

# FLAG phase 2: status and prospects

Gilberto Colangelo  
(on behalf of FLAG)

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*Albert Einstein Center for Fundamental Physics*

XXX Lattice Conference, Cairns, 29.6.2012

# Outline

Introduction: some history

FLAG-2

Current status of the review

Conclusions

Many thanks to all FLAG colleagues,  
in particular to Andreas Jüttner and Urs Wenger

# Lattice and precision physics

- ▶ search for new physics through precision studies more important than ever  
→ talks by J. Serrano and C. Tarantino
- ▶ main hurdle on the theory side: hadronic effects  
**examples:**  $(g - 2)_\mu$ ,  $\epsilon_K$ ,  $\epsilon'/\epsilon$ ,  $\Delta A_{CP}$ , ...  
→ T. Blum's talk  
more examples: talks by J. Serrano and C. Tarantino
- ▶ theoretical tools:  
**lattice**, sum rules,  $\chi$ PT, dispersion relations
- ▶ progress is sometimes slow and reaching the necessary accuracy may take several years
- ▶ pressure to provide an answer **now!** can be very high

# Lattice and precision physics

- ▶ digging into the lattice literature not easy for non-experts  
(ask the next-door lattice colleague?)
- ▶ this kind of situation occurred already in other fields
- ▶ **solution:** compilation of results ready-to-use for  
non-experts
- ▶ **examples:** PDG, HFAG, etc.
- ▶ recently two initiatives inside the lattice community:  
FLAG and Laiho-Lunghi-Van de Water  
⇒ now: **FLAG-2**

# What/Who is FLAG-1?

**FLAG = FLAVIAnet Lattice Averaging Group**

## Members:

Gilberto Colangelo (Bern)

Stephan Dürr (Jülich, BMW)

Andreas Jüttner (Southampton, RBC/UKQCD)

Laurent Lellouch (Marseille, BMW)

Heiri Leutwyler (Bern)

Vittorio Lubicz (Rome 3, ETM)

Silvia Necco (CERN, Alpha)

Chris Sachrajda (Southampton, RBC/UKQCD)

Silvano Simula (Rome 3, ETM)

Tassos Vladikas (Rome 2, Alpha and ETM)

Urs Wenger (Bern, ETM)

Hartmut Wittig (Mainz, Alpha)

# What/Who is FLAG-1?

**FLAG** = FLAVIAnet Lattice Averaging Group

## History and status:

- ▶ Beginning: FLAVIAnet meeting, Orsay, November 2007
- ▶ Start of the actual work: Bern, March 2008
- ▶ ...
- ▶ first paper appeared in November 2010  
updated and published in May 2011 on EPJC
- ▶ webpage made public in 2011:  
<http://itpwiki.unibe.ch/flag>

arXiv.1011.4408

# What exactly did FLAG-1 offer?

An answer to the questions

- ▶ what is the current lattice value for quantity  $X$ ?
- ▶ what is a reliable estimate of the uncertainty?

in a way easily accessible to non-experts

Quantities considered in the first edition:

- ▶ light quark masses
- ▶ LEC
- ▶ decay constants (of pions and kaons)
- ▶ form factors (of pions and kaons)
- ▶  $B_K$

# What exactly did FLAG-1 offer?

For each quantity we provided:

- ▶ complete list of references
- ▶ summary of relevant formulae and notation
- ▶ summary of the essential aspects of each calculation:
  - ▶ lattice action
  - ▶ number of dynamical quarks ( $N_f$ )
  - ▶ minimal value and range of quark masses
  - ▶ minimal value and range of lattice spacing
  - ▶ maximal value and range of lattice volumes
  - ▶ renormalization method (where applicable)

in a unified and easy to read (color coding) manner

- ▶ averages (if sensible)
- ▶ and a “lattice dictionary” for non-experts  
(details of lattice actions, etc.)

# What exactly did FLAG-1 offer?

We also offered some original contributions:

- ▶ thorough discussion and **parametrization** of electromagnetic contributions to meson masses (and their role in the determination of quark masses)
- ▶ some **new**  $\chi$ PT two-loop formulae (either completely new or written in a user-friendly way)
- ▶ a thorough consistency test of lattice calculations of  $f_+(0)$  and  $f_K/f_\pi$  **assuming** unitarity of the CKM matrix

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and begged readers to **always quote the original references** too

# Color coding – FLAG-1 definition

[subject to change before each new edition]

- ▶ chiral extrapolation
  - ★  $M_{\pi,\min} < 250 \text{ MeV}$
  - $250 \text{ MeV} \leq M_{\pi,\min} \leq 400 \text{ MeV}$
  - $M_{\pi,\min} > 400 \text{ MeV}$

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  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - 2 or more lattice spacings, at least 1 point below 0.1 fm
  - otherwise

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- ▶ finite volume effects
  - ★  $(M_{\pi} L)_{\min} > 4$  or at least 3 volumes
  - $(M_{\pi} L)_{\min} > 3$  and at least 2 volumes
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  - otherwise
- ▶ renormalization (where applicable)
  - ★ non-perturbative
  - 2-loop perturbation theory (well behaved series)
  - otherwise

