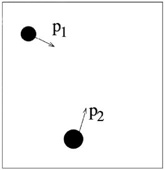
$$a_l = \frac{-\sqrt{s}\Gamma(s)}{s - s_R + i\sqrt{s}\Gamma(s)}$$

with resonance position $s_R = m_R^2$ and decay width Γ

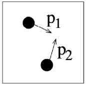
# The Lüscher method for elastic scattering

M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

$$E = E(p_1) + E(p_2)$$




L >> few fm




L= few fm

$$E = E(p_1) + E(p_2) + \Delta_E$$

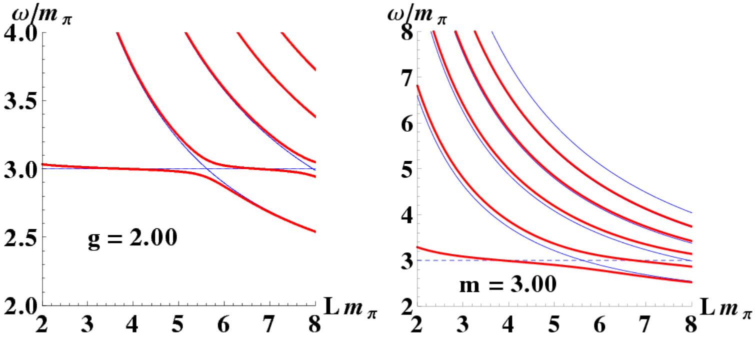
$$E_n(L) \quad \xrightarrow{(2)} \quad \delta_l \quad \xrightarrow{(3)} \quad m_R; \quad \Gamma_R \text{ or coupling } g$$

(1) Extract energy levels $E_n(L)$ in a finite box

(2) The Lüscher formula relates this spectrum to the phase shift of the continuum scattering amplitude

(3) Extract resonance parameters with some degree of modeling/approximation

# Energy levels in a box - an illustration




animations by C. B. Lang and DM

- **Left:** Expectations for $\rho$-like resonance at varying coupling $g_{\rho\pi\pi}$
- **Right:** Expectations for $\rho$-like resonance with physical $g_{\rho\pi\pi}$ and varying mass

# Multiple possibilities

- Lüscher method
  - Rest-frame calculation in multiple spatial volumes $L^3$

    M. Lüscher Commun. Math. Phys. 105 (1986) 153; Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.
  - Moving frames for equal mass hadrons $m_{h1} = m_{h2}$

    Rummukainen, Gottlieb, Nucl. Phys. B 450, 397 (1995);
    Kim, Sachrajda, Sharpe, Nucl. Phys. B 727, 218 (2005);
    Feng, Jansen, Renner, PoS LAT2010 104 (2010);
    Dudek, Edwards, Thomas, arXiv:1203.6041.
  - Moving frames for $m_{h1} \neq m_{h2}$: Even and odd $l$ mix

    Fu, PRD 85 014506 (2012); Döring et al. arXiv:1205.4838;
    Göckeler et al. arXiv:1206.4141; Leskovec, Prelovsek, PRD 85 114507 (2012);
  - Calculations in multiple asymmetric boxes i.e. $L^2 \times L_z$
- Alternative approaches
  - Histogram method

    Bernard, Lage, Meißner, Rusetsky, JHEP 0808 (2008) 024
  - Correlator method

    Meißner, Polejaeva, Rusetsky, Nucl. Phys. B 846,1 (2011)

# Comparing the Lüscher and histogram methods

- Toy model study in the O(4) non-linear sigma model

Giudice, McManus, Peardon, arXiv:1204.2745

- Compares Lüscher's method with the histogram method in the elastic and inelastic regions
- Histogram method

Bernard, Lage, Meißner, Rusetsky, JHEP 0808 (2008) 024

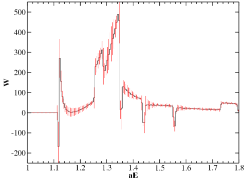
  - Interpolate energy levels to obtain $E_n(L)$ in $[L_0, L_M]$
  - Slice $[L_0, L_M]$ into $M$ parts of length $\Delta L = \frac{L_M - L_0}{M}$
  - Slice $[E_{min}, E_{max}]$ into bins of length $\Delta_E$
  - Make a histogram and normalize to get $W(E)$
  - Subtract the noninteracting background
  - Close to a resonance one gets

