
spectroscopy overview

Jozef Dudek
Old Dominion University
& Jefferson Lab

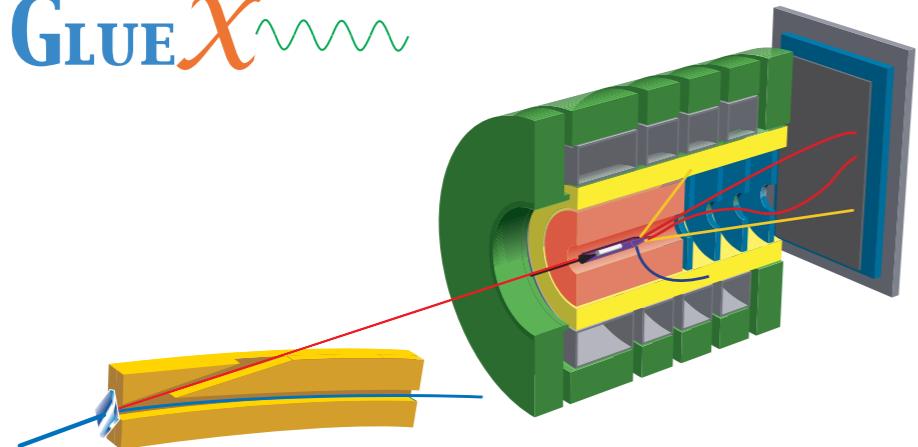
thanks for inviting a whining pom

spectroscopy ?

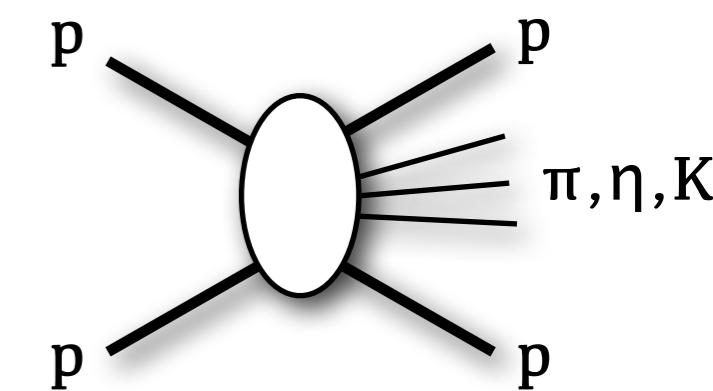
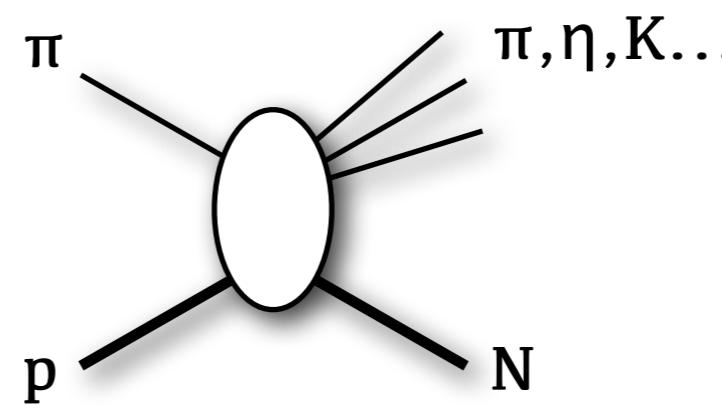
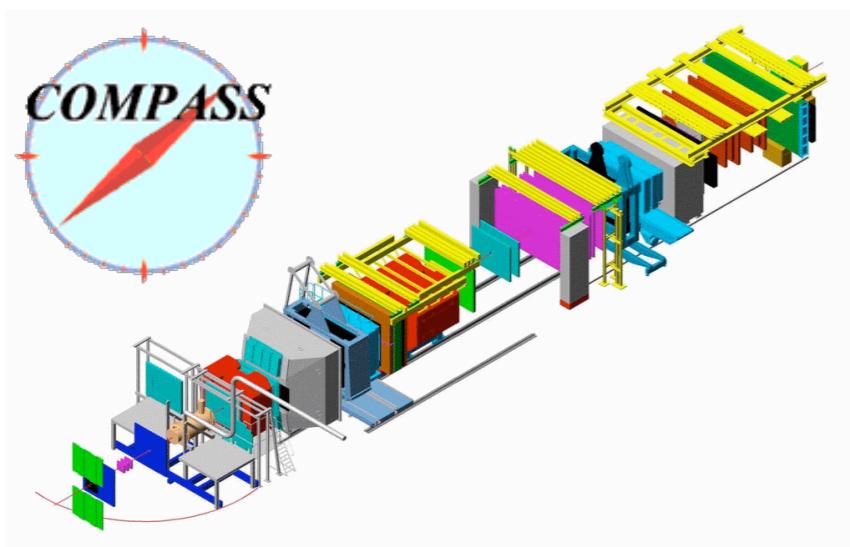
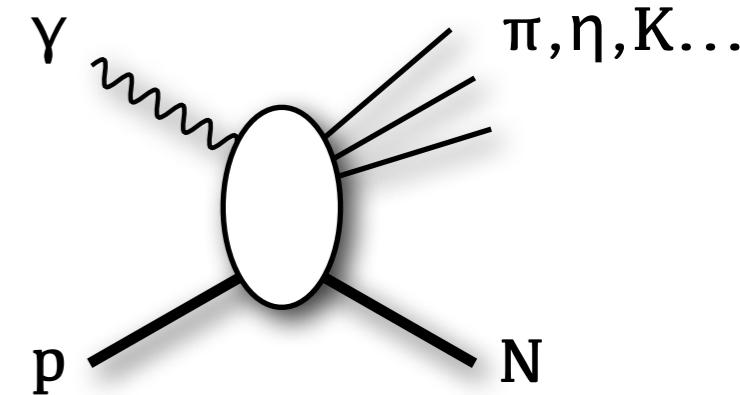
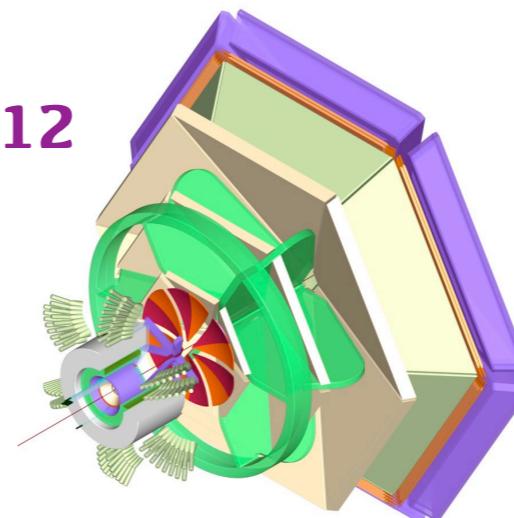
- will touch only lightly on ‘precision’ spectroscopy - masses of (QCD)-stable hadrons with control over systematic errors
 - obvious importance of this work
 - doesn’t need restating
- will rather focus on areas in which lattice QCD calculations are running alongside (or ahead of) experiments & models looking at the excited hadron spectrum
 - necessitates some (temporary) relaxation in rigour
 - (most interesting states are inelastic resonances)
 - at this stage looking for qualitative patterns in the QCD spectrum
 - (do what we can with the technology we have)
 - (be careful to assess the validity of the results)
 - maturing of finite-volume method for scattering will eventually lead to more rigorous studies
 - (Dan Mohler up next with a survey)

the light meson spectrum - experiments

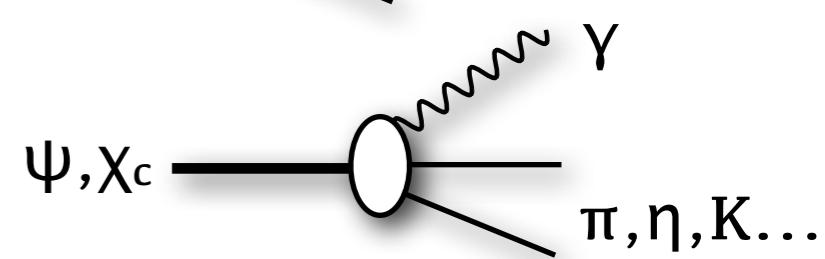
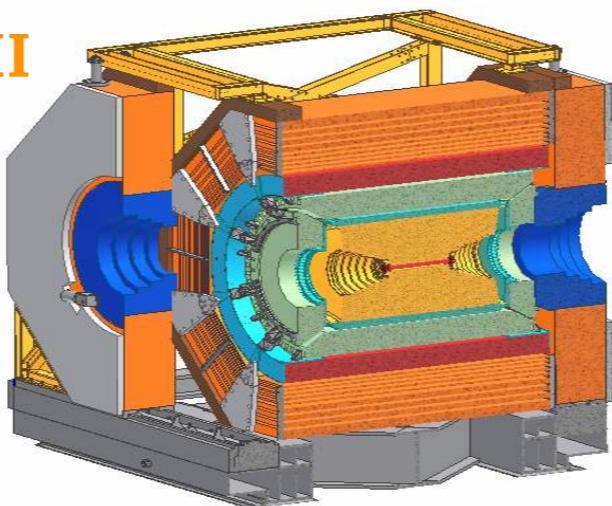
GLUE χ



CLAS12



BES III



the light meson spectrum

relatively simple models of hadrons:

bound states of **constituent** quarks and antiquarks

“the quark model”

$$M \sim q\bar{q} \quad B \sim qqq$$

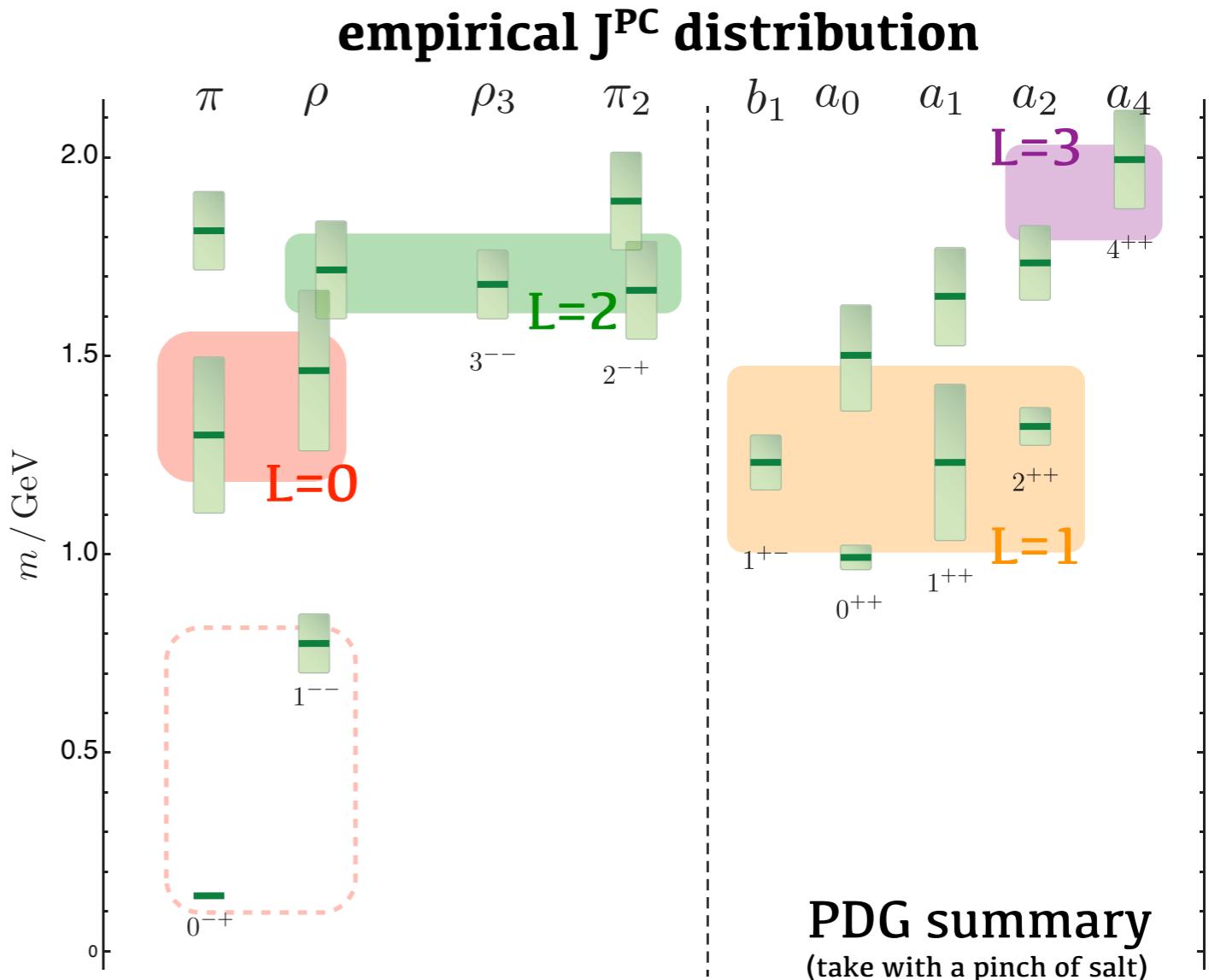
empirical meson flavour systematics

$I=0, S=0 : \eta, \varphi, \omega, f_J \dots$

$I=1, S=0 : \pi, \rho, b_1, a_J \dots$

$I=\frac{1}{2}, S=\pm 1 : K, K^* \dots$

$I \geq 1, |S| \geq 1$



the light meson spectrum

an example of states beyond minimal quark model configurations

hybrid mesons

states in which a gluonic excitation is present

smoking gun signature - J^{PC} outside the set accessible to $q\bar{q}$

$$J_{q\bar{q}}^{PC} \neq 0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \dots$$

PHYSICAL REVIEW D

VOLUME 17, NUMBER 3

1 FEBRUARY 1978

Model of mesons with constituent gluons*

D. Horn[†]

Calif.

Massach.

A model of mesons composed of constituents is provided by the model are described, and states.

NEW MESON CONFIGURATION IN THE BAG MODEL

(I). First order energy spectrum of $q\bar{q}g$ states

F. DE VIRON and J. WEYERS

Département de Physique Théorique, U

Re
(Final ver)

Perturbative QCD in the bag is
The meson spectrum is computed
 $J^{PC} = 1^{-+}, 0^{+-}, 0^{--}$ are found in th

A LIGHT EXOTIC $q\bar{q}g$ HERMAPHRODITE MESON?

Ted Barnes and F.E. Close

Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, UK

Received 15 April 1982

We suggest that $q\bar{q}g$ mesons may exist as low modes and perhaps be relatively stable. The bag splittings is computed analogously to Jaffe's $q\bar{q}$ cussed.

and many others since

many models, many different spectrum predictions ...

lots of operators + variational analysis ...

proves rather powerful method to extract a spectrum of excited states

e.g. a basis of fermion bilinears
with up to three covariant derivatives

$$\begin{array}{c} \bar{\psi} \Gamma \tilde{\psi} \\ \bar{\psi} \Gamma \overleftrightarrow{D} \tilde{\psi} \\ \bar{\psi} \Gamma \overleftrightarrow{D} \overleftrightarrow{D} \tilde{\psi} \\ \bar{\psi} \Gamma \overleftrightarrow{D} \overleftrightarrow{D} \overleftrightarrow{D} \tilde{\psi} \end{array}$$

one possible smearing methodology -
distillation

'distillation' smeared quarks
- allows for relevantly efficient
construction of a large operator
basis

$$\begin{aligned} \tilde{\psi} &= \square \psi = \sum_{n=1}^N f(\lambda_n) \xi_n \xi_n^\dagger \psi \\ -\nabla^2 \xi_n &= \lambda_n \xi_n \end{aligned}$$

expensive, but becomes good value for:
→ large operator basis disconnected diagrams
→ computing two-particle correlators with definite
inter-particle momentum

lots of ops + variational analysis ...

operators constructed to be irreducible (definite spin) in the continuum theory

$$\text{e.g. } \mathcal{O}^{J,M} = \langle 1m_1; 1m_2 | JM \rangle \bar{\psi} \gamma_{m_1} \overleftrightarrow{D}_{m_2} \tilde{\psi} \quad \gamma_m \equiv \vec{\epsilon}_m \cdot \vec{\gamma}$$

subduced into irreducible representations of the cubic group

$$\mathcal{O}_{\Lambda,\lambda}^{[J]} = \sum_M S_{\Lambda\lambda}^{JM} \mathcal{O}^{J,M}$$

e.g.

$$S_{\Lambda\lambda}^{JM} \quad \begin{array}{c|ccccc} & \multicolumn{5}{c} {(J=2) \rightarrow T_2} \\ \lambda & M & 2 & 1 & 0 & -1 & -2 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{c|ccccc} & \multicolumn{5}{c} {(J=2) \rightarrow E} \\ \lambda & M & 2 & 1 & 0 & -1 & -2 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \end{array}$$

$$(J=3) \rightarrow T_1 \quad \begin{array}{c|ccccccc} & \multicolumn{7}{c} {(J=3) \rightarrow T_1} \\ \lambda & M & 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ \hline 1 & 0 & 0 & \sqrt{\frac{3}{8}} & 0 & 0 & 0 & \sqrt{\frac{5}{8}} & \\ 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \\ 3 & \sqrt{\frac{5}{8}} & 0 & 0 & 0 & \sqrt{\frac{3}{8}} & 0 & 0 & \end{array}$$

$$(J=3) \rightarrow T_2 \quad \begin{array}{c|ccccccc} & \multicolumn{7}{c} {(J=3) \rightarrow T_2} \\ \lambda & M & 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ \hline 1 & 0 & 0 & \sqrt{\frac{5}{8}} & 0 & 0 & 0 & -\sqrt{\frac{3}{8}} & \\ 2 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \\ 3 & \sqrt{\frac{3}{8}} & 0 & 0 & 0 & -\sqrt{\frac{5}{8}} & 0 & 0 & \end{array}$$

$$(J=3) \rightarrow A_2 \quad \begin{array}{c|ccccccc} & \multicolumn{7}{c} {(J=3) \rightarrow A_2} \\ \lambda & M & 3 & 2 & 1 & 0 & -1 & -2 & -3 \\ \hline 1 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \end{array}$$

lots of ops + variational analysis ...