# Averages

Different lattice results were averaged *if*

- ▶ published
  - [lattice proceedings not enough]
- ▶ no red tags
- ▶ same  $N_f$ 
  - [no average of  $N_f = 2$  and  $N_f = 3$  calculations]

Final FLAG number:

- ▶ average or single *no-red-tag*  $N_f = 3$  number (if available)
- ▶ average or single *no-red-tag*  $N_f = 2$  number (if available)

If *both*  $N_f = 3$  *and*  $N_f = 2$  numbers available:

agreement  $\Rightarrow$  more confidence in the final number

## Similar initiative: Laiho, Lunghi and Van de Water

- ▶ began in 2009 to provide lattice-QCD inputs for the CKM unitarity-triangle analysis and other flavor-physics phenomenology
- ▶ main differences wrt FLAG-1:
  - ▶ only include  $N_f = 2 + 1$  flavor results
  - ▶ no strict publication-only rule provided complete and reasonable systematic error budgets
  - ▶ heavy-quark quantities included from the start
  - ▶ unitarity triangle fits with lattice input
  - ▶ whenever a source of error is at all correlated between two lattice calculations (e.g. use the same gauge configurations, same theoretical tools, or experimental inputs), conservatively assume that the degree-of-correlation is 100%
- ▶ web page ([www.latticeaverages.org](http://www.latticeaverages.org)) also popular

LLVdW, Phys.Rev. D81 (2010) 034503

# FLAG-2

FLAG = Flavour Lattice Averaging Group

has now entered its phase 2 and has been extended in various directions

- ▶ quantities to be reviewed  
main extension: light quarks → + heavy quarks
- ▶ represented lattice collaborations:  
Alpha, BMW, ETMC, RBC/UKQCD → + CLS, Fermilab, HPQCD, JLQCD, MILC, PACS-CS, SWME
- ▶ represented world regions: Europe → + Japan and US
- ▶ number of people: 12 → 28

# FLAG-2 organization

- ▶ Advisory Board:  
S. Aoki, C. Bernard, C. Sachrajda
- ▶ Editorial Board:  
GC, H. Leutwyler, T. Vladikas, U. Wenger
- ▶ Working Groups
  - ▶ Quark masses L. Lellouch, T. Blum, V. Lubicz
  - ▶  $V_{us}$ ,  $V_{ud}$  A. Jüttner, T. Kaneko, S. Simula
  - ▶ LEC S. Dürr, H. Fukaya, S. Necco
  - ▶  $B_K$  H. Wittig, J. Laiho, S. Sharpe
  - ▶  $\alpha_s$  R. Sommer, T. Onogi, J. Shigemitsu
  - ▶  $f_B$ ,  $B_B$  A. El Khadra, Y. Aoki, M. Della Morte
  - ▶  $B \rightarrow H\ell\nu$  R. Van de Water, E. Lunghi, C. Pena

## FLAG-2 plans and rules

- ▶ next review: end 2012
- ▶ regularly update the webpage
- ▶ new published review: every 2nd year
- ▶ some internal FLAG rules
  - ▶ members of the advisory board have a 4-year mandate
  - ▶ AB = EU+J+US
  - ▶ regular members can stay longer
  - ▶ replacements must keep/improve the balance of FLAG
  - ▶ WG members belong to 3 different lattice coll.
  - ▶ a paper is not reviewed (color-coded) by an author

## FLAG-2 status

- ▶ kick-off meeting: Les Houches, May 7-11 2012
- ▶ work on the update of the review is in progress
- ▶ WG for the new sections are working on defining their own quality criteria
- ▶ first draft (internal) of the new review: September 2012
- ▶ publication of the new review: early 2013
- ▶ it will cover all results published until December 31 2012

## FLAG-2 status

Two important policy changes wrt FLAG-1

- ▶ use of one-loop renormalization will **not necessarily mean a red tag**
- ▶ error calculation for the averages:  
**LLVdW procedure will be adopted as default**  
(final error may be stretched if not convincingly conservative)

One less important cosmetic change:

- ▶  → 

# Quark masses

Collaboration	publ.	$m_{ud} \rightarrow 0$	$a \rightarrow 0$	$F_V$	renorm.	$m_{ud}$	$m_s$
PACS-CS 12	P	★	■	■	★	3.12(24)(8)	83.60(0.58)(2.23)
RBC/UKQCD 12	C	★	○	★	★	3.39(9)(4)(2)(7)	94.2(1.9)(1.0)(0.4)(2.1)
LVdW 11	C	○	★	★	○	3.31(7)(20)(17)	94.2(1.4)(3.2)(4.7)
PACS-CS 10	A	★	■	■	★	2.78(27)	86.7(2.3)
MILC 10A	C	○	★	★	○	3.19(4)(5)(16)	—
HPQCD 10	A	○	★	★	★	3.39(6)	92.2(1.3)
BMW 10A, 10B <sup>+</sup>	A	★	★	★	★	3.469(47)(48)	95.5(1.1)(1.5)
RBC/UKQCD 10A	A	○	○	★	★	3.59(13)(14)(8)	96.2(1.6)(0.2)(2.1)
Blum 10	A	○	■	○	★	3.44(12)(22)	97.6(2.9)(5.5)
PACS-CS 09	A	★	■	■	★	2.97(28)(3)	92.75(58)(95)
HPQCD 09	A	○	★	★	★	3.40(7)	92.4(1.5)
MILC 09A	C	○	★	★	○	3.25 (1)(7)(16)(0)	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	A	○	★	★	○	3.2(0)(1)(2)(0)	88(0)(3)(4)(0)
PACS-CS 08	A	★	■	■	■	2.527(47)	72.72(78)
RBC/UKQCD 08	A	○	■	★	★	3.72(16)(33)(18)	107.3(4.4)(9.7)(4.9)
CP-PACS/ JLQCD 07	A	■	★	★	■	3.55(19)( <sup>+56</sup> <sub>-20</sub> )	90.1(4.3)( <sup>+16.7</sup> <sub>-4.3</sub> )
HPQCD 05	A	○	○	○	○	3.2(0)(2)(2)(0)	87(0)(4)(4)(0)
MILC 04, HPQCD/ MILC/UKQCD 04	A	○	○	○	■	2.8(0)(1)(3)(0)	76(0)(3)(7)(0)

$$N_f = 2 + 1$$

# Quark masses

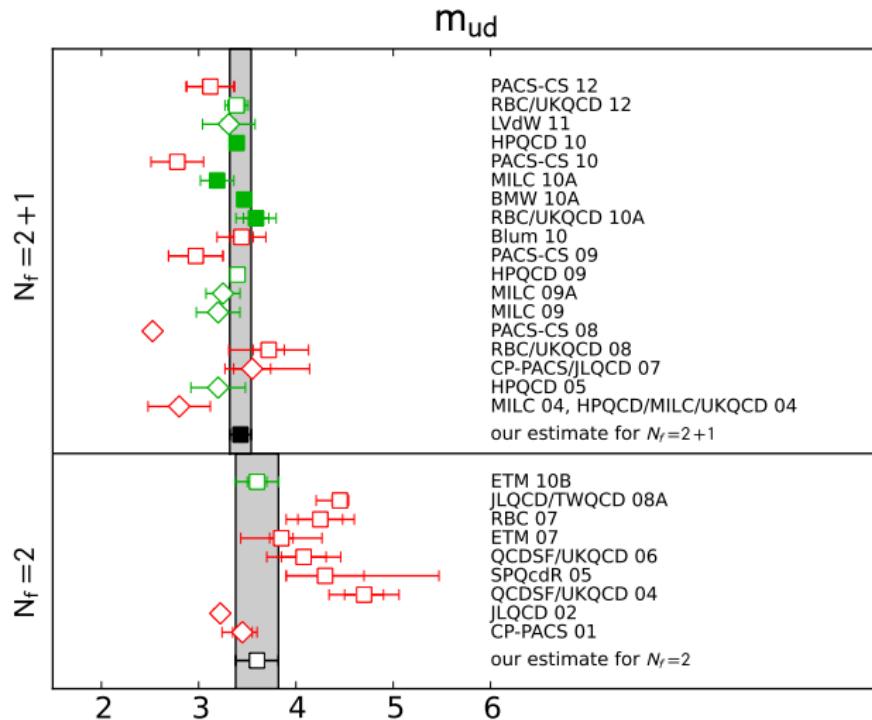
Collaboration	publ.	$m_{ud} \rightarrow 0$	$a \rightarrow 0$	$F_V$	renorm.	$m_{ud}$	$m_s$
ALPHA 12	P	○	★	★	★	—	102(3)(1)
ETM 10B	A	○	★	○	★	3.6(1)(2)	95(2)(6)
JLQCD/TWQCD 08A	A	○	■	■	★	$4.452(81)(38) \left( \begin{array}{l} +0 \\ -227 \end{array} \right)$	—
RBC 07 <sup>†</sup>	A	■	■	★	★	4.25(23)(26)	119.5(5.6)(7.4)
ETM 07	A	○	■	○	★	3.85(12)(40)	105(3)(9)
QCDSF/ UKQCD 06	A	■	★	■	★	4.08(23)(19)(23)	111(6)(4)(6)
SPQcdR 05	A	■	○	○	★	$4.3(4) \left( \begin{array}{l} +1.1 \\ -0.0 \end{array} \right)$	101(8)( <sup>+25</sup> <sub>-0</sub> )
ALPHA 05	A	■	○	★	★	—	97(4)(18)
QCDSF/ UKQCD 04	A	■	★	■	★	4.7(2)(3)	119(5)(8)
JLQCD 02	A	■	■	○	■	$3.223 \left( \begin{array}{l} +46 \\ -69 \end{array} \right)$	84.5( <sup>+12.0</sup> <sub>-1.7</sub> )
CP-PACS 01	A	■	■	★	■	$3.45(10) \left( \begin{array}{l} +11 \\ -18 \end{array} \right)$	89(2)( <sup>+2</sup> <sub>-6</sub> ) <sup>*</sup>