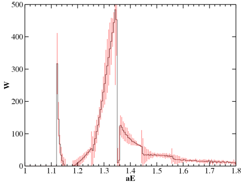
$$W(p) - W_0(p) \propto \frac{1}{[E(p)^2 - M_r^2]^2 + M_r^2 \Gamma^2} \qquad W(p) = W(E)\frac{\partial E}{\partial p}$$

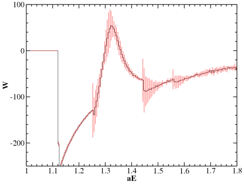
# Results from the histogram method

Giudice, McManus, Peardon, arXiv:1204.2745

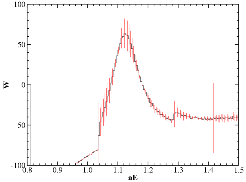
- Narrow resonance with different strategies for the background subtraction




- Example from the inelastic case

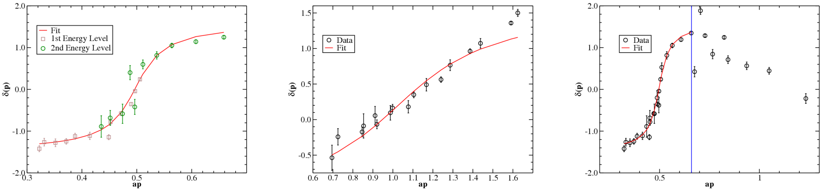



- No theoretical support for the inelastic case

# Results from the Lüscher method

Giudice, McManus, Peardon, arXiv:1204.2745

- Narrow, wide and inelastic cases with the Lüscher method




- Table comparing both methods

| | Lüscher | | histogram | |
|---|---|---|---|---|
| parameter set | $aM_\sigma$ | $a\Gamma_\sigma$ | $aM_\sigma$ | $a\Gamma_\sigma$ |
| set A | 1.35(2) | 0.115(8) | 1.33(5) | 0.10(5) |
| set B | 2.03(2) | 0.35(2) | 2.01(2) | 0.35(10) |
| set C | 3.1(7) | 1.2(5) | - | - |

# QCD results - Technically demanding

- Observation: In practical calculations $\bar{q}q$ interpolators couple very weakly to multi-hadron states

McNeile & Michael, Phys. Lett. B 556, 177 (2003); Engel et al. PRD 82, 034505 (2010);
Bulava et al. PRD 82, 014507(2010); Dudek et al. PRD 82, 034508(2010);

- This is not unlike observations in string breaking studies

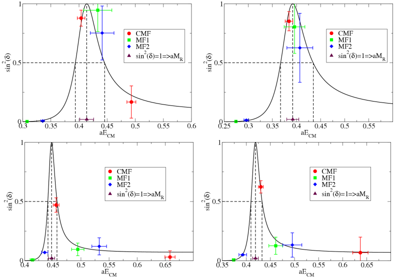
Pennanen & Michael hep-lat/0001015;Bernard et al. PRD 64 074509 2001;

- This necessitates the inclusion of hadron-hadron interpolators

- The distillation method is especially well suited

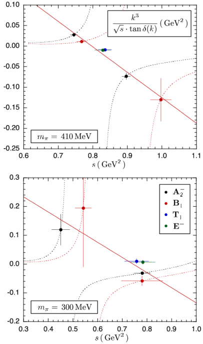
Peardon et al. PRD 80, 054506 (2009)
Morningstar et al. PRD 83, 114505 (2011)

# The $\rho$ resonance - previous studies



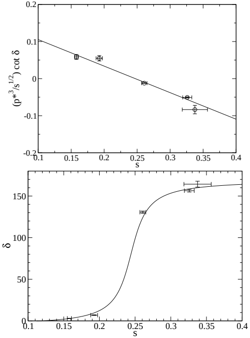

Feng, Jansen, Renner

PRD 83 094505 (2011)




Aoki et al., PRD 84

094505 (2011)




Lang et al., PRD 84

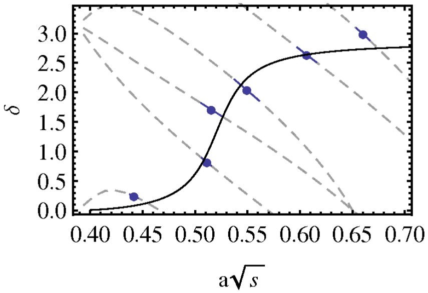
054503 (2011)