up to three-derivatives gives 26 operators in T_1^{--}

19 subduced from J=1
6 subduced from J=3
1 subduced from J=4

compute the full 26×26 correlation matrix

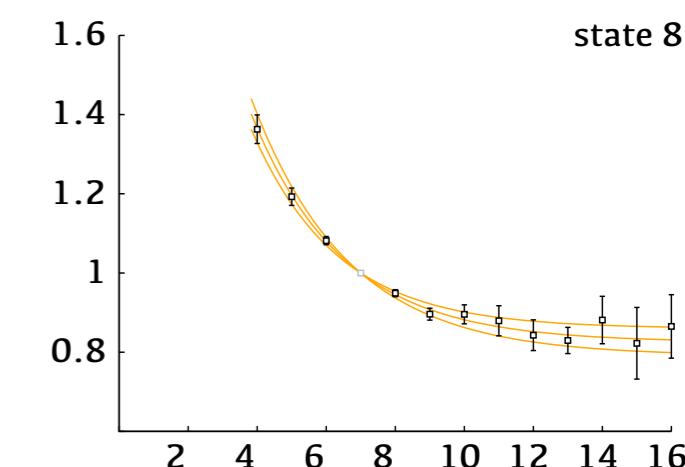
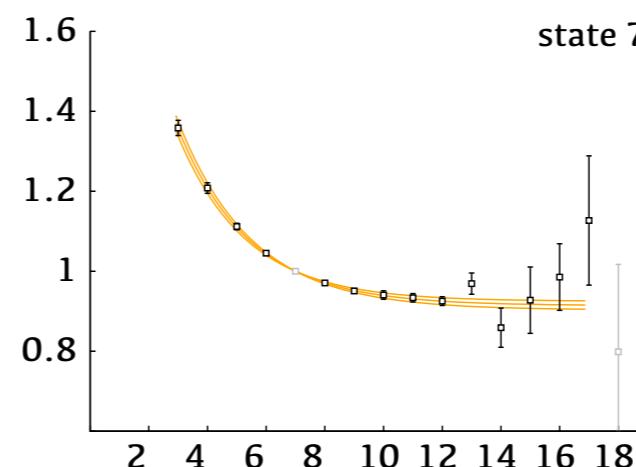
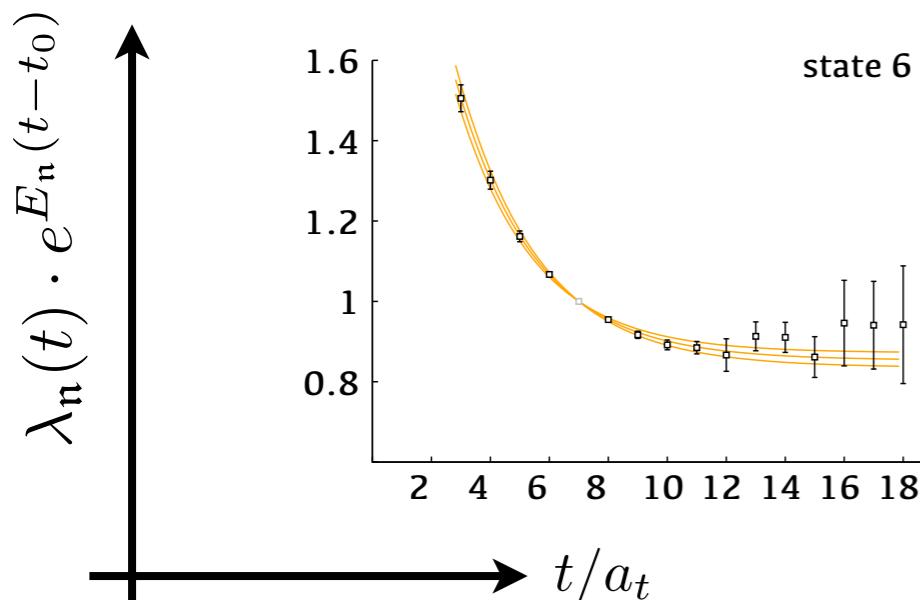
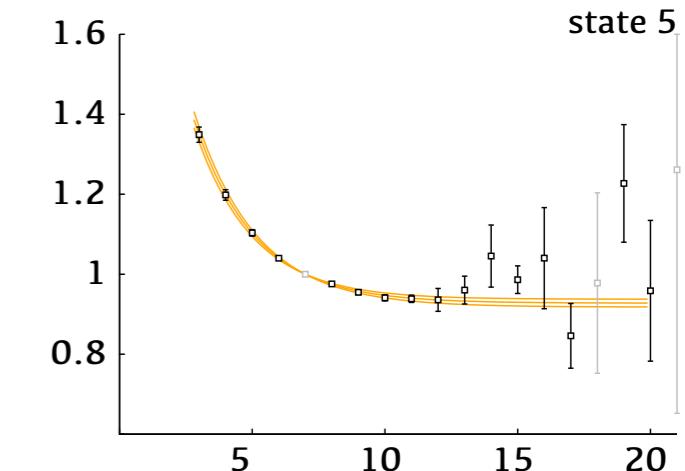
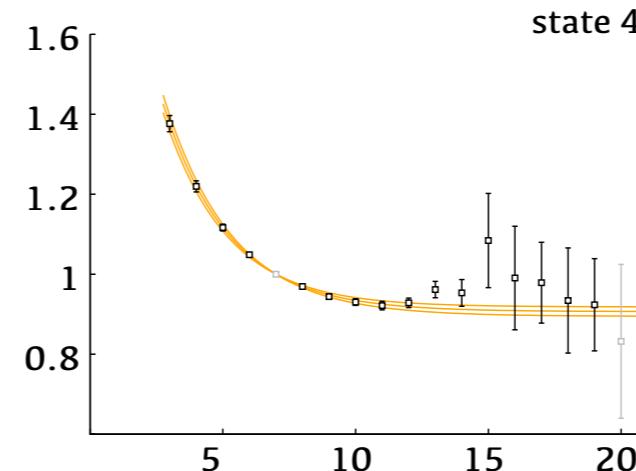
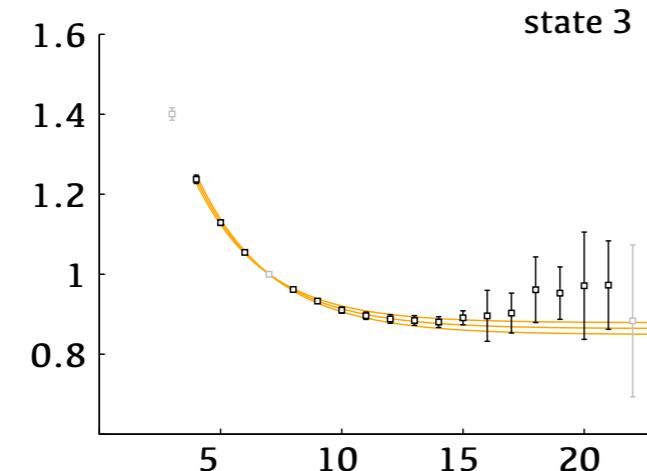
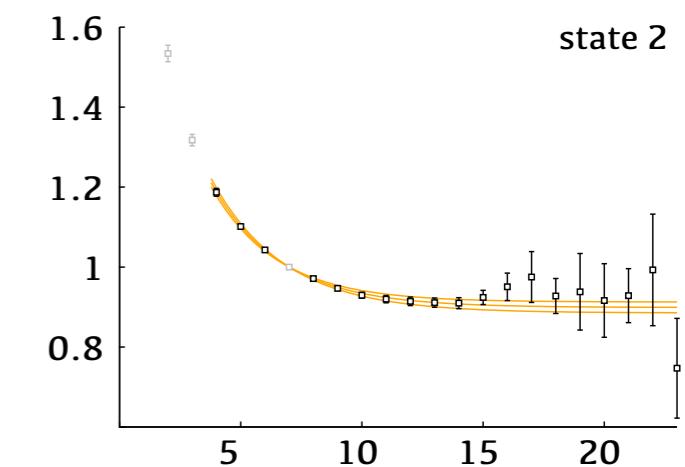
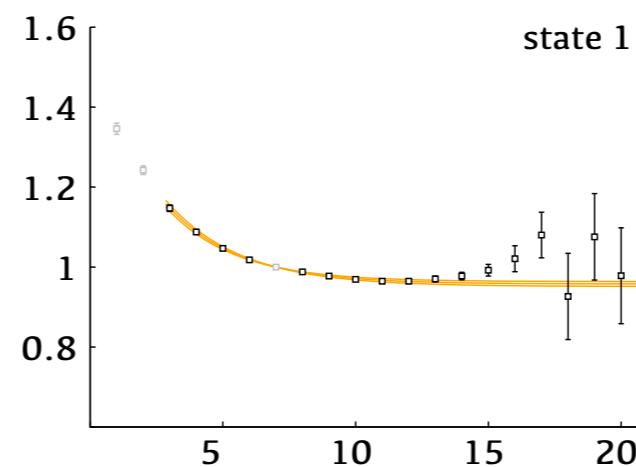
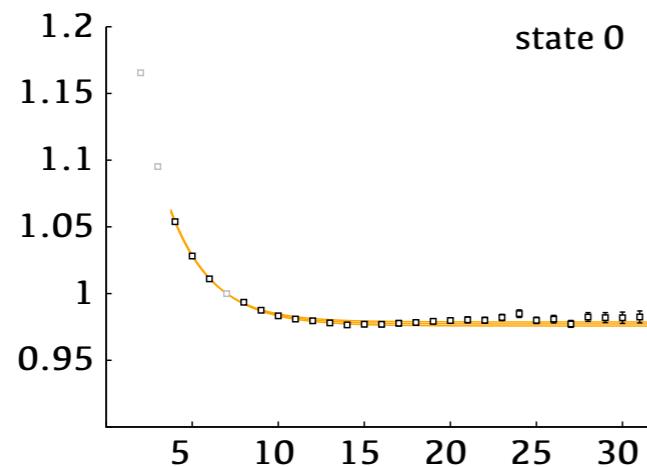
solve the generalised eigenvalue problem $C(t)v_{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C(t_0)v_{\mathfrak{n}}$

anisotropic 2+1 Clover
 $(24^3 \times 128)$
 $m_\pi \sim 400$ MeV

principal correlators - T_1^{--}

$$\lambda(t) = (1 - A)e^{-m(t-t_0)} + Ae^{-m'(t-t_0)}$$

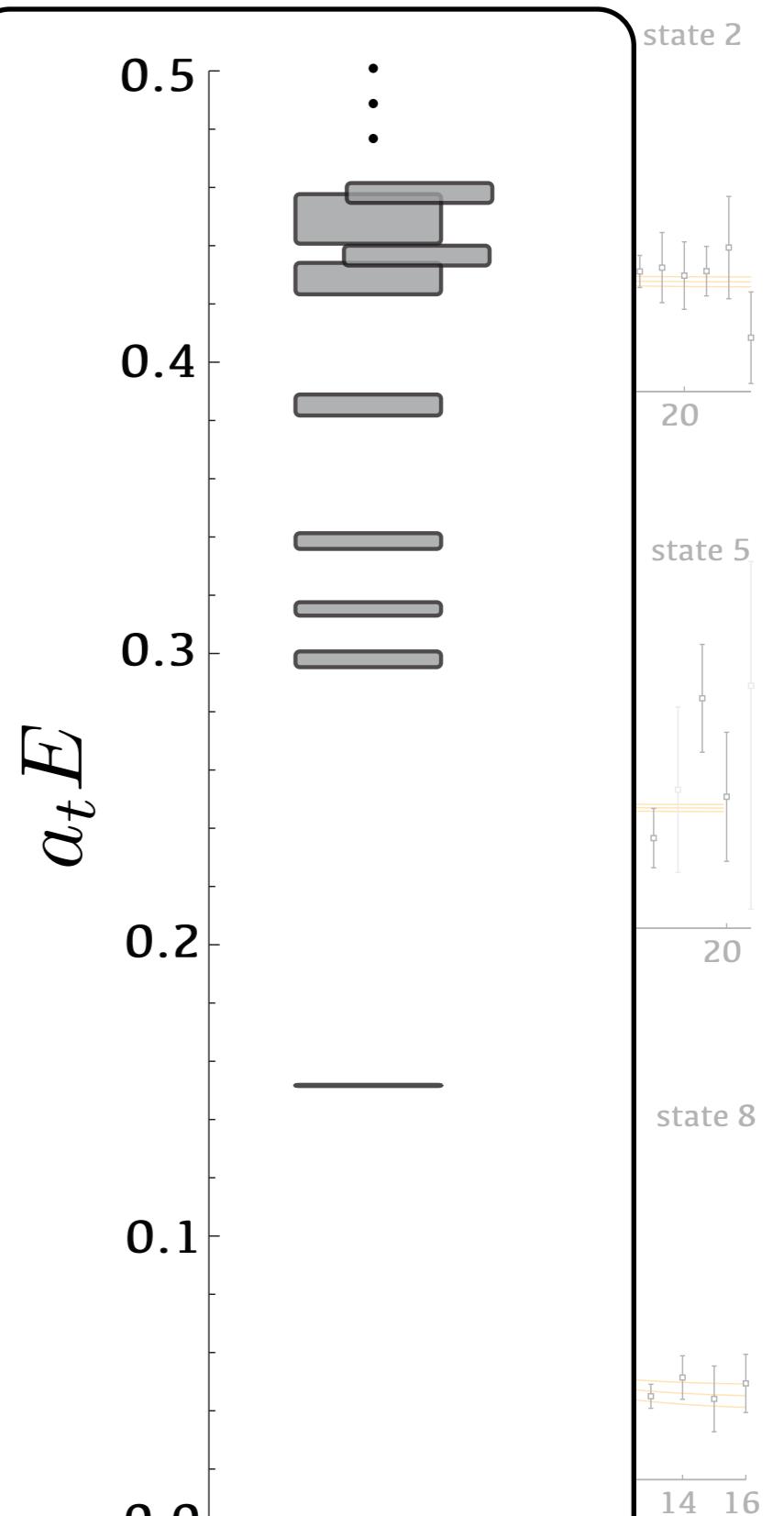
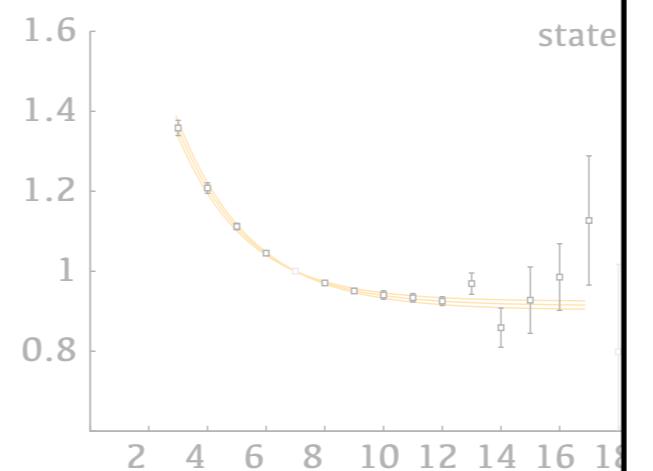
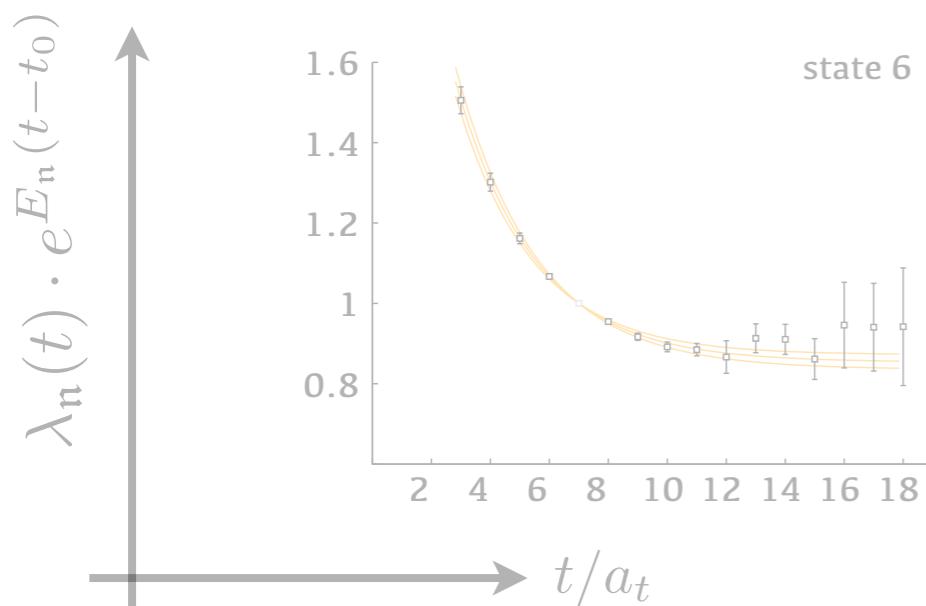
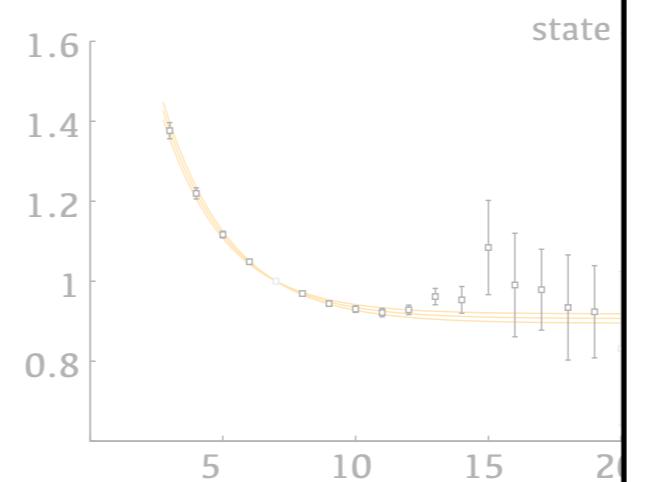
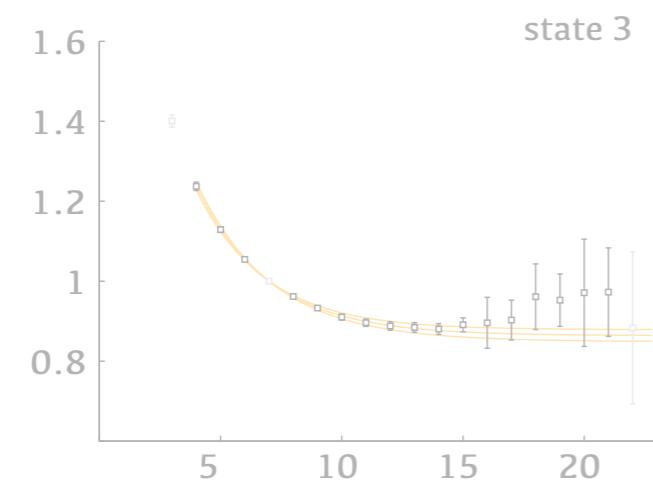
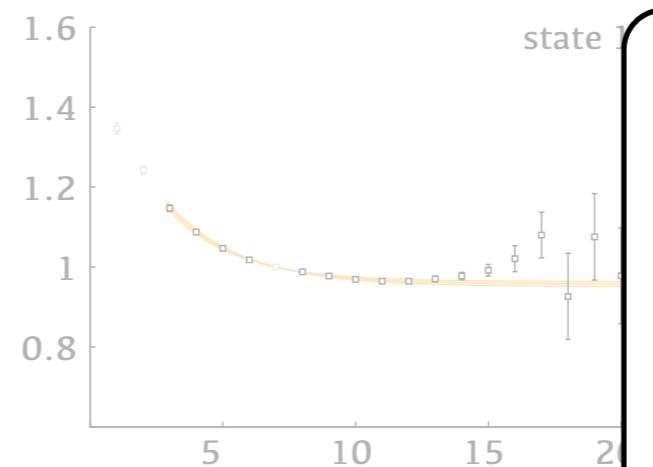
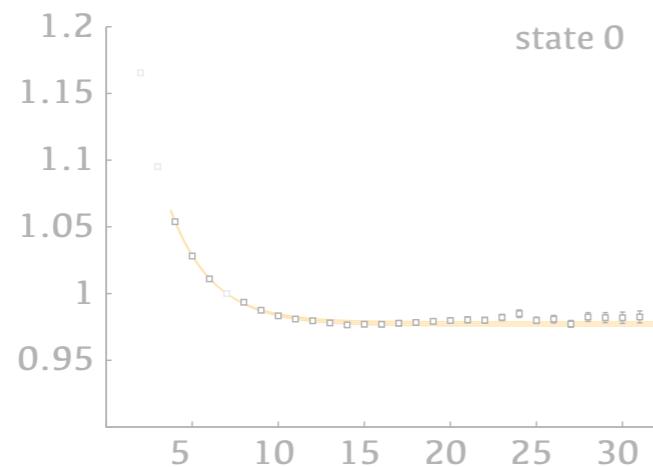
$t_0/a_t = 7$