$$N_f = 2$$

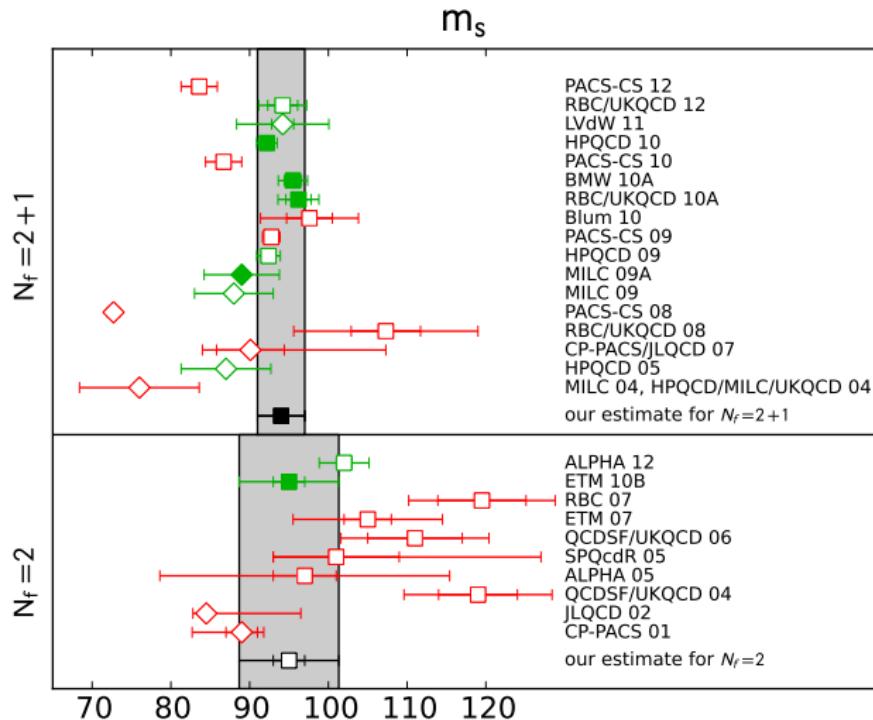
# Quark masses

Collaboration	$N_f$	$\mu_{\text{refl.}}$	$m_{ud} \rightarrow 0$	$a \rightarrow 0$	$F_V$	$m_s/m_{ud}$
PACS-CS 12	2+1	P	★	■	■	26.8(2.0)
LVdW 11	2+1	C	○	★	★	28.4(0.5)(1.3)
BMW 10A, 10B	2+1	A	★	★	★	27.53(20)(8)
RBC/UKQCD 10A	2+1	A	○	○	★	26.8(0.8)(1.1)
Blum 10	2+1	A	○	■	○	28.31(0.29)(1.77)
PACS-CS 09	2+1	A	★	■	■	31.2(2.7)
MILC 09A	2+1	C	○	★	★	27.41(5)(22)(0)(4)
MILC 09	2+1	A	○	★	★	27.2(1)(3)(0)(0)
PACS-CS 08	2+1	A	★	■	■	28.8(4)
RBC/UKQCD 08	2+1	A	○	■	★	28.8(0.4)(1.6)
MILC 04, HPQCD/ MILC/UKQCD 04	2+1	A	○	○	○	27.4(1)(4)(0)(1)
ETM 10B	2	A	○	★	○	27.3(5)(7)
RBC 07	2	A	■	■	★	28.10(38)
ETM 07	2	A	○	■	○	27.3(0.3)(1.2)
QCDSF/UKQCD 06	2	A	■	★	■	27.2(3.2)

