The muon anomalous magnetic moment

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Collaborators

Work on g-2 done in collaboration with

HVP	HLbL
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	Eigo Shintani (RBRC)
	Norikazu Yamada (KEK)

Outline

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) = -\vec{\mu} \cdot \vec{B}_{\mathrm{ext}}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin $(c = \hbar = 1)$

$$\vec{\mu} = g\left(\frac{e}{2m}\right)\vec{S}$$

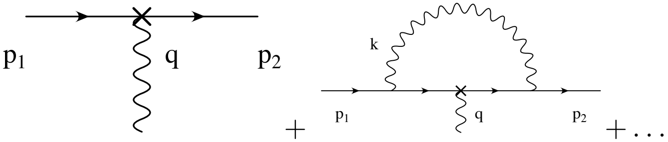
The Landé g-factor is predicted from the free Dirac eq. to be

$$g = 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



$$\gamma^\mu \quad \rightarrow \quad \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2) \right)$$

which results from Lorentz and gauge invariance when the muon is on-mass-shell.

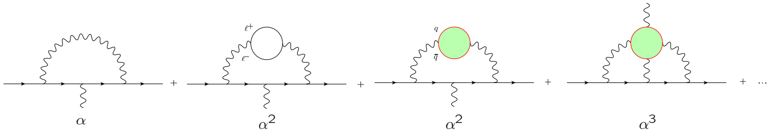
$$F_2(0) \quad = \quad \frac{g-2}{2} \equiv a_\mu \qquad\qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$

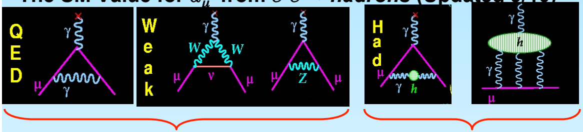


Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

hadronic contributions $\sim 6 \times 10^{-5}$ times smaller (leading error).

Value in the standard model

The SM Value for a_μ from $e^+e^- \rightarrow$ *hadrons* (Updated 9/10)



CONTRIBUTION	RESULT ($\times 10^{-11}$) UNITS
QED (leptons)	$116\ 584\ 718.09 \pm 0.14 \pm 0.04_\alpha$
HVP(lo)	$6\ 914 \pm 42_{\rm exp} \pm 14_{\rm rad} \pm 7_{\rm pQCD}$
HVP(ho)	$-98 \pm 1_{\rm exp} \pm 0.3_{\rm rad}$
HLxL	105 ± 26
EW	$152 \pm 2 \pm 1$
Total SM	$116\ 591\ 793 \pm 51$

A. Höcker Tau 2010, U. Manchester September 2010

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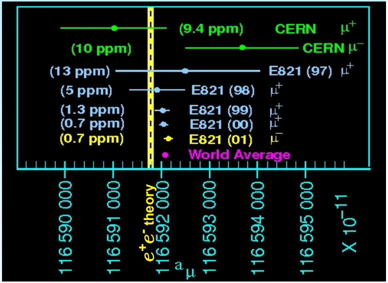
- new: QED thru $O(\alpha^5)$ [Aoyama, *et al.*, 2012]

Experimental value (dominated by BNL E821)

E821 achieved ± 0.54 ppm. The e^+e^- based theory is at the ~0.4 ppm level. Difference is ~3.6 σ



$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \ (0.54\,\mathrm{ppm})$$

$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}$$

Theory: arXiv:1010.4180v1 [hep-ph] Davier, Hoecker, Malaescu, and Zhang, Tau2010

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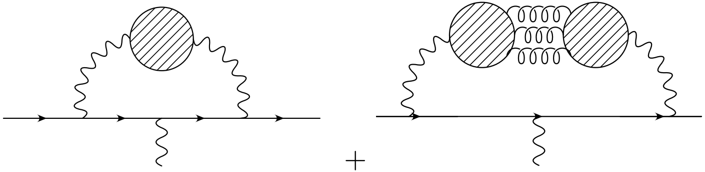
New experiments + new theory