principal correlators - T_1^{--}

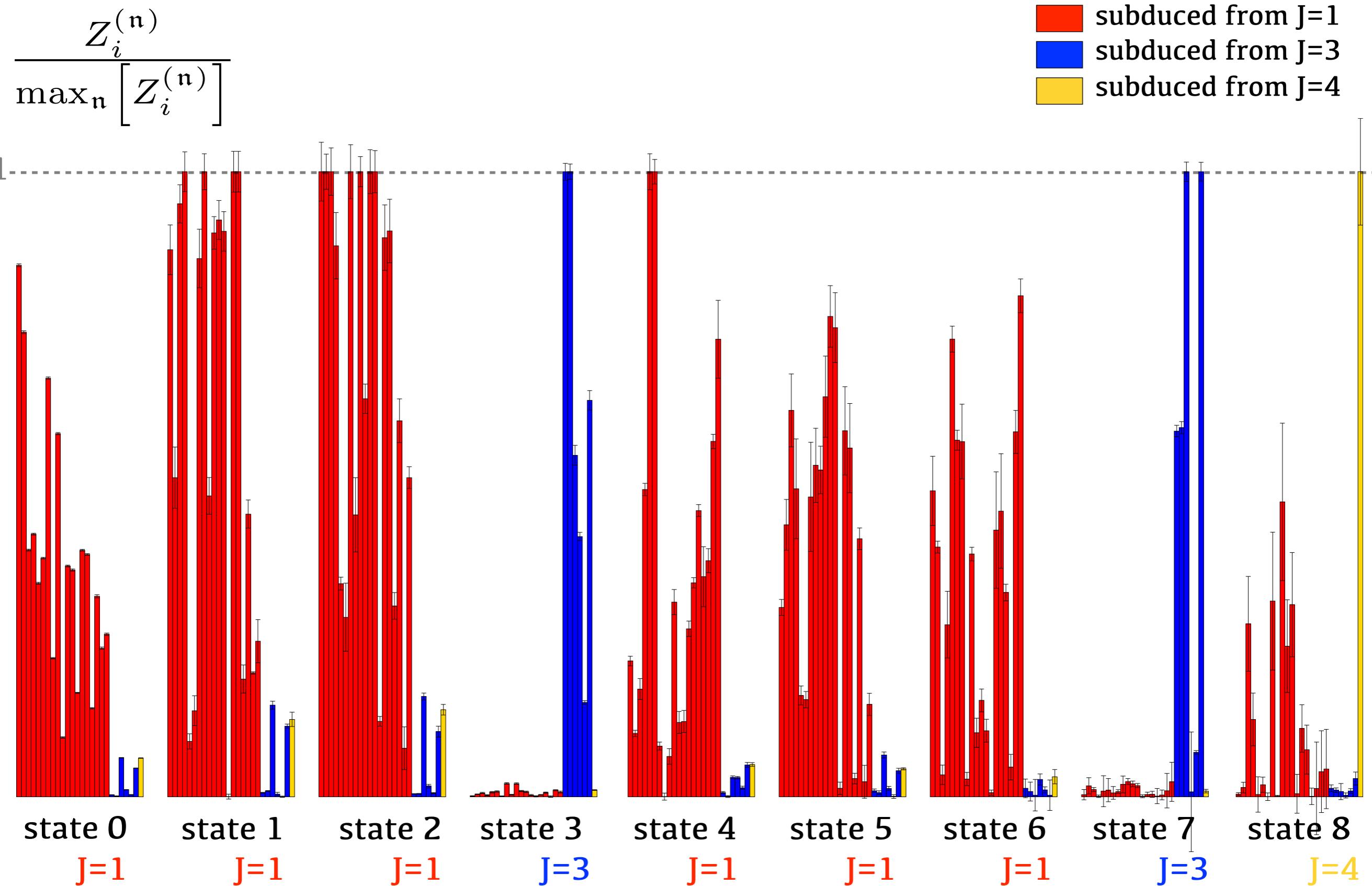
$$\lambda(t) = (1 - A)e^{-m(t-t_0)} + Ae^{-m'(t-t_0)}$$