# Quark masses



# Quark masses



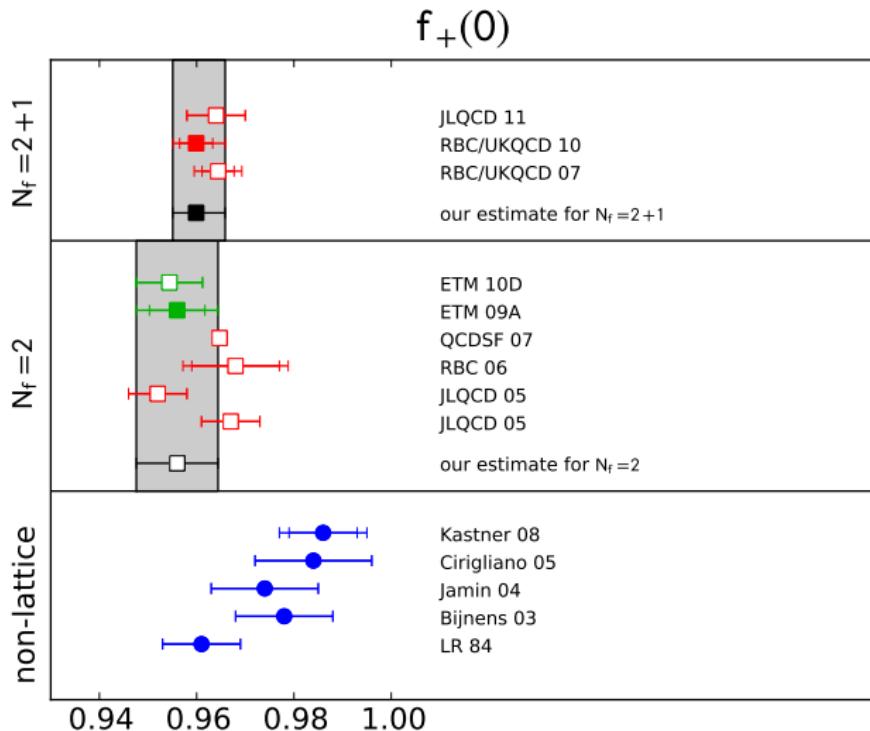
## $V_{us}$ and $V_{ud}$ – tables

Collaboration	$N_f$	Publ.	$m_{ud} \rightarrow 0$	$a \rightarrow 0$	$F_V$	$f_+(0)$
JLQCD 11	2+1	C	○	■	★	0.964(6)
RBC/UKQCD 10	2+1	A	○	■	★	0.9599(34)( $^{+31}_{-47}$ )(14)
RBC/UKQCD 07	2+1	A	○	■	★	0.9644(33)(34)(14)
ETM 10D	2	C	○	★	○	0.9544(68) <sub>stat</sub>
ETM 09A	2	A	○	○	○	0.9560(57)(62)
QCDSF 07	2	C	■	■	★	0.9647(15) <sub>stat</sub>
RBC 06	2	A	■	■	★	0.968(9)(6)
JLQCD 05	2	C	■	■	★	0.967(6), 0.952(6)

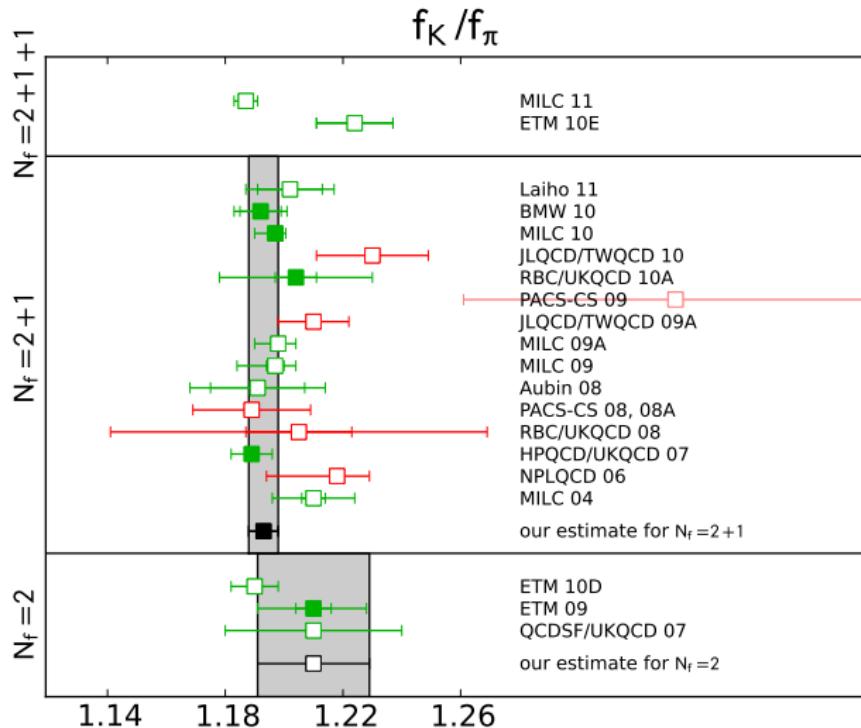
# $V_{us}$ and $V_{ud}$ – tables

Collaboration	$N_f$	Publ.	$m_{ud} \rightarrow 0$	$a \rightarrow 0$	$F_V$	$f_K/f_\pi$
ETM 10E	2+1+1	C	○	○	○	1.224(13) <sub>stat</sub>
MILC 11	2+1+1	C	○	○	○	1.1872(42) <sup>†</sup> <sub>stat.</sub>
Laiho 11	2+1	C	○	○	○	1.202(11) <sub>stat</sub> (9) $\chi_{\text{PT}}$ (2) <sub>scale</sub> (5) $m_q$
MILC 10	2+1	C	○	★	★	1.197(2)( <sup>+3</sup> <sub>-7</sub> )
JLQCD/TWQCD 10	2+1	C	○	■	★	1.230(19)
RBC/UKQCD 10A	2+1	A	○	○	★	1.204(7)(25)
PACS-CS 09	2+1	A	★	■	■	1.333(72)
BMW 10	2+1	A	★	★	★	1.192(7)(6)
JLQCD/TWQCD 09A	2+1	C	○	■	■	1.210(12) <sub>stat</sub>
MILC 09A	2+1	C	○	★	★	1.198(2)( <sup>+6</sup> <sub>-8</sub> )
MILC 09	2+1	A	○	★	★	1.197(3)( <sup>+6</sup> <sub>-13</sub> )
Aubin 08	2+1	C	○	○	○	1.191(16)(17)
PACS-CS 08,	2+1	A	★	■	■	1.189(20)
RBC/UKQCD 08	2+1	A	○	■	★	1.205(18)(62)
HPQCD/UKQCD 07	2+1	A	○	★	○	1.189(2)(7)
NPLQCD 06	2+1	A	○	■	■	1.218(2)( <sup>+11</sup> <sub>-24</sub> )
MILC 04	2+1	A	○	○	○	1.210(4)(13)
ETM 10D	2	C	○	★	○	1.190(8) <sub>stat</sub>
ETM 09	2	A	○	★	○	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	○	○	★	1.21(3)