- Fermilab E989, $\sim$ 5 years away, 0.14 ppm
- J-PARC E34 ? (recently, lower priority than $\mu \to e$)
- $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) = 287(63)(51)$ ($\times 10^{-11}$), or $\sim 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\to\sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\to\sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\to\sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations crucial
- a_μ good for constraining and explaining BSM physics

Outline

Hadronic vacuum polarization (HVP)



The blobs, which represent all possible intermediate hadronic states, are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for $e^+e^-(\text{and } \tau) \to$ hadrons $a_\mu^{\mathrm{had}(2)} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\sigma_{\mathrm{total}}(s)$
- first principles using lattice QCD, $a_\mu^{(2)\mathrm{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\, \Pi(Q^2)$ [Lautrup and de Rafael 1969, Blum 2002]

$a_\mu(\mathrm{HVP})$ from e^+e^- and $\tau \to \mathrm{hadrons}$

- Many expts. contributing: BaBar, Belle, BES, KLOE, VEPP2000, ...
- Current precision (Davier, *et al.*, 2011, or Hagiwara, *etal*. 2011)
  - $a_\mu(\mathrm{HVP}) = 692.3 \pm 4.2 \times 10^{-10}$ (e^+e^-)
  - $a_\mu(\mathrm{HVP}) = 701.5 \pm 4.7 \times 10^{-10}$ $(e^+e^- + \tau)$
  - 0.61 and 0.67 %, 3.6 and 2.4 σ
- Expected precision in 3-5 years 2-3$\times 10^{-10}$ (0.3-0.4%)

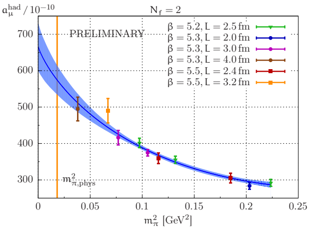
(S. Eidelman, private communication)

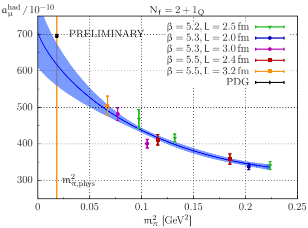
$a_\mu(\mathrm{HVP})$, lattice reg.

a_μ	N_f	errors	action	group
713(15)	2+1	stat.	Asqtad	Aubin, Blum (2006)
748(21)	2+1	stat.	Asqtad	Aubin, Blum (2006)
641(33)(32)	2+1	stat., sys.	DWF	UKQCD (2011)
572(16)	2	stat.	TM	ETMC (2011)
618(64)	2+1[1]	stat., sys.	Wilson	Mainz (2011)

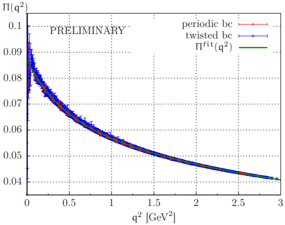
[1] strange quark is quenched

a_μ(HVP) [talk by Benni Jaeger (Mainz group)]



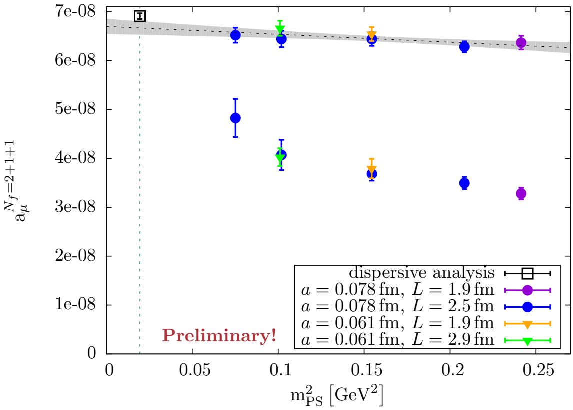


- $N_f = 2$
- $N_f = 2 + 1$ (str. qnch'd)



- $\Pi(q^2)$ with Twisted BC's
- $m_\pi \geq 195$ MeV
- several lattice spacings
- increasing statistics