$t_0/a_t = 7$



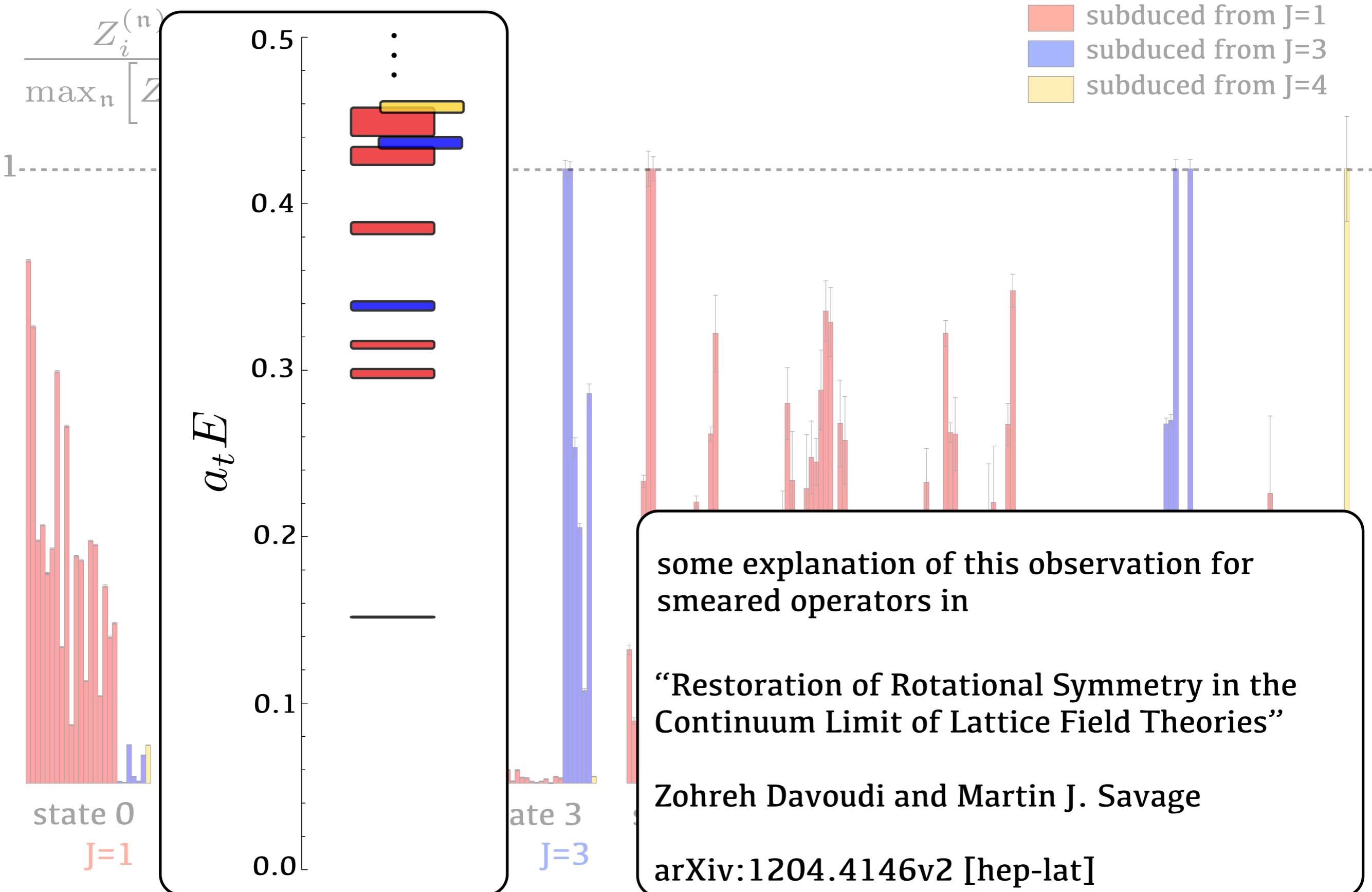
overlap matrix elements - T_1^{--}

$$Z_i^{(n)} = \langle n | O_i | 0 \rangle$$

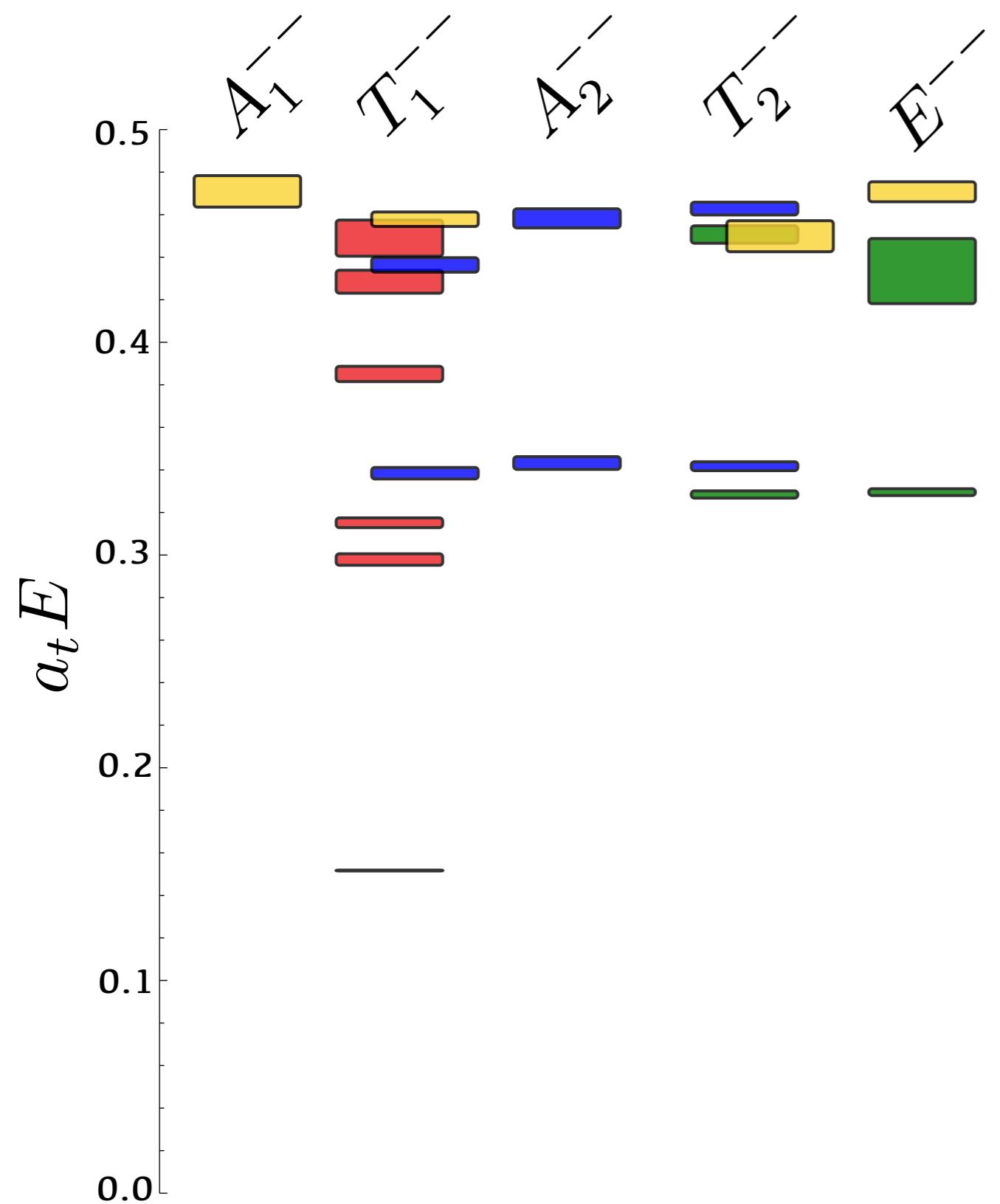


overlap matrix elements - T_1^{--}

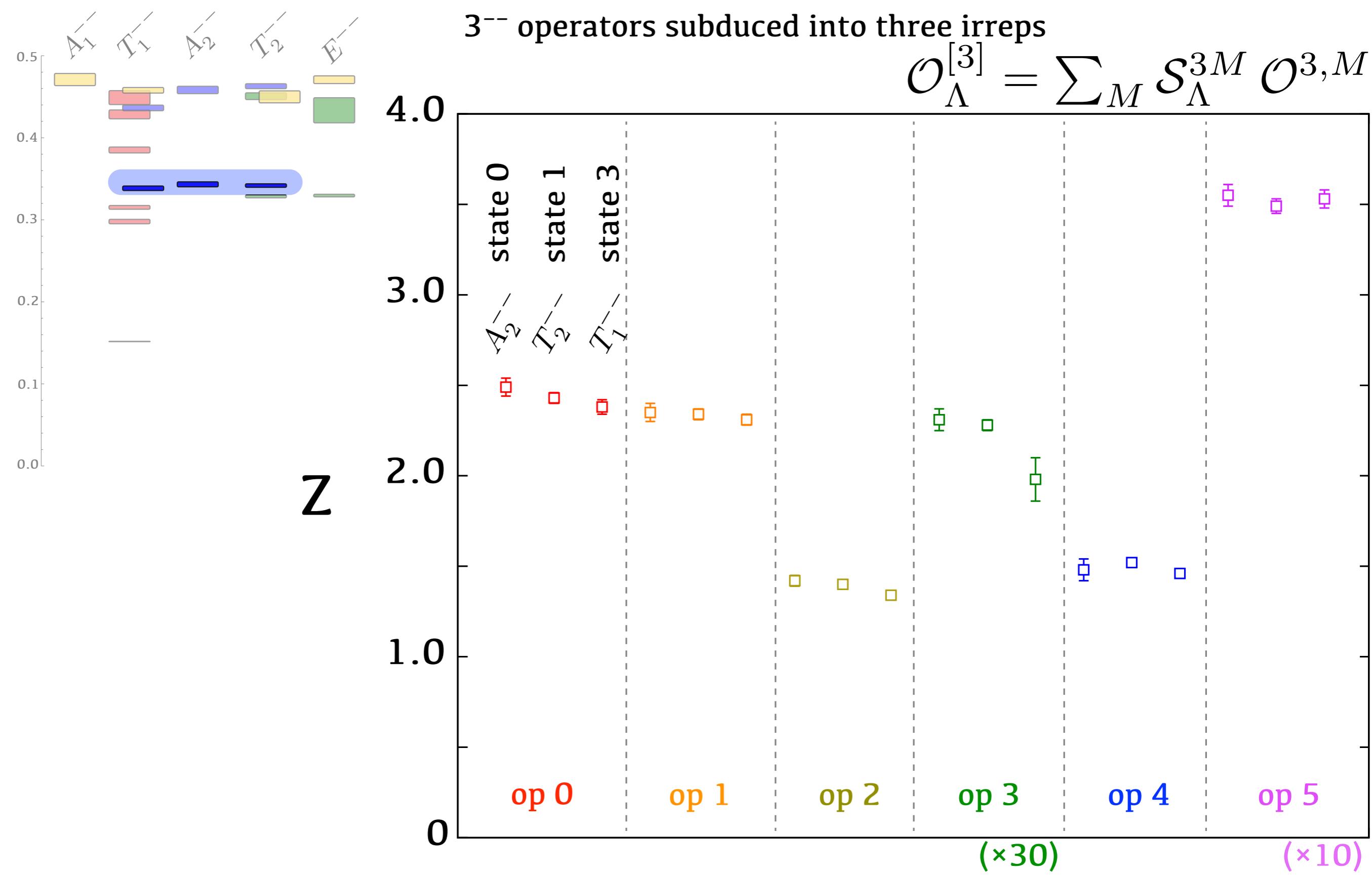
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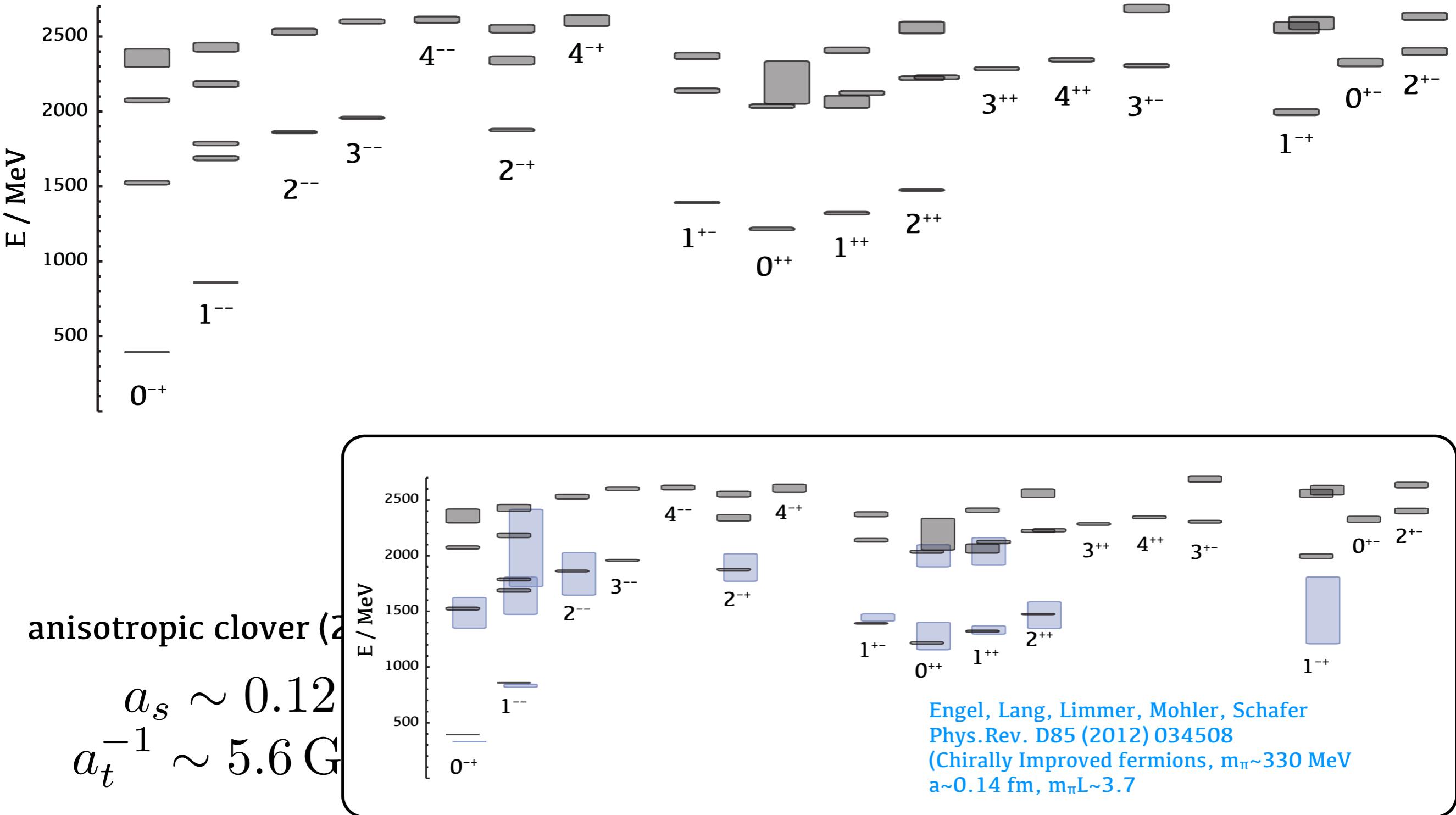
spin identification



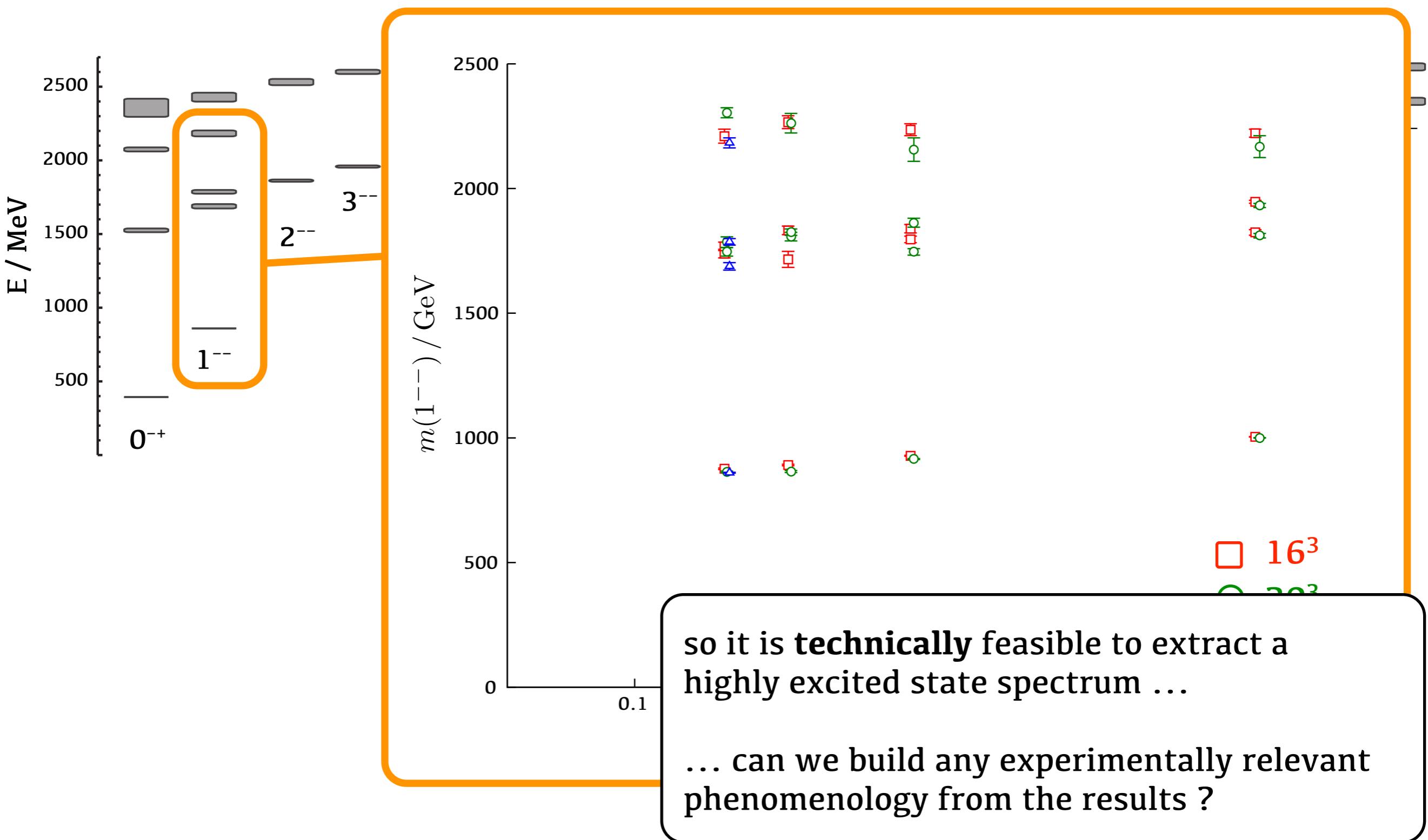
spin identification



a meson spectrum

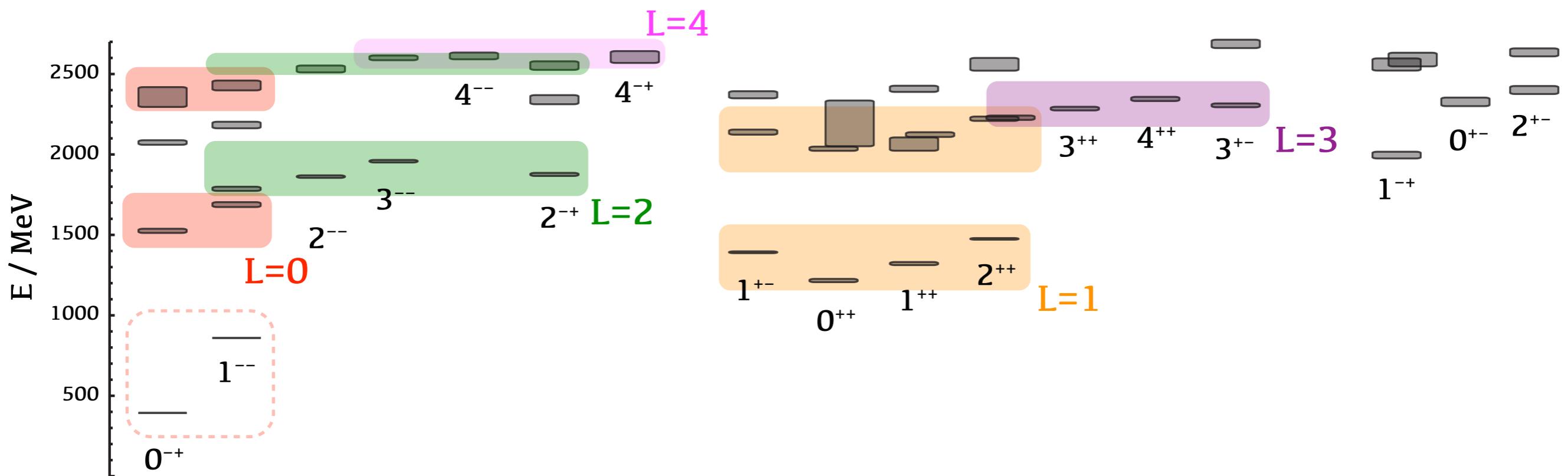


a meson spectrum



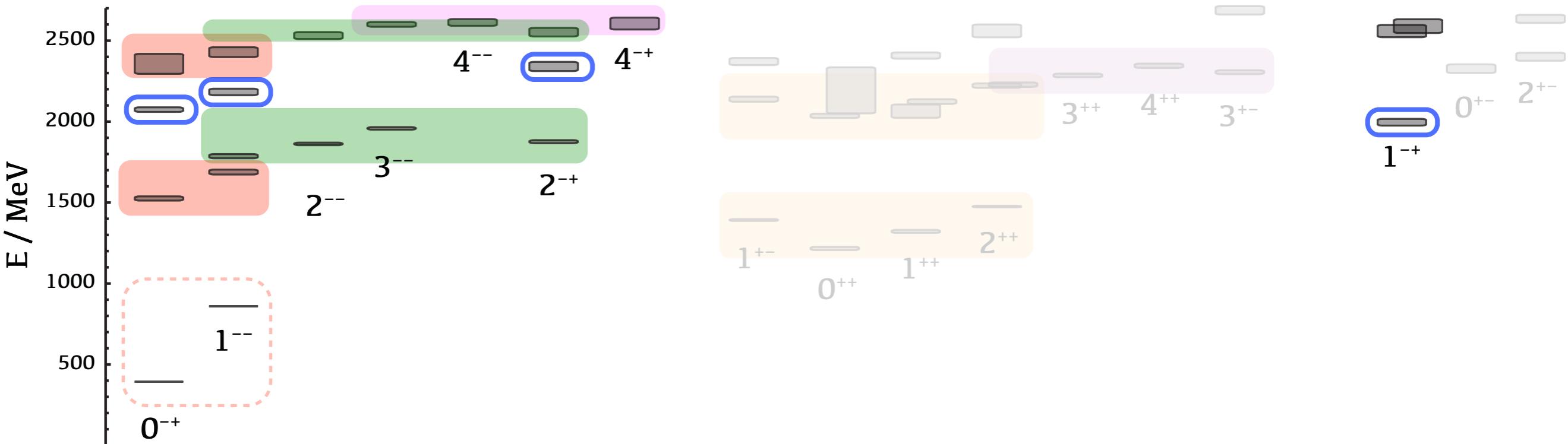
a meson spectrum

$q\bar{q}$ $2S+1L_J$ degeneracy pattern ?



a meson spectrum

what are these 'extra' states ?



some (model-dependent) interpretation

consider 3 of the 19 $J^{PC}=1^{--}$ operators

$$\rightarrow \bar{\tilde{\psi}} \gamma_i \tilde{\psi} \xrightarrow{\Lambda_{\frac{1}{2}[1-\gamma_0]}} {}^3S_1$$

upper component projector
“non-relativistic”

two-derivative constructions : $D_{J,m}^{[2]} = \langle 1m_1; 1m_2 | Jm \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$

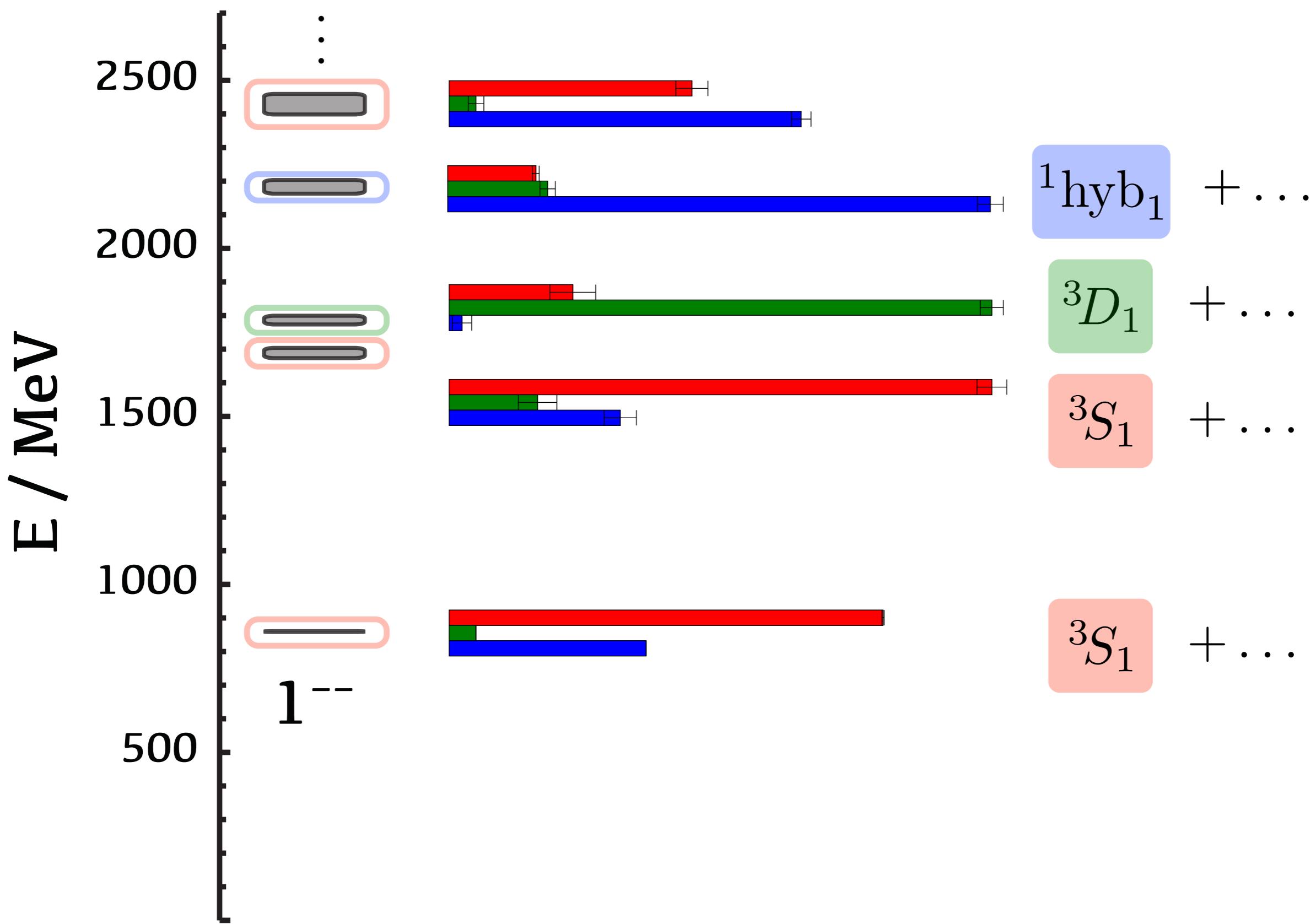
$$\rightarrow \langle 1m_1; 2m_2 | 1m \rangle \bar{\tilde{\psi}} \gamma_{m_1} D_{J=2, m_2}^{[2]} \tilde{\psi} \xrightarrow{\Lambda_{\frac{1}{2}[1-\gamma_0]}, \text{ignoring the gauge-field}} {}^3D_1$$

gauge-invariant version of a D-wave ?