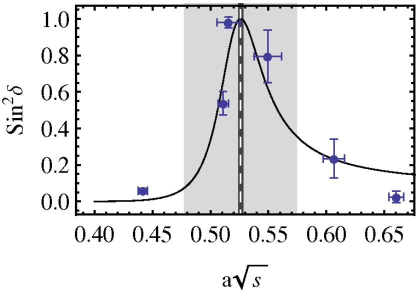
- Further results

Aoki et al. PRD 76 094506 (2007); Göckeler et al. PoS LATTICE2008, 136 (2008)
Frison et al. PoS LATTICE2010, 139 (2010); Pelissier et al. PoS LATTICE2011 134 (2011)

C. S. Pelissier and A. Alexandru, private communication

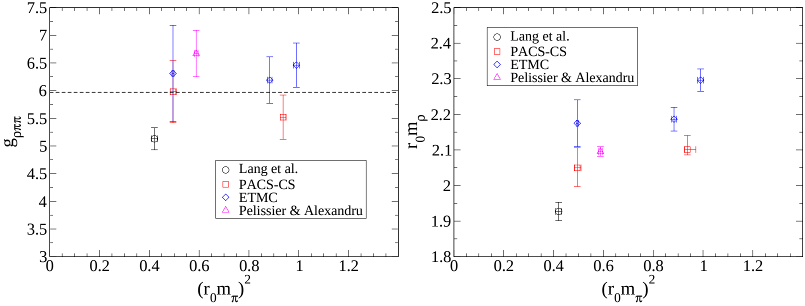
- Three ensembles with $N_f = 2$ nHYP-smeared clover fermions, $a = 0.1225(7)$fm and $m_\pi = 304(2)$MeV
- Lattice sizes $24^2 \times L_z \times 48$ with $L_z = 24, 32, 48$
- Variational basis with one $\bar{q}q$ and one meson-meson interpolating field



# The $\rho$ resonance - comparing results

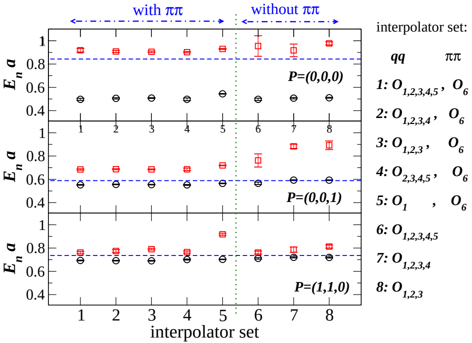
- To compare the masses I use the values for $\frac{r0}{a}$ for each ensemble




My apologies to ETMC for not finding one value

# Something I would like to stress - choice of basis

Interpolator basis dependence on the example of $\rho$ meson results




From Lang, DM, Prelovsek, Vidmar PRD 84 054503 (2011)

# Meson - meson scattering in the $K\pi$, $D\pi$ and $D^{\star}\pi$ channels

- $N_f = 2$ flavors of nHYP smeared Wilson-clover quarks

| $N_L^3 \times N_T$ | $\kappa_l$ | $\beta$ | $a$[fm] | $L$[fm] | #configs | $m_\pi$[MeV] | $m_K$[MeV] |
|---|---|---|---|---|---|---|---|
| $16^3 \times 32$ | 0.1283 | 7.1 | 0.1239(13) | 1.98 | 280/279 | 266(3)(3) | 552(2)(6) |

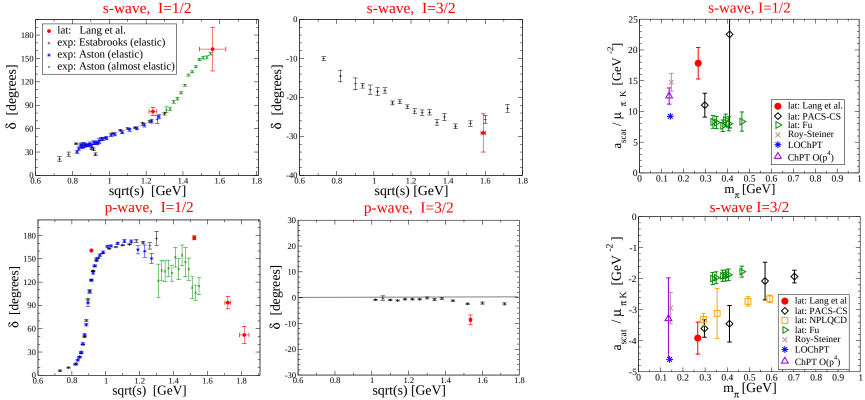
`Gauge ensemble from Hasenfratz et al. PRD 78 054511 (2008)`
`Hasenfratz et al. PRD 78 014515 (2008)`