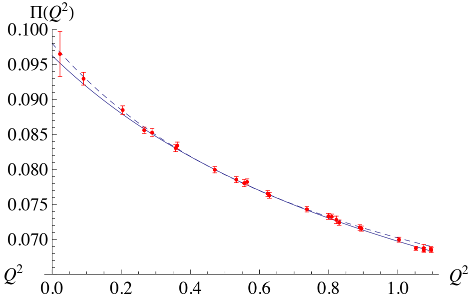
$a_\mu(\mathrm{HVP})$ 2+1+1f [talk by Grit Hotzel (ETMC)]



- 2+1+1 Twisted Mass fermions
- c-quark contribution $\sim O(\mathrm{HLbL})$
- ETMC: $\sim 2\times$

preliminary result: $a_\mu^{\mathrm{hvp}} = 6.67(14) \cdot 10^{-8}$

HVP: Pade approximants [talk by M. Golterman (ABGP)]

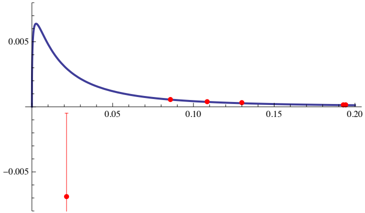


[1,1] corr. (solid) and VMD uncorr. (dashed)

- Pade: model independent. Stieltjes function constrains Pade approximants
- Pade: 350(8)
- VMD: 413(8)
- 17% diff.
- both good fits
- tendency to undershoot low Q^2 points

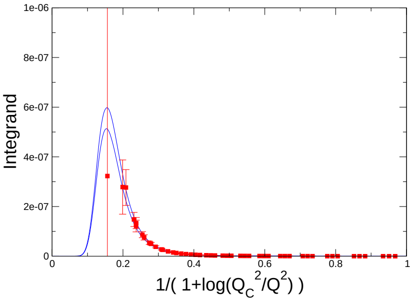
$a = 0.06$ fm, $m_\pi \approx 400$ MeV, MILC Asqtad ensemble

a_μ(HVP) integrand: low momentum region



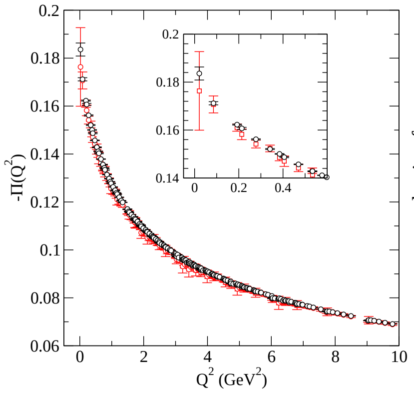
Integrand of $a_\mu^{\rm HLO}/(4\alpha^2)$ compared with data
(MILC, $a = 0.06$ fm , $m_\pi = 220$ MeV)

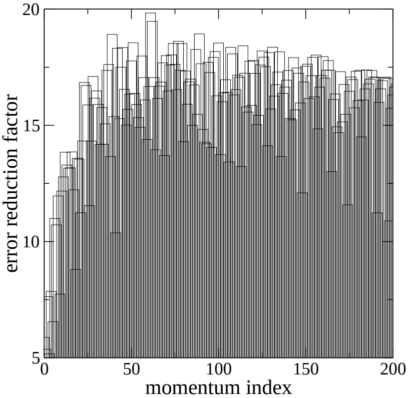
⇒ need more data at low Q^2 with smaller errors! In progress...



Talk by M. Golterman [Aubin, *et al.*, arXive:1205:3695] UKQCD [arXive:1107.1497]

a_μ(HVP) Reducing statistical errors (preliminary)





Use AMA (Blum, Izubuchi, Shintani), 1400 LM / 704 sources, $48^3 \times 144$ (MILC), 20 configs, 5-20 $\times$ error reduction!