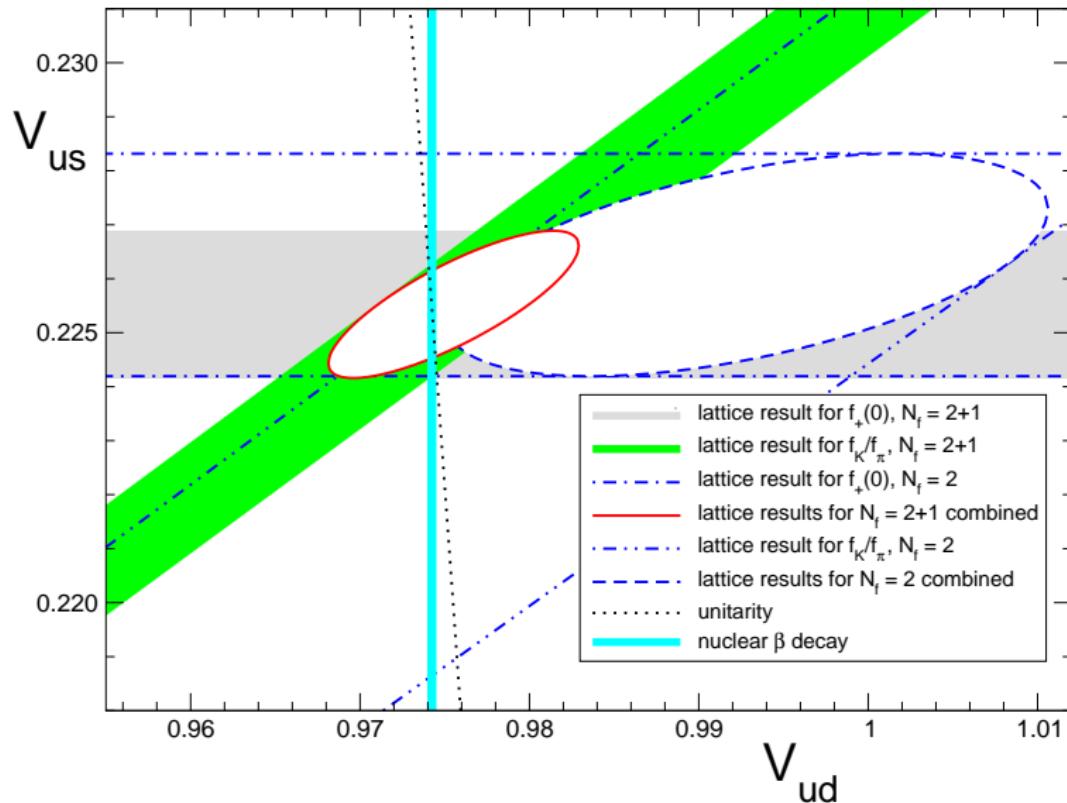
# $V_{us}$ and $V_{ud}$ – figures



# $V_{us}$ and $V_{ud}$ – figures



# $V_{us}$ and $V_{ud}$ – figures



# Analysis assuming CKM unitarity

Unitarity + experiment:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad [ |V_{ub}| = 3.89(44) \cdot 10^{-3}, \text{PDG (10)} ]$$

Experiment: FLAVIAnet Kaon WG (10)