$$\rightarrow \bar{\tilde{\psi}} \gamma_5 D_{J=1, m}^{[2]} \tilde{\psi} \xrightarrow{\Lambda_{\frac{1}{2}[1-\gamma_0]}, \text{ignoring the gauge-field}, [D, D] \rightarrow [\partial, \partial] = 0} {}^1\text{hyb}_1$$

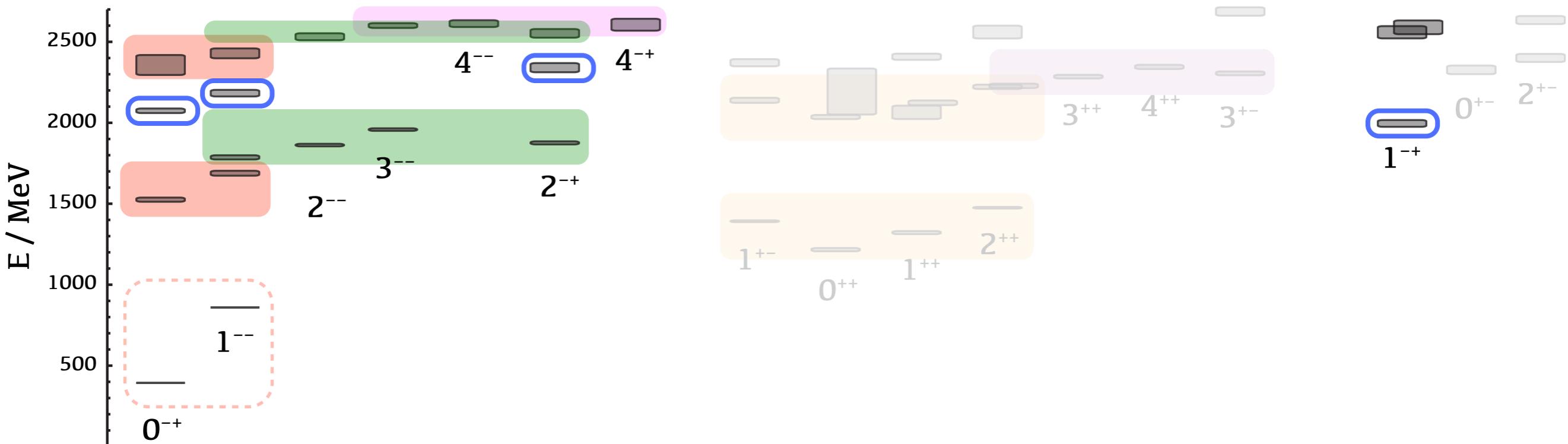
Phys. Rev. D84 074023 (2011)

some (model-dependent) interpretation

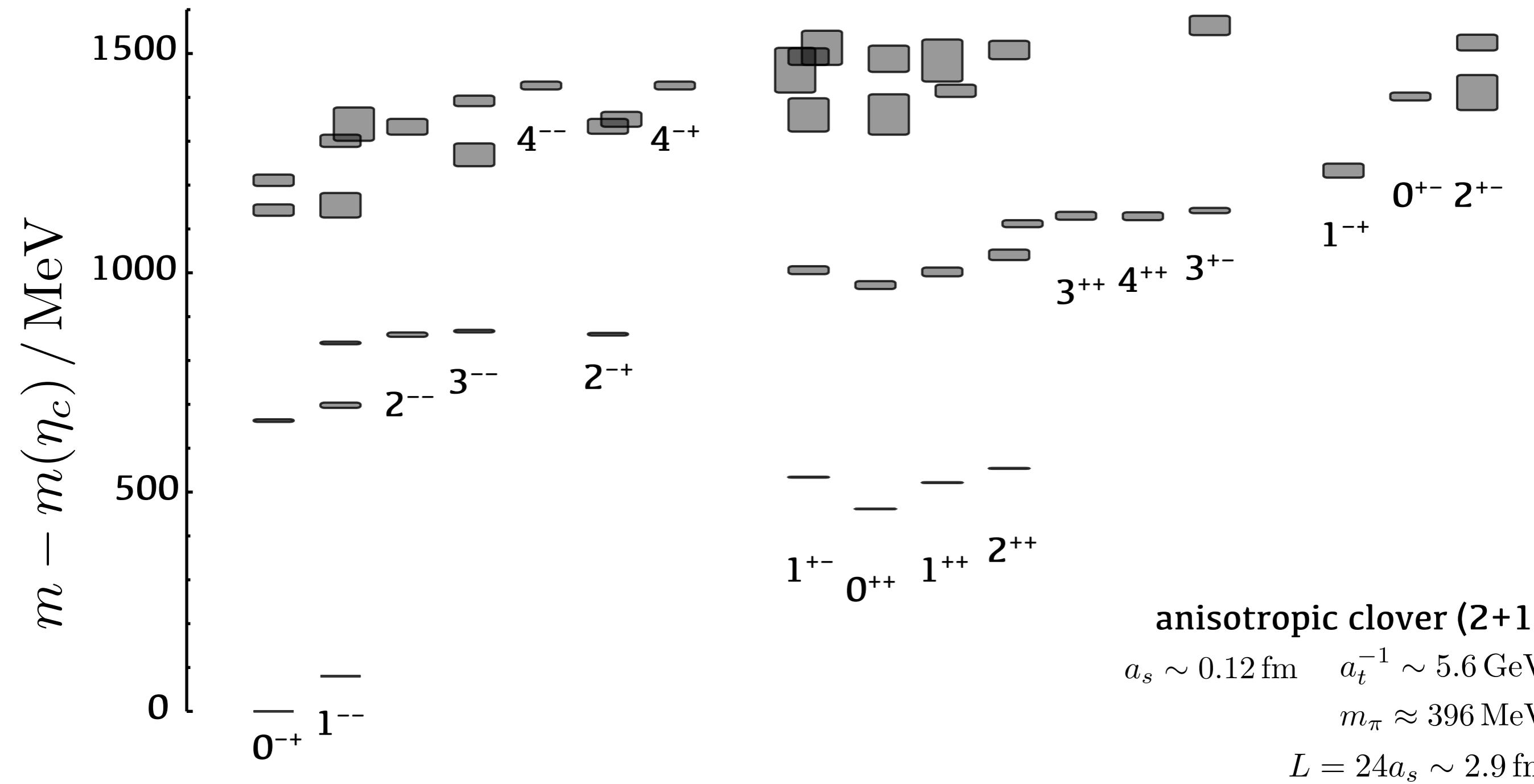


a meson spectrum

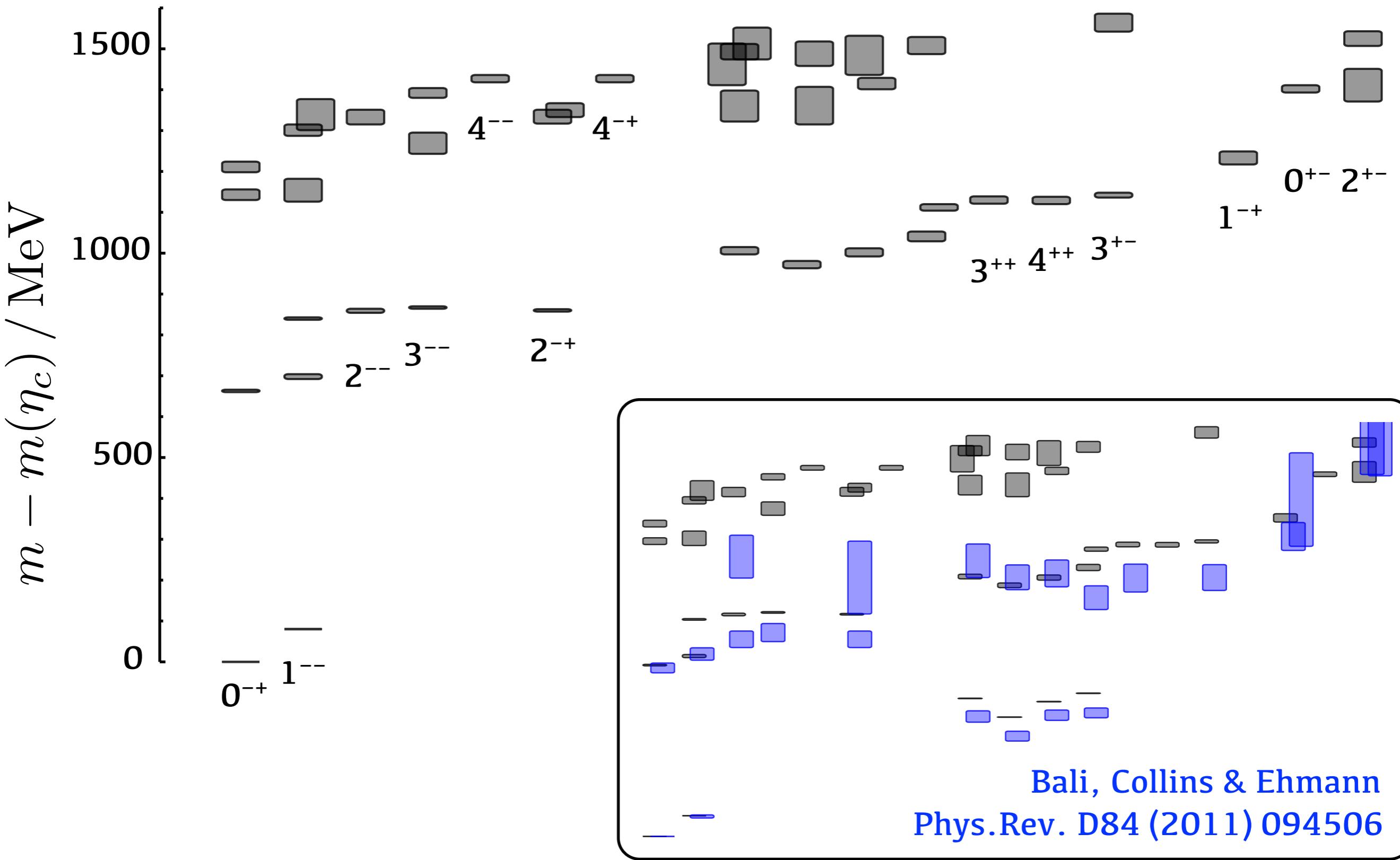
“extra” states are
hybrid mesons ?



wouldn't this ‘non-relativistic’ logic be
better justified in charmonium ... ?

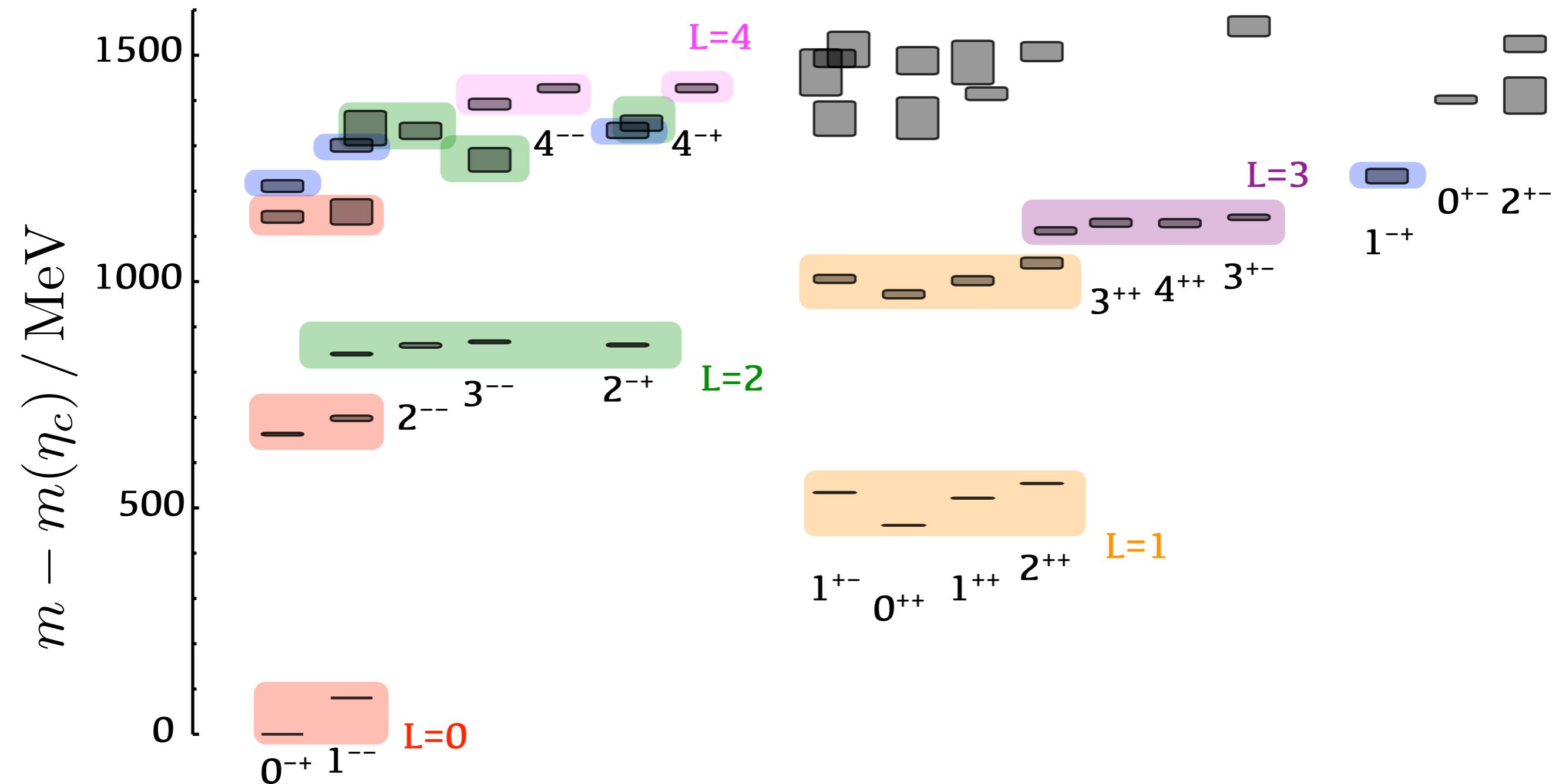


charmonium



charmonium

$c\bar{c}$ $2S+1L_J$ degeneracy & overlap pattern



a QCD phenomenology of hybrid mesons

replace model ‘guesswork’ with something motivated by these lattice calcs ...