[AMA method: poster by E. Shintani]

$a_\mu(\mathrm{HVP})$ Summary

Controlling errors at the 1% level

- Q^2 dependence
  - LMA/AMA
  - Twisted BC's
  - Pade approximants for model independent fits
- physical quark masses / large boxes
- disconnected diagrams / isospin breaking
- charm contribution

Will give confidence that dispersive calculation is right

Outline

Introduction

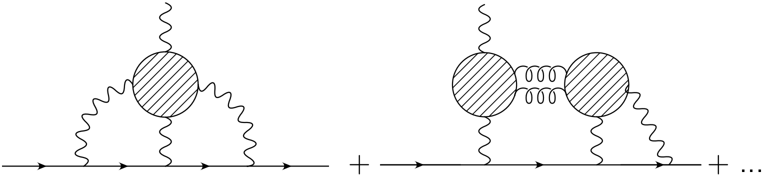
The hadronic vacuum polarization (HVP) contribution (O(α^2))

The hadronic light-by-light (HLbL) contribution (O(α^3))

a_μ Implications for new physics

a_μ(HLbL) Summary/Outlook

HLbL



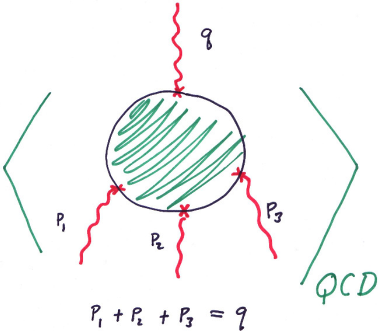
Blobs: all possible hadronic states

Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty

No dispersion relation *a'la* vacuum polarization

Lattice regulator: model independent, approximations systematically improvable

Lattice QCD: conventional approach



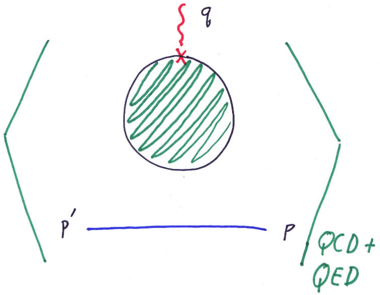
Correlation of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$

Two independent momenta +external mom q

Compute for all possible values of p_1 and p_2, ($O(V^2)$) four index tensor (32 Lorentz structures for g-2!)

several q,(extrap $q \to 0$), fit, plug into perturbative QED two-loop integrals

New approach (QCD+QED on the lattice)



Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons

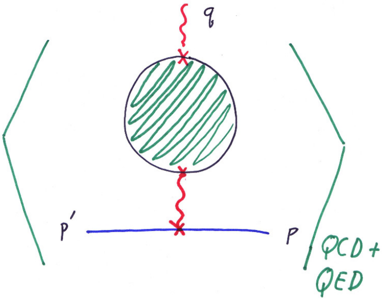
muon coupled to photons

[Hayakawa, *et al.* hep-lat/0509016;

Chowdhury *et al.* (2008);

Chowdhury Ph. D. thesis (2009)]

New approach (QCD+QED on the lattice)



Attach one photon by hand (see why in a minute)

Correlation of hadronic loop and muon line

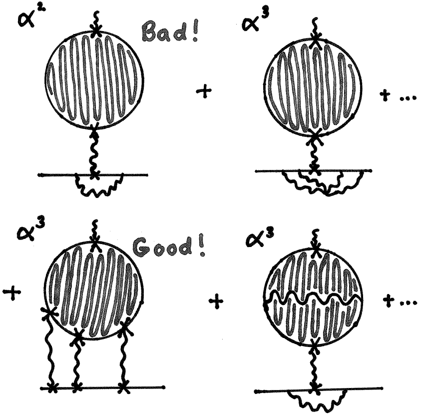
[Hayakawa, *et al.* hep-lat/0509016;

Chowdhury *et al.* (2008);