- Distillation method
- Basis of several $\bar{q}q$ and meson-meson interpolators
- Currently only total momentum zero
- Two separate studies
  - $K\pi$ scattering for isospin $\frac{1}{2}$ and $\frac{3}{2}$, s-wave and p-wave

    `Lang, Leskovec, DM, Prelovsek - to be published`
  - $D$ mesons including $D\pi$ and $D^{\star}\pi$ with Fermilab charm quarks

    `DM, Prelovsek, Woloshyn - to be published`

# $K\pi$ scattering for isospin $\frac{1}{2}$ and $\frac{3}{2}$



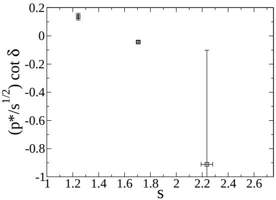

Lang, Leskovec, DM, Prelovsek - to be published

# $D\pi$ and $D^{\star}\pi$ scattering

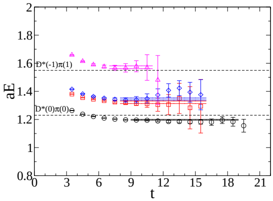
DM, Prelovsek, Woloshyn - to be published

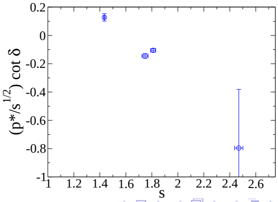
- In the $J^P = 0^+$ $D_0^{\star}$ channel we extract three levels

$$\frac{p^{\star}}{\sqrt{s}} \cot\delta = \frac{1}{g^2}(s - m_{res}^2)$$



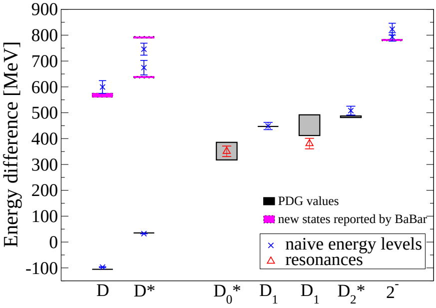

- For the $J^P = 1^+$ channel there are two resoances $D_1(2420)$ and $D_1(2430)$



DM, Prelovsek, Woloshyn - to be published

- Motivated by the heavy quark limit, We assume one $D_1$ state is given by the naive energy level and fit the remaining data to obtain




| | $D_0^\star(2400)$ | $D_1(2430)$ |
|---|---|---|
| $g^{lat}$ [GeV] | $2.55 \pm 0.21$ | $2.01 \pm 0.15$ |
| $g^{exp}$ [GeV] | $\leq 1.92 \pm 0.14$ | $\leq 2.50 \pm 0.40$ |

# $\kappa$ and $\sigma$ resonances from staggered simulations

Z. Fu, JHEP 1201 (2012) 017 and arXiv:1202.5834

- Simulation on $16^3 \times 48$ asqtad MILC lattices with $m_{u/d} = 0.2 m_s$ and $a \approx 0.15$fm
- A basis of one $\bar{q}q$ and one meson-meson interpolator, picking the goldstone pion/kaon $\pi_5/K_5$
- Assumption of a Breit-Wigner resonance:

| $I = \frac{1}{2} \quad \pi K$ | $I = 0 \quad \pi\pi$ |
|---|---|
| $g_{\kappa\pi K} = 4.54(76)$GeV<br>$M_R = 0.779(27)a$ | $g_{\sigma\pi\pi} = 2.69(44)$GeV<br>$M_R = 0.691(37)a$ |

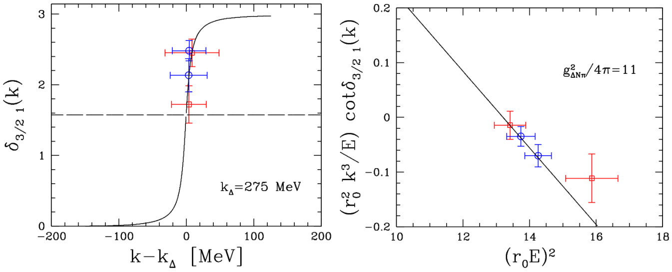
- Caveat: $\bar{q}q$ interpolators inevitably couple to all taste combinations