$$\rightarrow D_{J=1,m}^{[2]} = \langle 1m_1; 1m_2 | 1m \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \sim B^a$$

a chromomagnetic field configuration is lowest excitation

$$q\bar{q}_8(^1S_0)B_8 \sim 0^{-+} \otimes 1^{+-} = 1^{--}$$

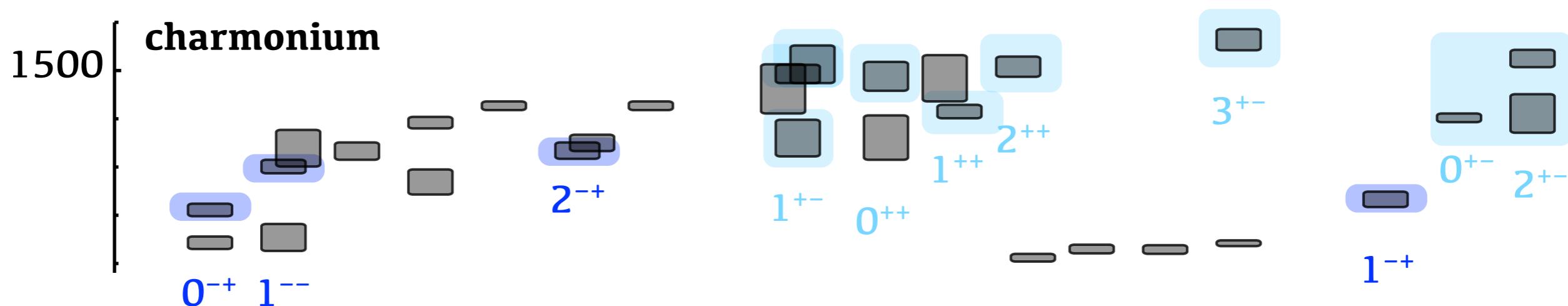
$$q\bar{q}_8(^3S_1)B_8 \sim 1^{--} \otimes 1^{+-} = (0, 1, 2)^{-+}$$

$$q\bar{q}_8(^1P_1)B_8 \sim 1^{+-} \otimes 1^{+-} = (0, 1, 2)^{++}$$

$$q\bar{q}_8(^3P_0)B_8 \sim 0^{++} \otimes 1^{+-} = 1^{+-}$$

$$q\bar{q}_8(^3P_1)B_8 \sim 1^{++} \otimes 1^{+-} = (0, 1, 2)^{+-}$$

$$q\bar{q}_8(^3P_2)B_8 \sim 2^{++} \otimes 1^{+-} = (1, 2, 3)^{+-}$$



a QCD phenomenology of hybrid mesons

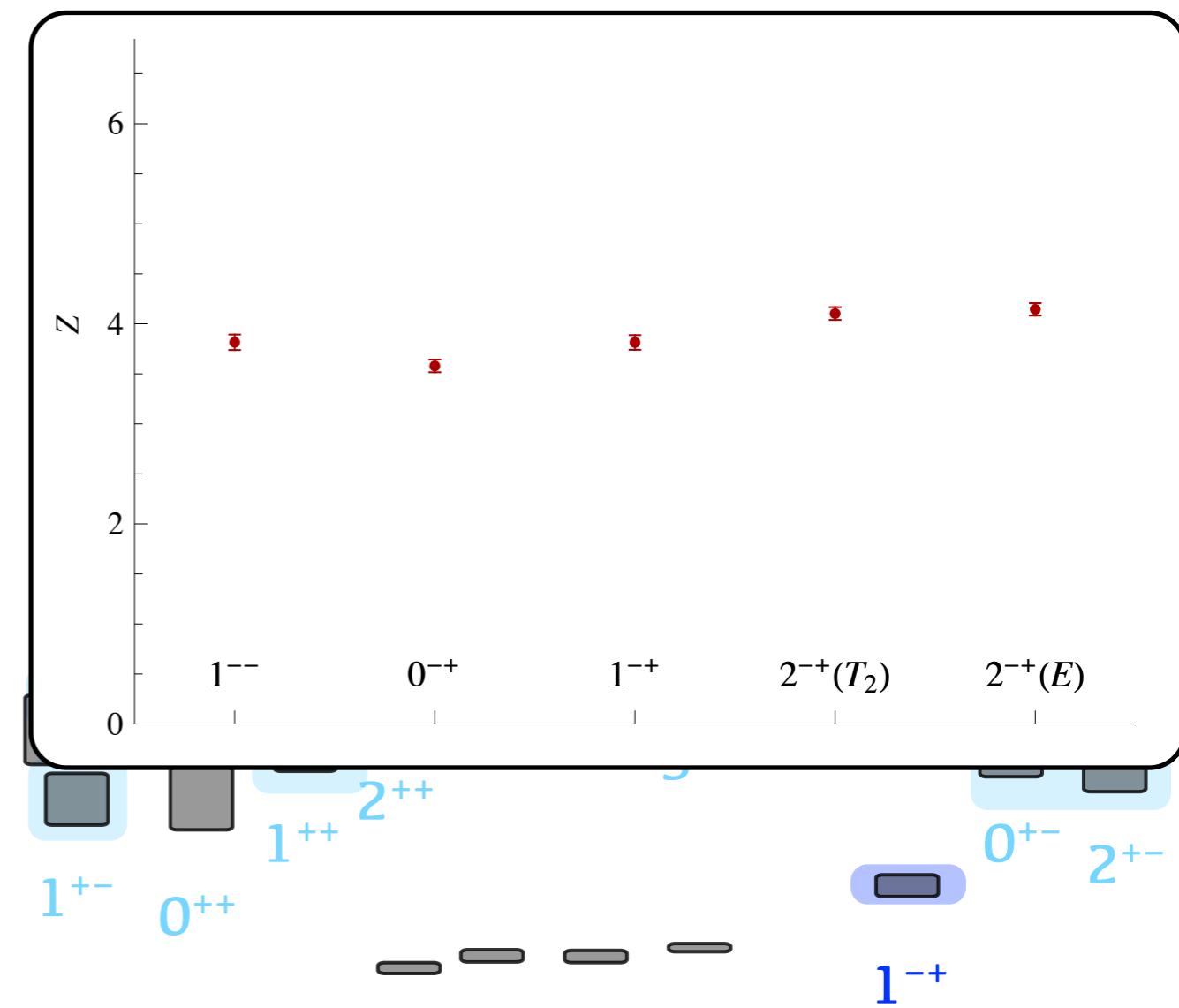
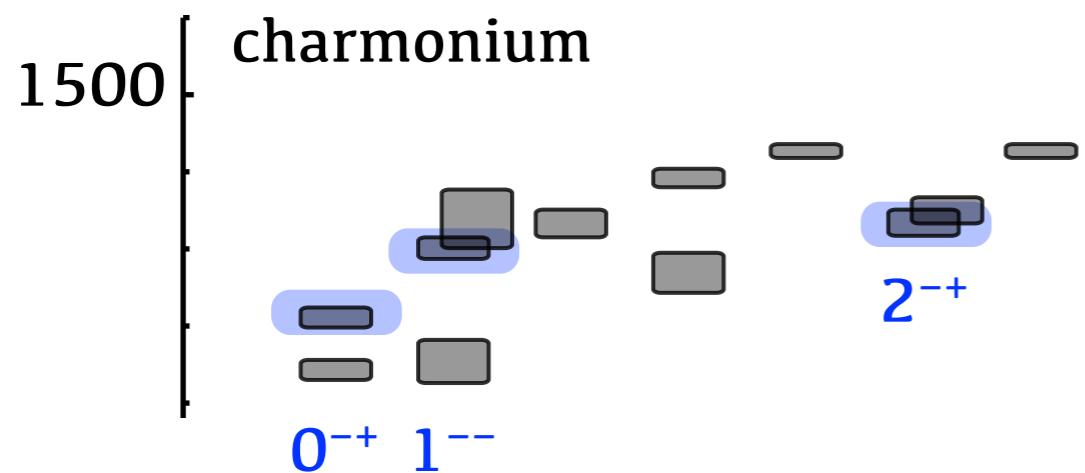
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$$q\bar{q}_8(^1S_0)B_8 \sim 0^{-+} \otimes 1^{+-} = 1^{--}$$

$$q\bar{q}_8(^3S_1)B_8 \sim 1^{--} \otimes 1^{+-} = (0, 1, 2)^{-+}$$



a QCD phenomenology of hybrid mesons

some candidate states in the experimental spectrum

$\pi(1800)$

$I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 1812 \pm 12$ MeV ($S = 2.3$)
Full width $\Gamma = 208 \pm 12$ MeV

$\pi_2(1880)$

$I^G(J^{PC}) = 1^-(2^{-+})$

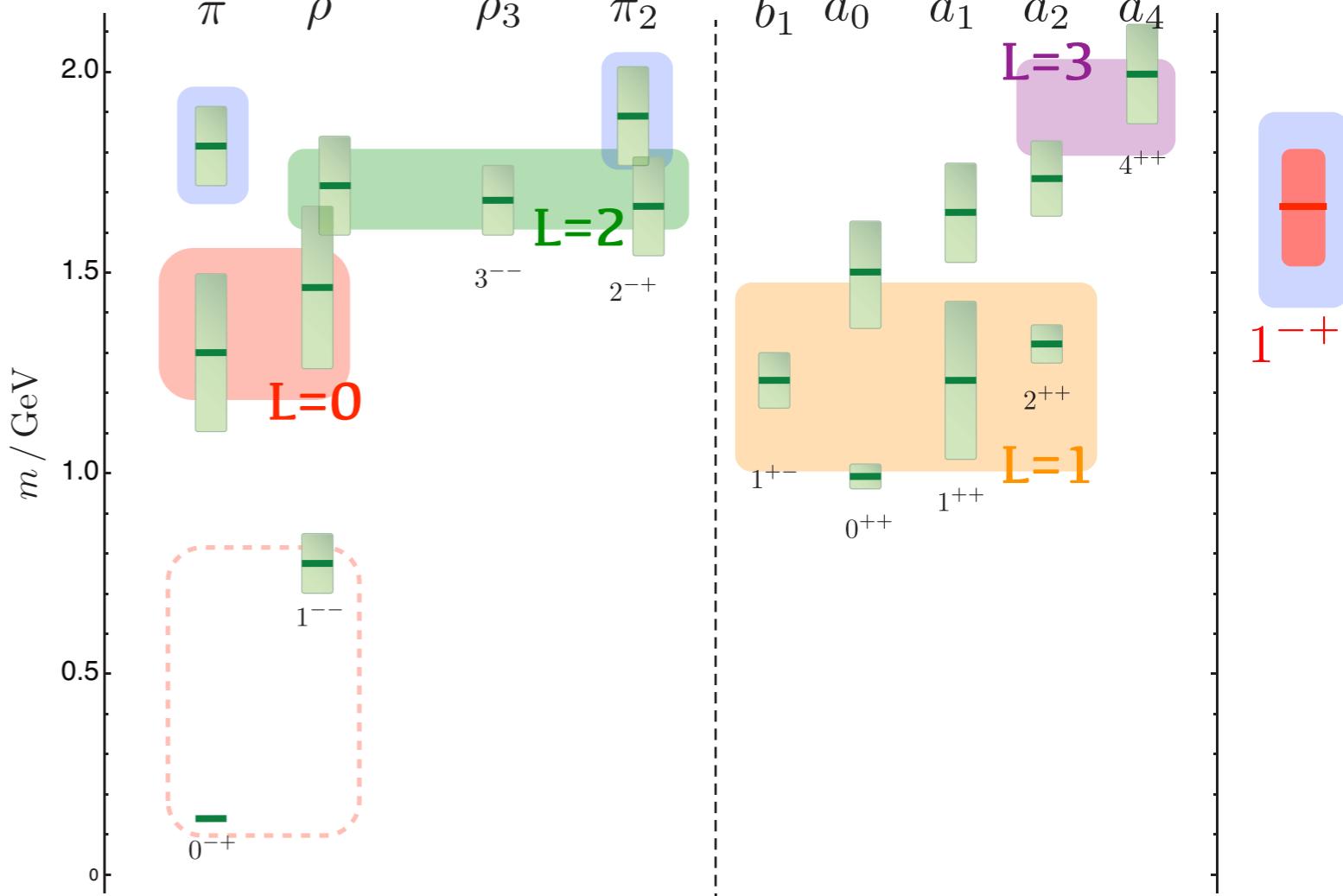
Mass $m = 1895 \pm 16$ MeV
Full width $\Gamma = 235 \pm 34$ MeV

$\pi_1(1600) [m]$

$I^G(J^{PC}) = 1^-(1^{-+})$

Mass $m = 1662_{-9}^{+8}$ MeV
Full width $\Gamma = 241 \pm 40$ MeV ($S = 1.4$)

empirical J^{PC} distribution



in charmonium the “Y”(4260)

$X(4260)$

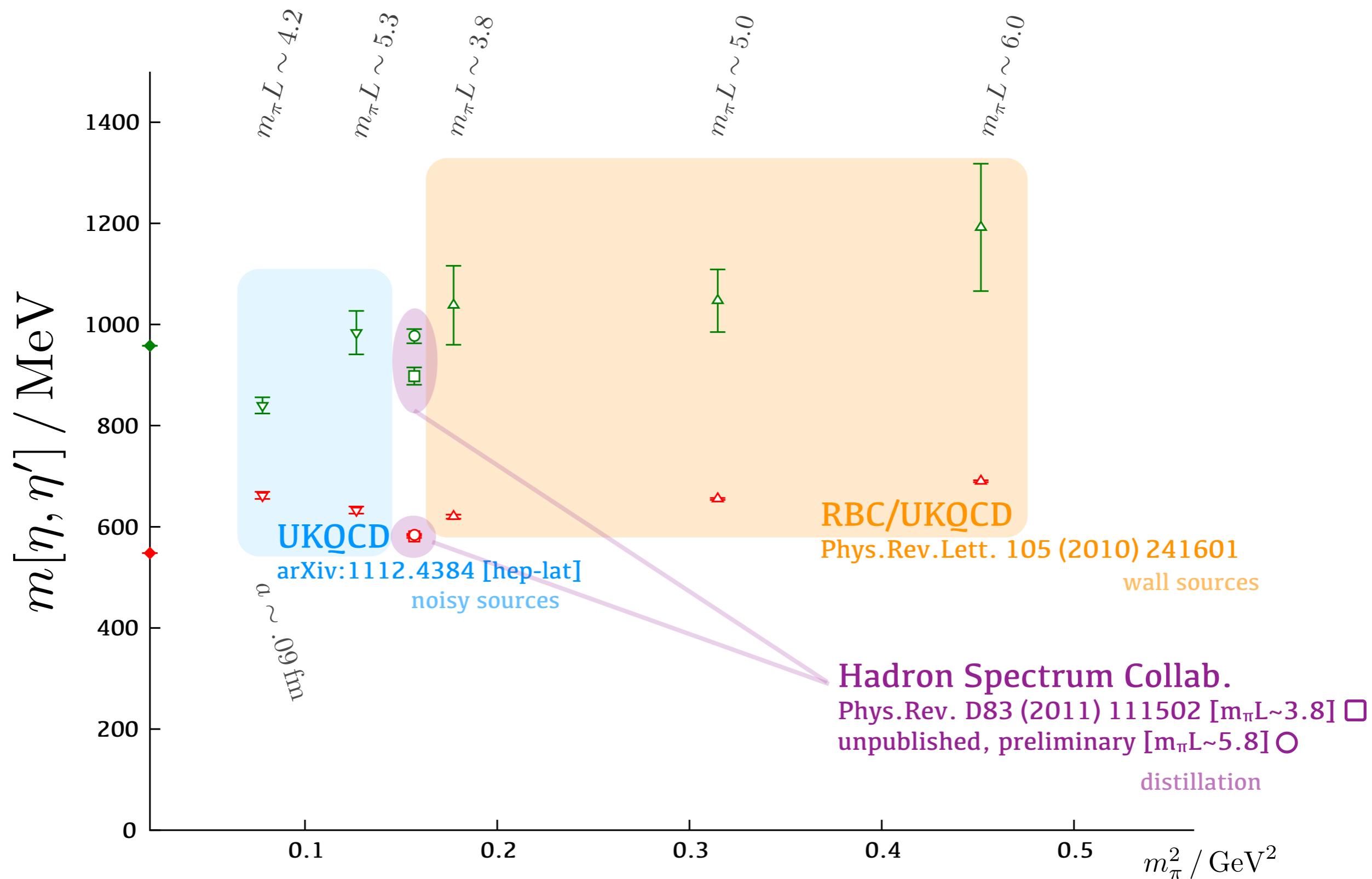
$I^G(J^{PC}) = ??(1^{--})$

Mass $m = 4263_{-9}^{+8}$ MeV ($S = 1.1$)
Full width $\Gamma = 95 \pm 14$ MeV

mass scale looks OK [$m(\eta_c) + 1300$ MeV]
but produced in e^+e^- - must have 3S_1 component
(need to compute the vector decay constant)