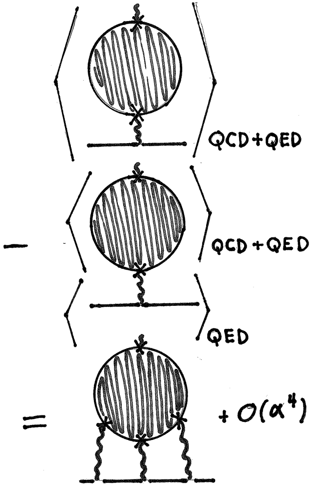
Chowdhury Ph. D. thesis (2009)]

New approach: Formally expand in α

The leading and next-to-leading contributions in α to magnetic part of correlation function come from



New approach: Subtraction of lowest order piece

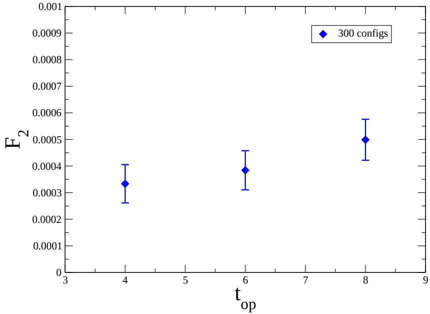


Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

F_2 ($m_\mu/m_e = 40$, QED only) (Chowdhury Ph. D. thesis, UConn, 2009)



$F_2 = (3.96 \pm 0.70) \times 10^{-4}$

- $e = 1$
- $16^3 \times 32$ lattice size
- lowest non-zero momentum only
- **stat error only**

- Expected size of enhancement (compared to $m_\mu/m_e = 1$)
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ ($e = 1$)
- roughly consistent with PT result, large finite volume effect

F_2, $m_\mu/m_e = 40$, finite volume study (QED only)

- Repeat calculation with 24^3 lattice volume
- Bigger box $F_2 = (1.19 \pm 0.32) \times 10^{-4}$
- Small box $F_2 = (3.96 \pm 0.70) \times 10^{-4}$
- finite volume effects manageable
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ $(e = 1)$
- Roughly consistent with PT result

$a_\mu(\mathrm{HLbL})$ in 2+1f QCD+QED (PRELIMINARY)

- Same as before, but with $U = U(1) \times SU(3)$ [Duncan, *et al*.]
- QCD in the loop only (same in subtraction)
- QED in both loop and line
- 2+1 flavors (u, d, s) of DWF (RBC/UKQCD)
- $a = 0.114$ fm, $16^3 \times 32\,(\times 16)$, $a^{-1} = 1.73$ GeV
- $m_q \approx 0.013$, $m_\pi \approx 420$ MeV
- $m_\mu \approx 692$ MeV ($m_\mu^{\mathrm{phys}} = 105.658367(4)$ MeV)
- 100 configurations (one QED conf. for each QCD conf.)
- $(N_s/4)^3 = 64$ (loop) propagator calculations/configuration

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

- $a_\mu(\mathrm{HLbL}) = (-15.7 \pm 2.3) \times 10^{-5}$ (lowest non-zero mom, $e = 1$)
- HLBL amplitude depends strongly on m_μ (m_μ^2 in models)
- Magnitude 5-10 times bigger, sign opposite from models
- models not expected to be accurate in this regime
- Check subtraction is working by varying $e = 0.84$, 1.19
  - HLbL amplitude ($\sim e^4$) changes by ~ 0.5 and 2 ✓
  - while unsubtracted amplitude stays the same ✓

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

- Easy to lower muon mass (muon line is cheap)
- Try $m_\mu \approx 190$ MeV
- $a_\mu(\mathrm{HLbL}) = (-2.2 \pm 0.8) \times 10^{-5}$ (lowest non-zero mom, $e = 1$). Right direction...