$$|V_{us} f_+(0)| = 0.2163(5)$$

$$\left| \frac{V_{us} f_K}{V_{ud} f_\pi} \right| = 0.2758(5)$$

3 relations and 4 unknowns

determine anyone of  $V_{ud}$ ,  $V_{us}$ ,  $f_+(0)$  or  $f_K/f_\pi$

⇒ get the other three

# Analysis assuming CKM unitarity

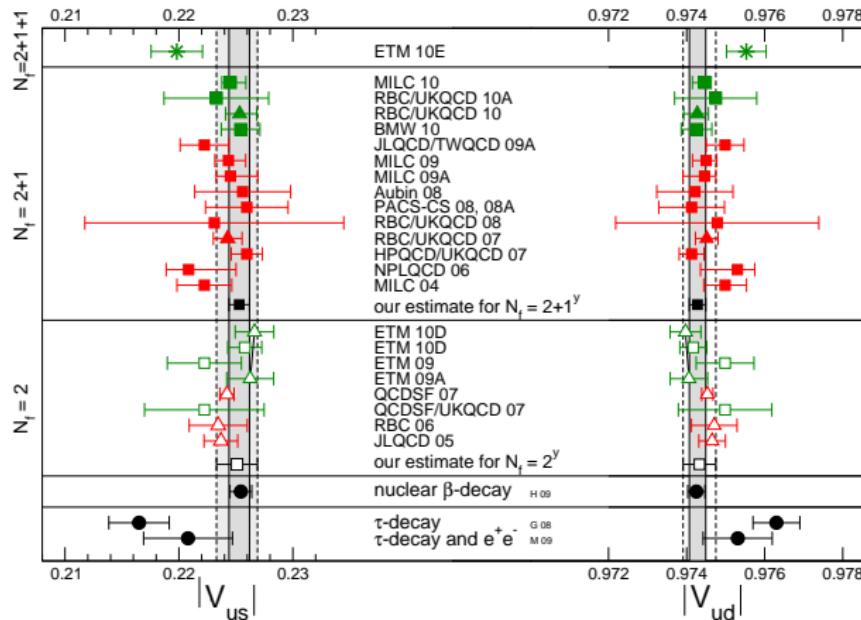
	$ V_{us} $	$ V_{ud} $	$f_+(0)$	$f_K/f_\pi$
$N_f = 2+1$	0.2253(9)	0.97428(21)	0.9599(38)	1.1927(50)
$N_f = 2$	0.2251(18)	0.97433(42)	0.9604(75)	1.194(10)
$\beta$ -dec. <sup>1</sup>	0.22544(95)	0.97425(22)	0.9595(46)	1.1919(57)
$\tau$ -dec. <sup>2</sup>	0.2165(26)	0.9763(6)	0.999(12)	1.244(16)
$\tau$ -dec. <sup>3</sup>	0.2208(39)	0.9753(9)	0.980(18)	1.218(23)

<sup>1</sup> Hardy & Towner

<sup>2</sup> Gamiz et al.

<sup>3</sup> Maltman

# Analysis assuming CKM unitarity



Assuming unitarity lattice predicts  $|V_{ud}|$  with the same precision as super-allowed Fermi  $\beta$ -decays

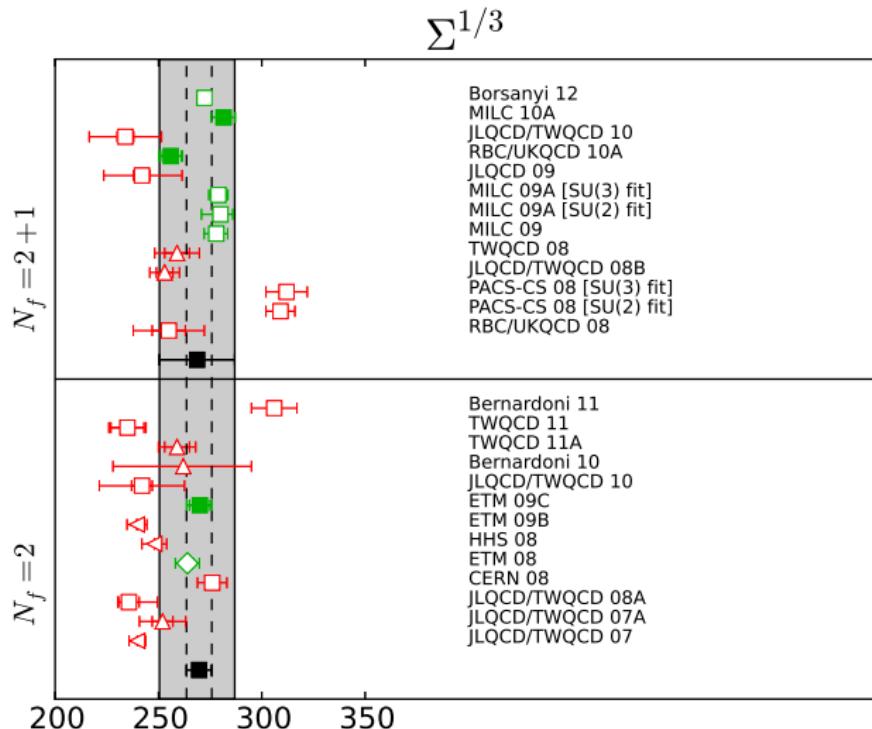
# LEC: $\Sigma$

Collaboration	$N_f$	publ.	$mud \rightarrow 0$	$a \rightarrow 0$	$F_V$	renormal.	$\Sigma^{1/3}$ [MeV]
Borsanyi 12	2+1	P	★	★	○	★	272.3(1.2)(1.4)
MILC 10A	2+1	C	○	★	★	○	281.5(3.4) $\left(\begin{array}{l} +2.0 \\ -5.9 \end{array}\right)$ (4.0)
JLQCD/TWQCD 10	2+1	A	★	■	○	★	234(4)(17)
RBC/UKQCD 10A	2+1	A	○	○	★	★	256(5)(2)(2)
JLQCD 09	2+1	A	★	■	○	★	242(4) $\left(\begin{array}{l} +19 \\ -18 \end{array}\right)$
MILC 09A	2+1	C	○	★	★	○	279(1)(2)(4)
MILC 09A	2+1	C	○	★	★	○	280(2) $\left(\begin{array}{l} +4 \\ -8 \end{array}\right)$ (4)
MILC 09	2+1	A	○	★	★	○	278(1) $\left(\begin{array}{l} +2 \\ -3 \end{array}\right)$ (5)
TWQCD 08	2+1	A	○	■	■	★	259(6)(9)
JLQCD/TWQCD 08B	2+1	C	○	■	■	★	253(4)(6)
PACS-CS 08	2+1	A	★	■	■	■	312(10)
PACS-CS 08	2+1	A	★	■	■	■	309(7)
RBC/UKQCD 08	2+1	A	○	■	★	★	255(8)(8)(13)