Prelovsek, PRD 73, 014506 (2006); Bernard et al., PRD 76, 094504 (2007)

- Variational analysis may render other taste combination $K_b \pi_b$ and $\pi_b \pi_b$ as excited states
- Experimental data makes Breit-Wigner assumption questionable

# $\Delta(1232) \leftrightarrow N\pi$ by the QCDSF collaboration

- $m_\pi \approx 250$ MeV on $32^3 \times T$, $40^3 \times T$ and $48^3 \times T$ boxes
- Preliminary results for the coupling: $\frac{g^2_{\Delta\pi N}}{4\pi} = 11\pm^4_3$
- This should be compared to $\frac{g^2_{\Delta\pi N,exp}}{4\pi} \approx 14.4$




Göckeler, Horsley, Lage, Meißner, Rakow, Rusetsky, Schierholz, Zanotti

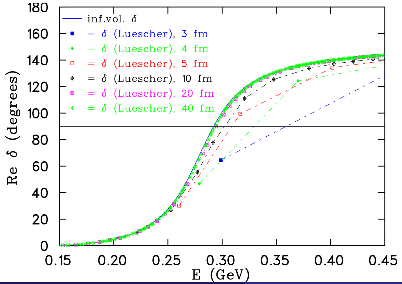
- private communication

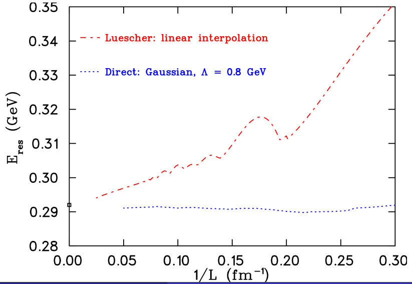
# Baryon interactions in a matrix Hamiltonian model

Hall, Hsu, Leinweber, Thomas, Young. - to be published

parallel talk Wednesday 10:40

- In Lüscher's method finite volume errors of the resonance energy $E_{res}$ should be exponentially suppressed as a function of L
- This assumes that hadrons at the boundary of the box are in the asymptotic region, otherwise the error may scale as $\frac{1}{L}$
- The authors investigate this in a matrix Hamiltonian model for $\Delta \leftrightarrow N\pi$

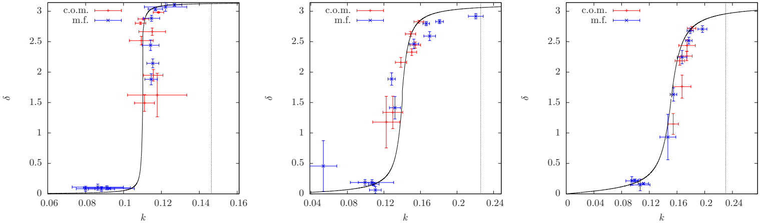


# Higgs boson resonance from a chiral Higgs-Yukawa model

`Gerhold, Jansen, Kallarackal Phys.Lett.  B710 697 (2012)`

- Pure Higgs-Yukawa sector of the EW standard model
- Complex scalar Higgs doublet and a mass-degenerate fermion doublet (top, bottom) coupled in a chirally invariant way
- Goal: Treat the Higgs as a true resonance using the Lüscher method




- Interesting observation: Even for large quartic coupling the width stays small!

# Conclusions and challenges

- Studies of QCD resonances are still in their infancy
- The Lüscher method is restricted to the elastic case
- So far alternatives/extensions are not well developed or require modeling

Looking at experiment, even some low lying states seem quite difficult

- $a_0(980)$
  Goes to $\eta\pi$ and $\bar{K}K$
- $K_1(1230)$
  Goes to $K\rho$, $K^\star(872)\pi$, $K\omega$ and $K_0^\star(1430)\pi$ in experiment
  Mixes with nearby $K_1(1400)$
- X,Y,Z states in the Charmonium spectrum?

$\longrightarrow$ There is much work ahead!

- Don't miss further related talks and posters:

Talk: Friday 2:50 by Sho Ozaki on $J/\Psi\phi$ scattering
Talk: Friday 3:10 by Jan Daldrop on $a_0(980)$ and $\kappa$
Poster: Bruno Charron on $\pi\pi$ interaction in the $I = 1$ channel
Poster: Chi-Pin Hsu on resonant phase shifts in finite volume

# Thank you!

. . . to the organizers
. . . to everyone sending me material
. . . to my collaborators
. . . to all of you for listening