$a_\mu(\mathrm{HLbL})$ in 2+1 flavor lattice QCD+QED

- Try larger lattice size, 24^3 ($(2.7\text{ fm})^3$)
- Pion mass is smaller too, $m_\pi = 329$ MeV
- Same muon mass
- two lowest values of Q^2 (0.11 and 0.18 GeV^2)
- Use All Mode Averaging (AMA)
  - 6^3 point sources/configuration (216)
  - AMA approximation: "sloppy CG", $r_{\mathrm{stop}} = 10^{-4}$

$a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

Signal may be emerging in the model ballpark:

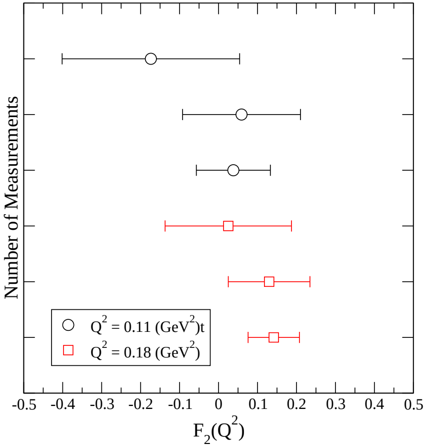
- $F_2(0.18\ \mathrm{GeV}^2) = (0.142 \pm 0.067) \times \left(\frac{\alpha}{\pi}\right)^3$
- $F_2(0.11\ \mathrm{GeV}^2) = (0.038 \pm 0.095) \times \left(\frac{\alpha}{\pi}\right)^3$
- $a_\mu(\mathrm{HLbL/model}) = (0.084 \pm 0.020) \times \left(\frac{\alpha}{\pi}\right)^3$

Lattice size 24^3, $m_\pi = 329$ MeV, $m_\mu \approx 190$ MeV

model value/error is "Glasgow Consensus" (arXiv:0901.0306 [hep-ph])

a_μ(HLbL) in 2+1f lattice QCD+QED (PRELIMINARY)

$F_2(Q^2)$ stable with additional measurements (20 → 40 → 80 configs)



24^3 lattice size

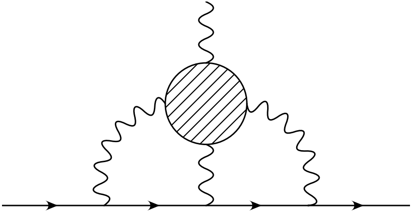
$Q^2 = 0.11$ and 0.18 GeV^2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

$a_\mu(\mathrm{HLbL})$ Systematic error

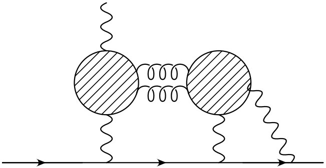
Calculated:



$$\times \left(\left(\frac{2}{3}\right)^4 + \left(-\frac{1}{3}\right)^4 \right)$$

i.e., contribution from fractionally charged pions and neutral pion (partial)

$a_\mu(\mathrm{HLbL})$ Systematic error



"Disconnected" diagrams (quark loops connected by gluons) not calculated yet (not suppressed).

Several possibilities,

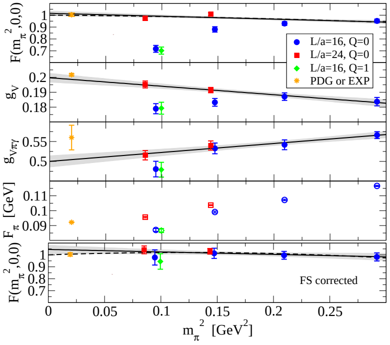
1. Use multiple valence quark loops (qQED)
2. Re-weight in α (T. Ishikawa) or dynamical QED in HMC
3. "$\not{A}$ SeqSrc" (see Izubuchi's talk) (no subtraction)

$a_\mu(\mathrm{HLbL})$ more systematic errors

Need to address

- Finite volume
- $q^2 \to 0$ exptrap
- $m_q \to m_{q,\,\mathrm{phys}}$
- $m_\mu \to m_{\mu,\,\mathrm{phys}}$
- excited states/"around the world" effects
- $a \to 0$
- QED renormalization
- $\ldots$

$a_\mu(\mathrm{HLbL})$ Related calculations



- $\pi \to \gamma^*\gamma^*$ (talk by Xu Feng (JLQCD)[arXive 1206.1375])
- four point hadronic vector correlation function to check models
- magnetic susceptibility to improve model calculation
- ...