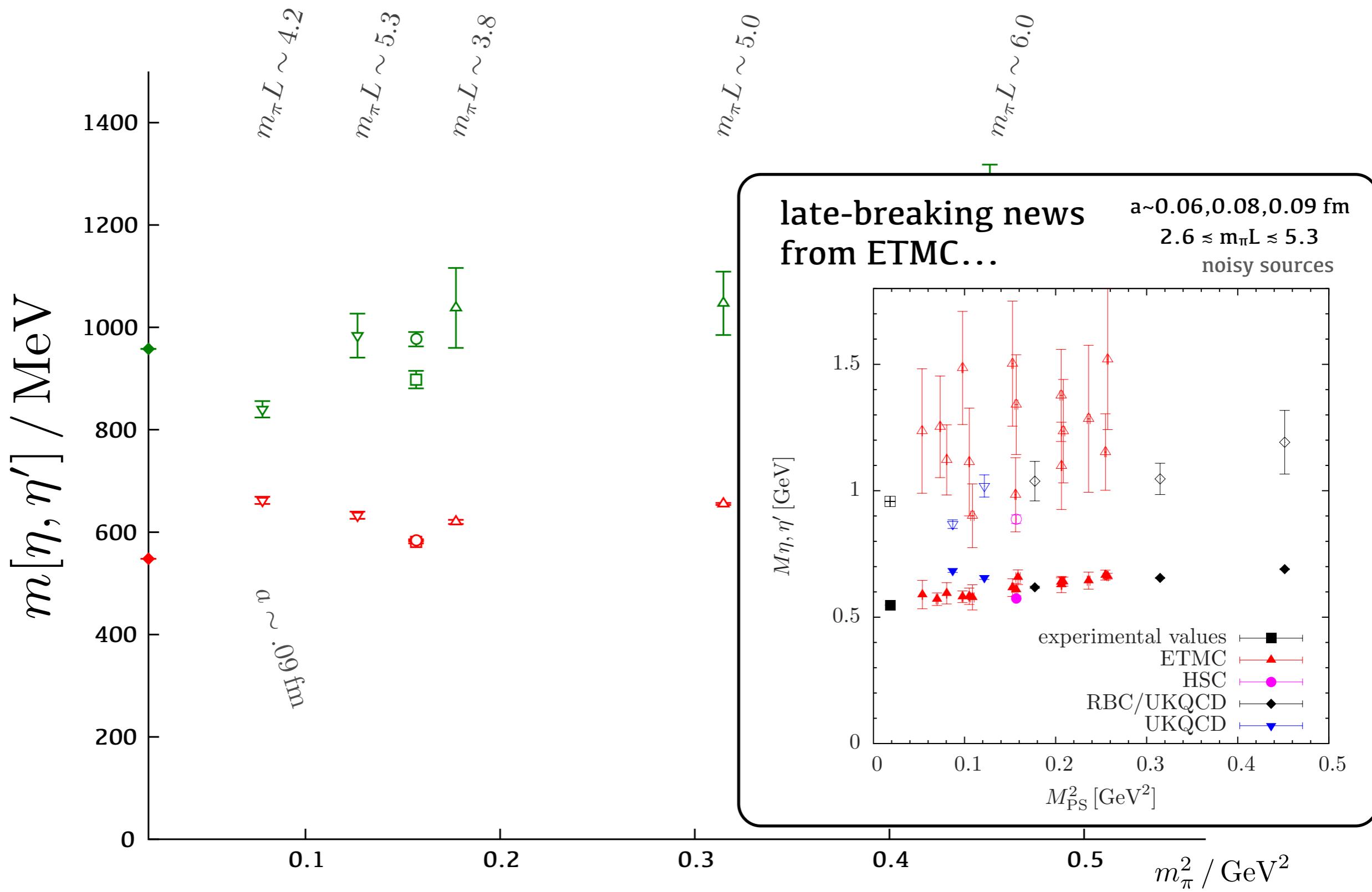
isoscalars - η & η'

mostly $a \sim 0.12$ fm

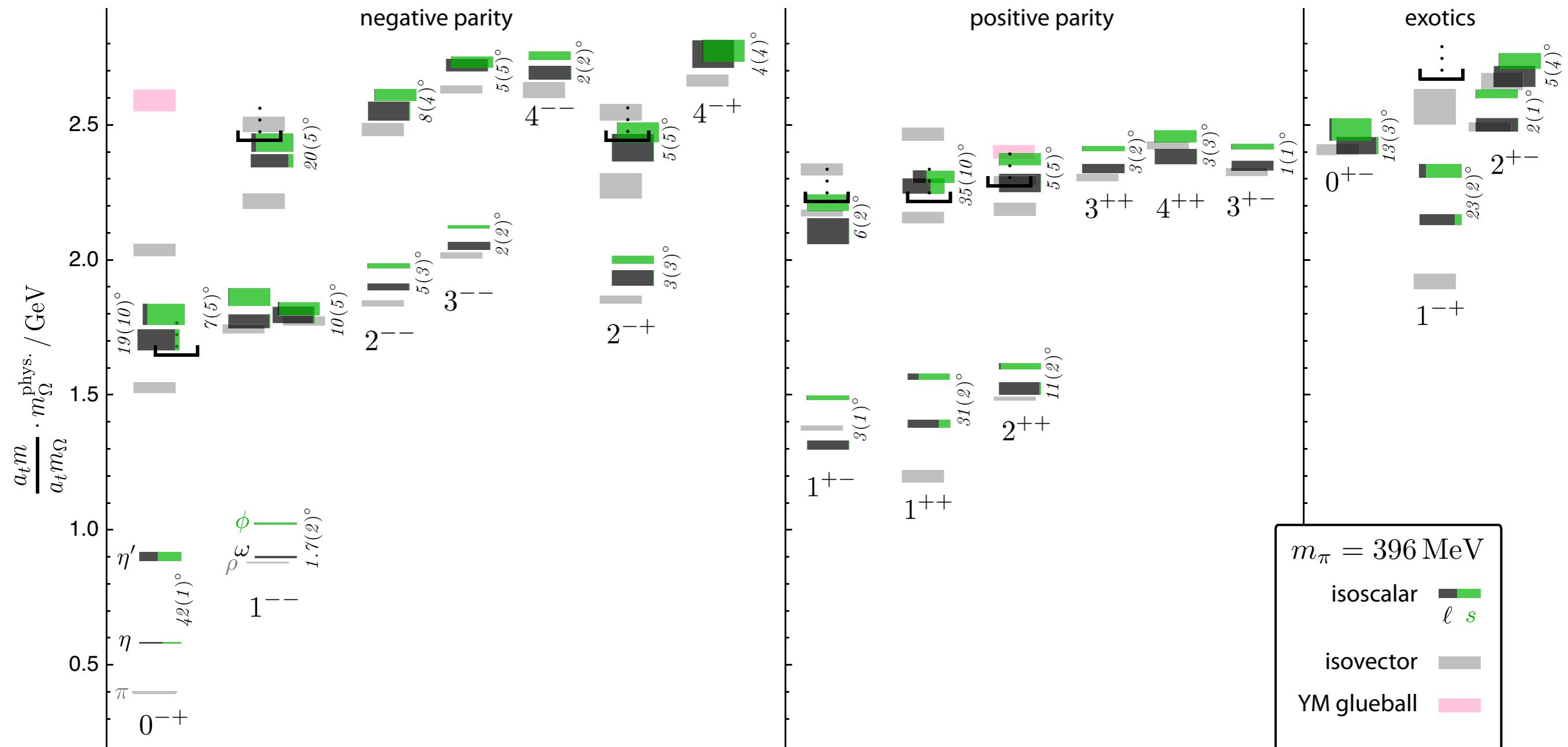


isoscalars - η & η'

mostly $a \sim 0.12$ fm



isoscalars - excited state spectrum

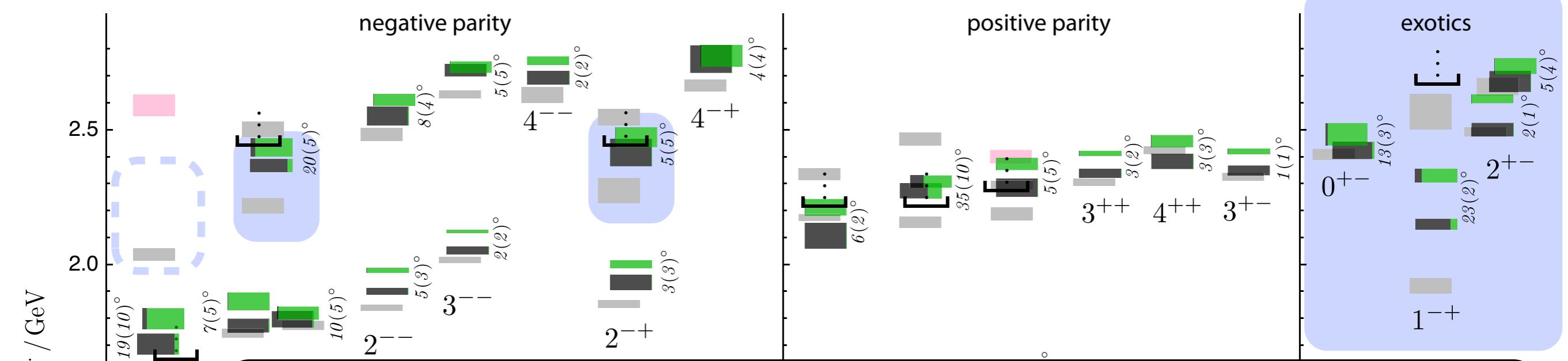


**small volume ($16^3 \times 128$)
(analysing $24^3 \times 128$ now) $m_\pi L \sim 3.8$**

distillation inversion cost is spread over as large an operator basis as you like

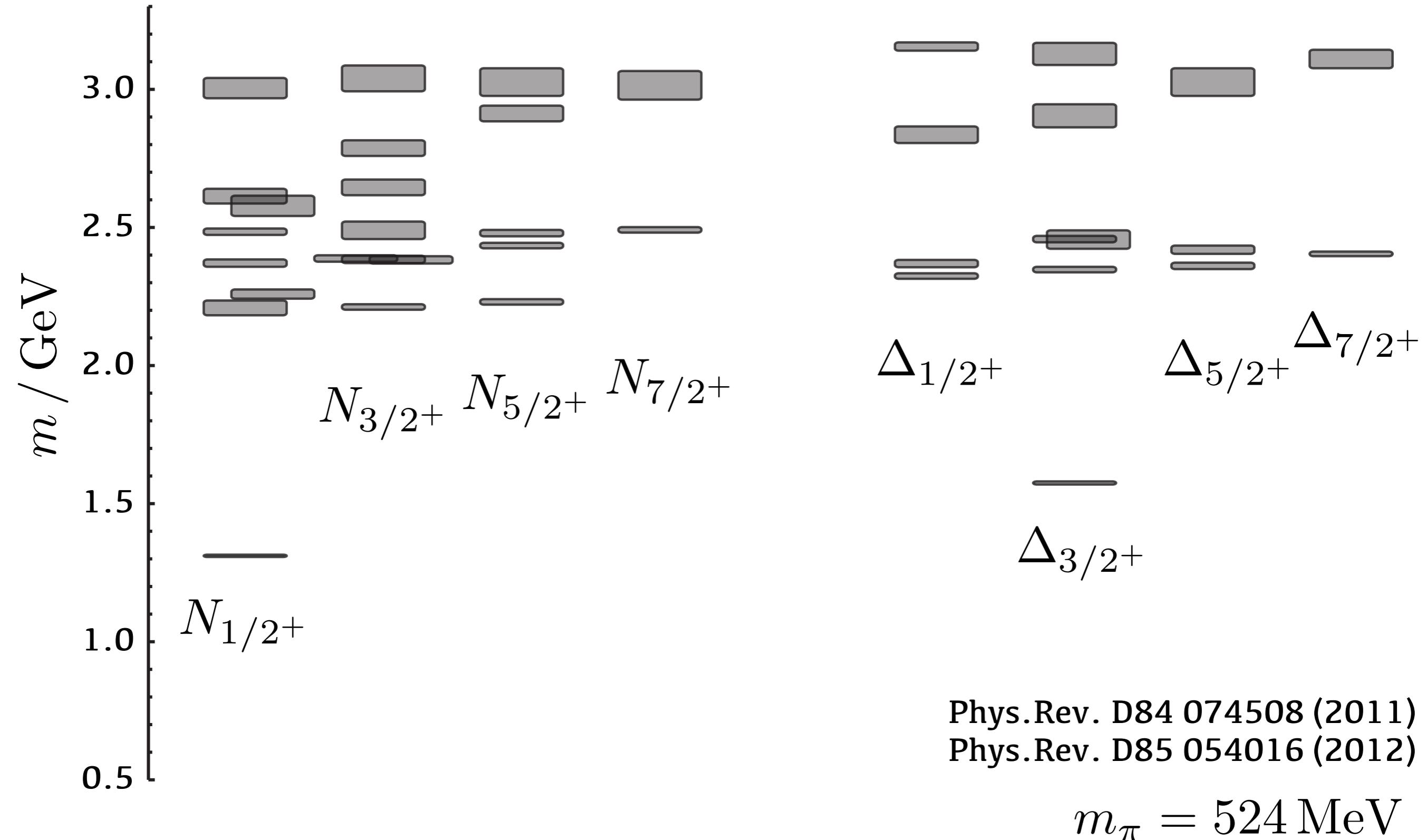
glueball operators too noisy ?

impact on experiment

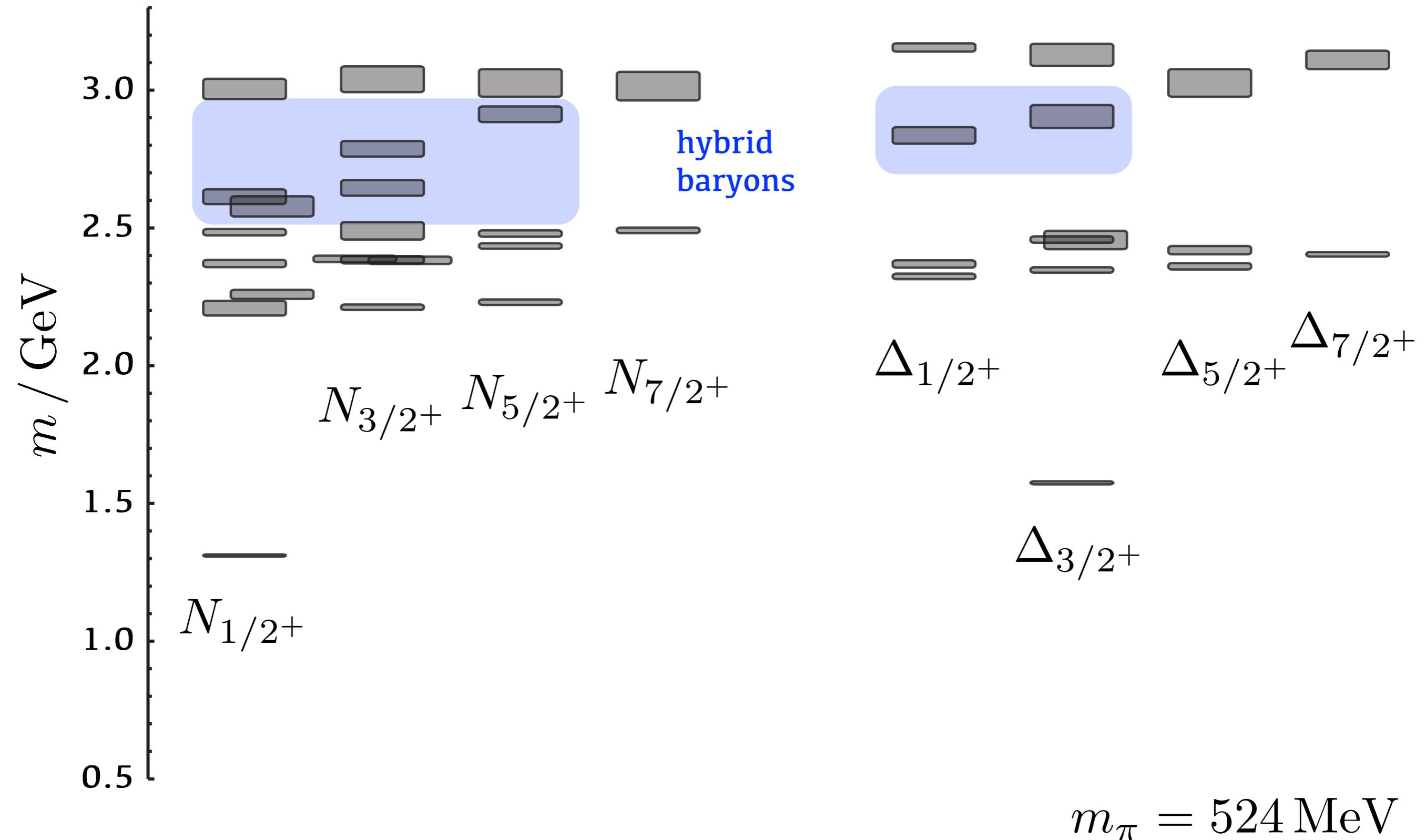


a light baryon spectrum

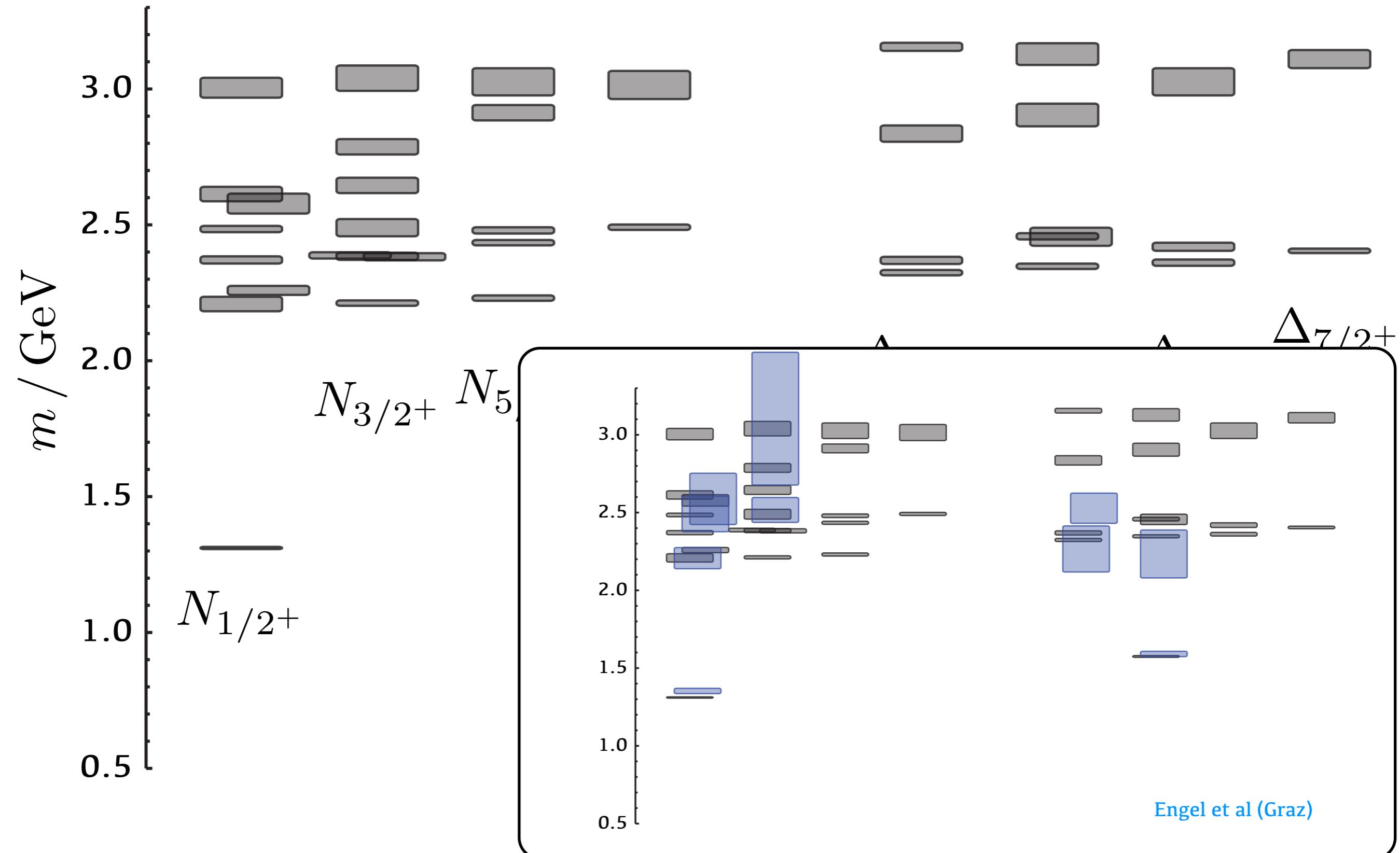
three-quark field operator basis with up to two derivatives