Outline

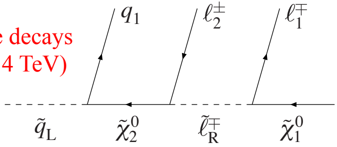
a_μ discrepancy [Fundamental Physics at the Intensity Frontier WS/WP (arXive1205.2671)]

1. Different BSM physics affect a_μ very differently
2. Constraints complimentary to other BSM observables
3. a_μ versatile in providing contraints!

- $a_{\mu,\,\mathrm{new\ physics}} = O(1) \times \left(\frac{m_\mu}{\Lambda}\right)^2 \times \frac{\delta m_\mu}{m_\mu}$ [Czarnecki and Marciano]

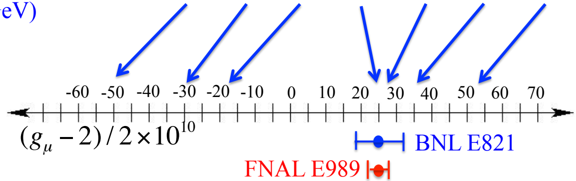
a_μ Discriminating among SUSY scenarios at the LHC

LHC cascade decays
(300 fb^{-1} @ 14 TeV)



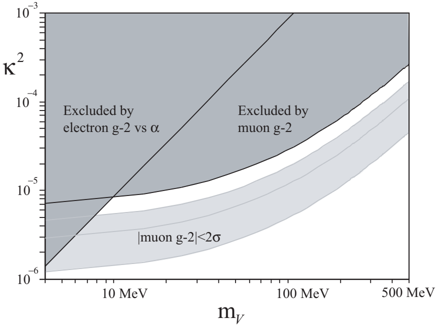
Degenerate SUSY breaking parameters (GeV)

μ:	-78	-358	-184	186	349	355	369
M_1:	369	98	102	109	100	185	136
M_2:	144	187	355	350	196	106	83



Dark photon: $U(1)'$ extension(s) of SM ("dark charge")

- Explanation for astrophysical obs. of excess positrons (Arkani-Hamed) (PAMELA, INTEGRAL,...)



- $\gamma' - \gamma$ Mixing couples SM, Dark sectors
- Like Schwinger term
- $m_V = 10 - 1000$ MeV
- coupling $\kappa = 10^{-8} - 10^{-2}$
- Pospelov (2008): explains $g-2$ discrepancy
- Search at JLab, Mainz, ...

(Pospelov, 2008)

Theories with (several) Higgs particles, New $U(1)_{L\mu-L\tau}$ symmetry (Z'), ...

- Heavy Z', $m_{Z'} \lesssim 100$ GeV
  - Gauged $U(1)_{L\mu-L\tau}$ (Ma and Roy, Heeck and Rodejohann)
  - Breaks $e-\mu-\tau$ universality
- Very light scalar, $m = 1 - 100$ MeV (Kinoshita and Marciano, 1990)
- Very light scalar or vector simul. solves proton radius problem (need violation of lepton universality)
- Multi-Higgs models
- Extra dimensions

Outline

a_μ(HLbL) Summary/Outlook

- Demanding, but straightforward calculation
- Early HLbL lattice calculation encouraging
- Optimistic lattice+models+expt can reach 10% goal in $\sim$ 5 years (INT WS on HLbL, Feb. 2011)
- White papers, prospects for lattice QCD:
  - USQCD white-paper (http://www.usqcd.org/collaboration.html)
  - Fundamental physics at the Intensity Frontier white-paper (arXiv:1205.2671 [hep-ex])
  - Project X Physics Study 2012 (Fermilab) (to appear)
- Expected precision
  - E989: 0.14 PPM (factor of 3-4 better than E821)
  - SM theory, HVP: 0.3% (factor of 2)
  - SM theory, HLbL 10% or better (?)
  - Same central values, a_μ discrepancy $\rightarrow$ 5-8 σ

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