a light baryon spectrum

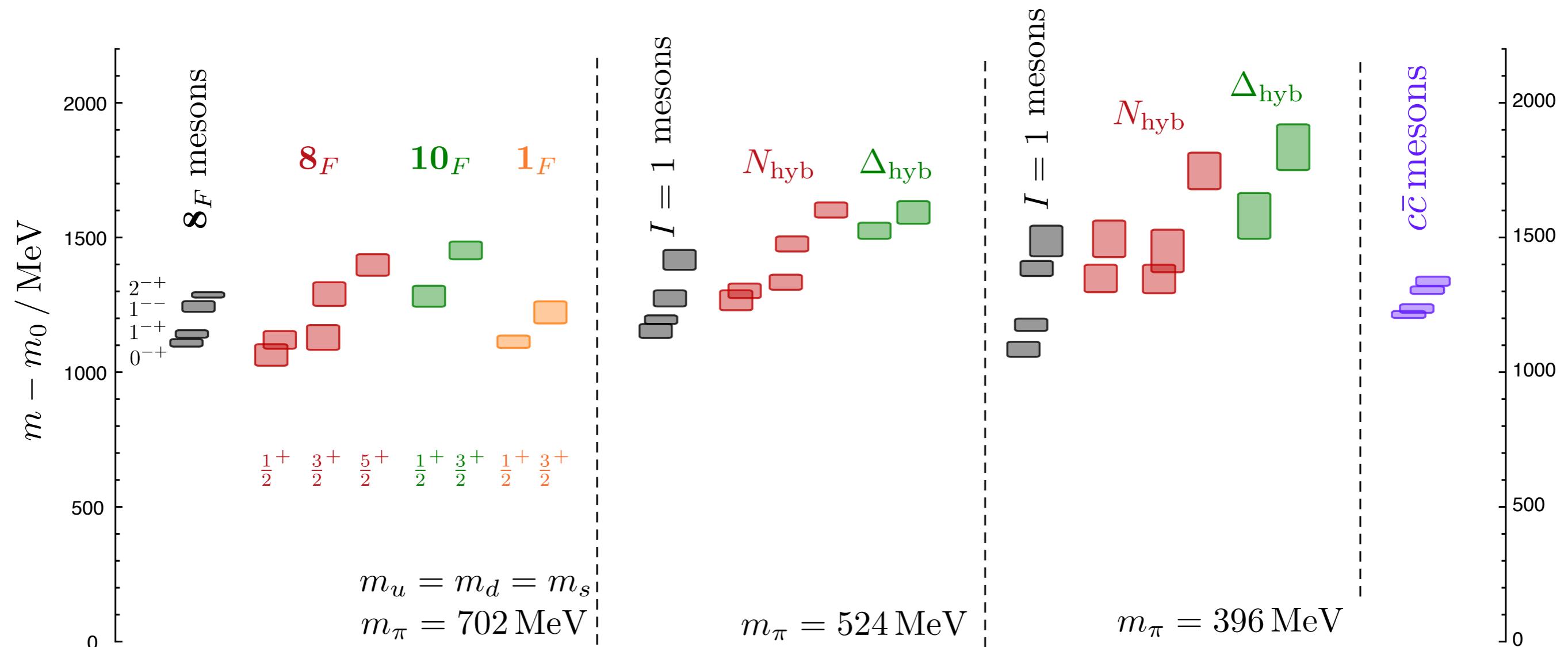


a light baryon spectrum



hybrids (mesons and baryons)

approximately remove the ‘quark mass’ contribution to the hybrid mass



light hybrid mesons - m_ρ

light hybrid baryon - m_N

charmonium hybrids - m_{η_c}

chromomagnetic gluonic excitation scale
~ 1.3-1.4 GeV ?

a model dependent interpretation of lattice QCD calculations

including multi-meson operators

next step is to include operators that resemble multi-mesons into the basis

$$\sum_{\vec{x}} e^{-i\vec{p}_1 \cdot \vec{x}} \bar{\psi} \Gamma_1 \tilde{\psi}_{\vec{x},t} \sum_{\vec{y}} e^{-i\vec{p}_2 \cdot \vec{y}} \bar{\psi} \Gamma_2 \tilde{\psi}_{\vec{y},t}$$

$M(\vec{p}_1) M(\vec{p}_2)$

projected into definite little-group irreps

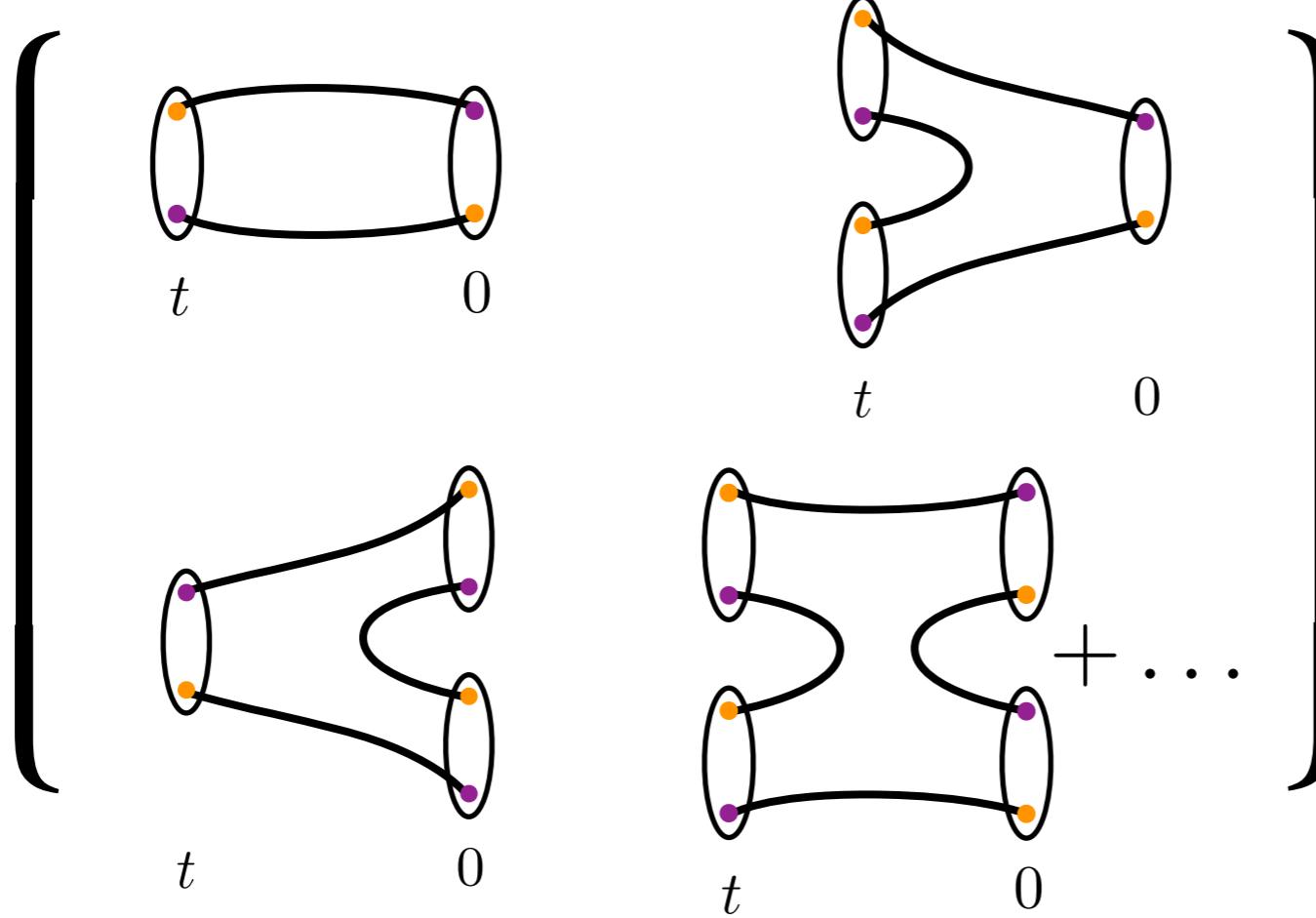
(arXiv:1203.6041 [hep-ph])

basis of various

$|\vec{p}_1|, |\vec{p}_2|$

(relatively
straightforward
with distillation)

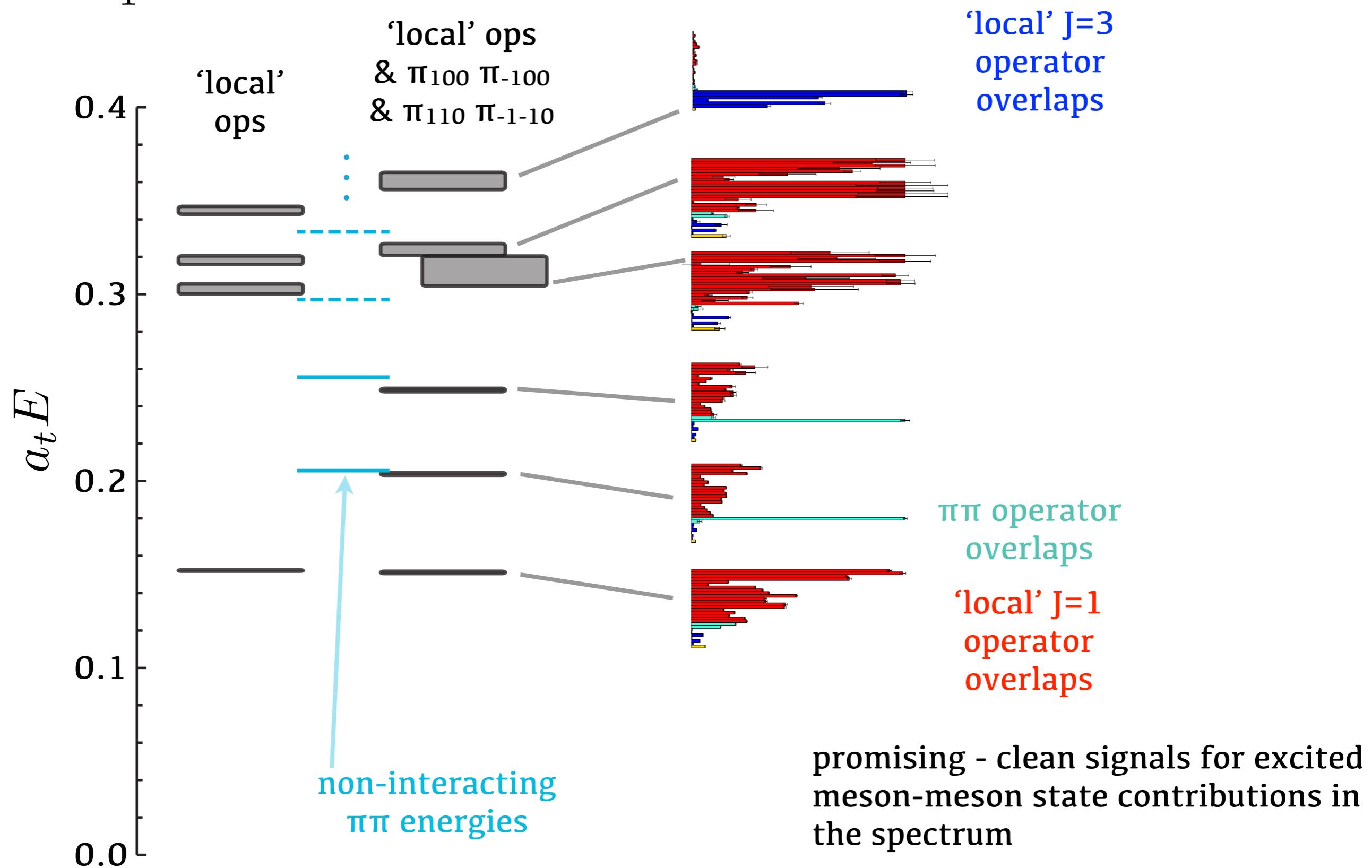
$C(t) =$



solve variational problem in this extended basis ...

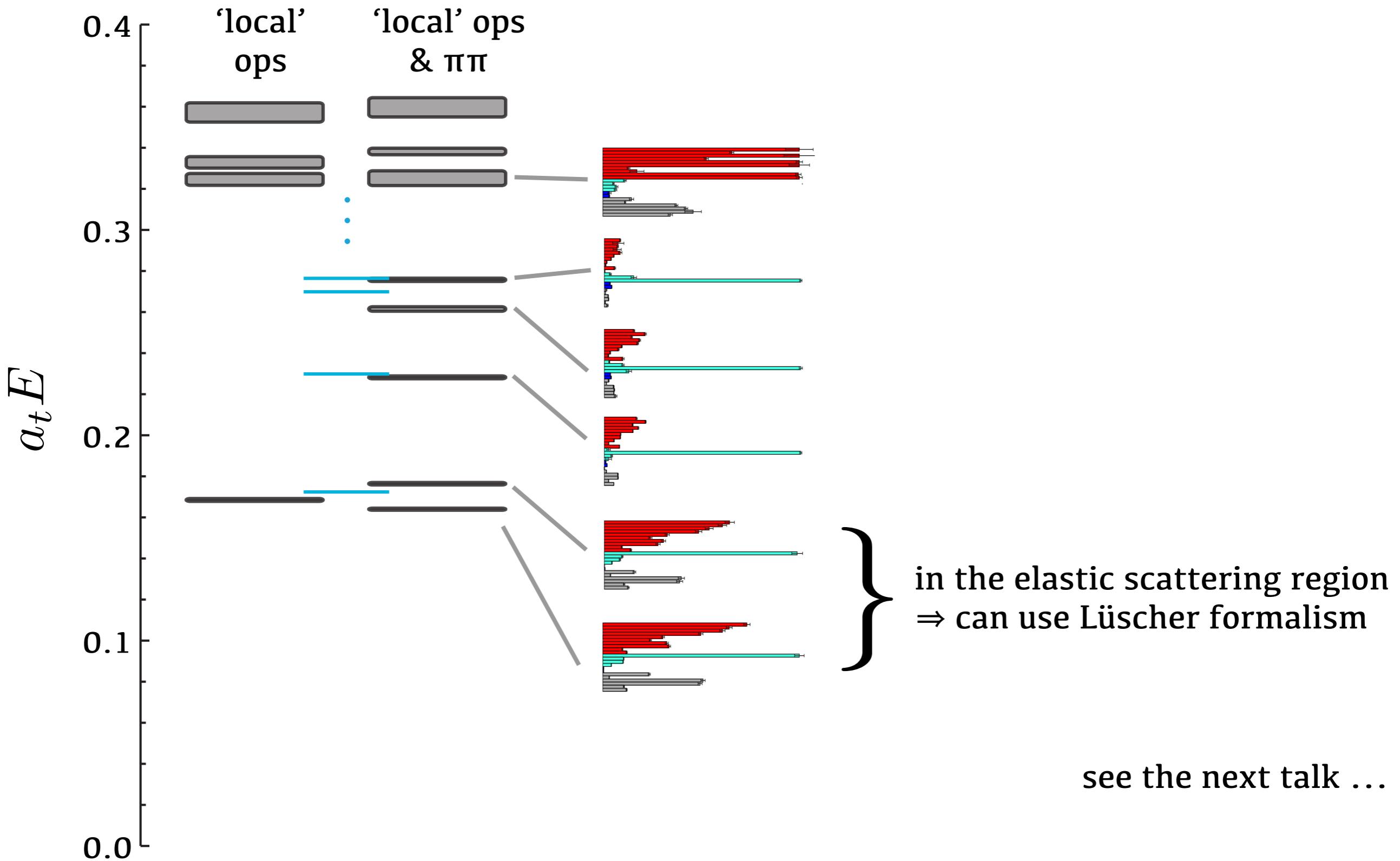
including multi-meson operators

e.g. T_1^{--} on 24^3



including multi-meson operators

e.g. $\text{Dic}_4 A_1^-$ $P=[100]$ on 24^3 ("in-flight helicity zero")



summary

- a large basis of ‘local’ operators + variational analysis
 - surprisingly good returns on investment
 - clean excited state spectra
 - possible interpretation of gross structures including hybrids
 - first directly QCD-based phenomenology of hybrids
news for COMPASS, JLab, BES ...
 - these calculations not complete
 - finite-volume spectrum contains states not well interpolated by these ops
 - including ‘meson-meson’ ops reveals their presence
 - signals can be statistically very clean