LEC:  $\Sigma$ 

Collaboration	$N_f$	publ.	$mud \rightarrow 0$	$a \rightarrow 0$	$FV$	renorm.	$\Sigma^{1/3}$ [MeV]
Bernardoni 11	2	C	○	■	■	○	306(11)
TWQCD 11	2	P	★	■	■	★	235(8)(4)
TWQCD 11A	2	A	★	■	■	★	259(6)(7)
Bernardoni 10	2	A	○	■	■	★	$262 \begin{pmatrix} +33 \\ -34 \end{pmatrix} \begin{pmatrix} +4 \\ -5 \end{pmatrix}$
JLQCD/TWQCD 10	2	A	★	■	■	★	242(5)(20)
ETM 09C	2	A	○	★	○	★	$270(5) \begin{pmatrix} +3 \\ -4 \end{pmatrix}$
ETM 08	2	A	○	○	○	★	264(3)(5)
CERN 08	2	A	○	■	○	★	276(3)(4)(5)
JLQCD/TWQCD 08A	2	A	○	■	■	★	$235.7(5.0)(2.0) \begin{pmatrix} +12.7 \\ -0.0 \end{pmatrix}$
JLQCD/TWQCD 07A	2	A	○	■	■	★	252(5)(10)
<hr/>							
ETM 09B	2	C	★	○	■	★	239.6(4.8)
HHS 08	2	A	★	■	○	★	248(6)
JLQCD/TWQCD 07	2	A	★	■	■	★	239.8(4.0)

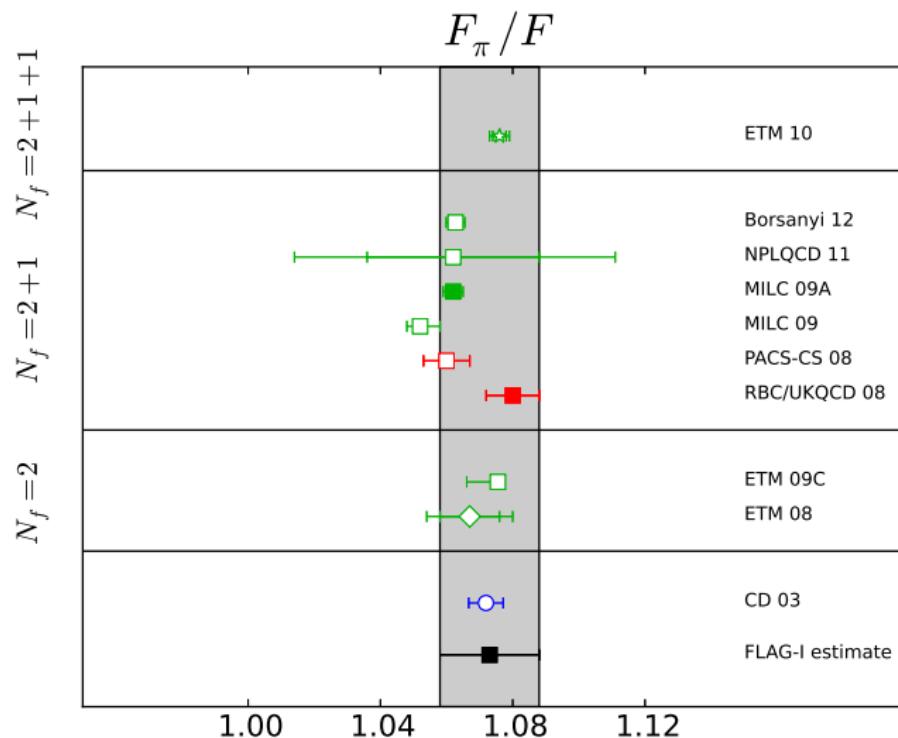
upper table:  $p$ -regime  
lower table:  $\epsilon$ -regime

LEC:  $\Sigma$ 

# LEC: $F_\pi$ and $F$

Collaboration	$N_f$	Publ.	$mud \rightarrow 0$	$a \rightarrow 0$	$F_V$	renormal.	$F$ [MeV]	$F_\pi / F$
ETM 11	2+1+1	C	○	★	○	★	85.60(4)	
ETM 10	2+1+1	A	○	○	○†	★	85.66(6)(13)	1.076(2)(2)
Borsanyi 12	2+1	P	★	★	○	★	86.78(05)(25)	1.0627(06)(27)
NPLQCD 11	2+1	P	★	○	★			1.062(26)( <sup>+42</sup> <sub>-40</sub> )
MILC 10A	2+1	C	○	★	★	○	87.5(1.0)( <sup>+0.7</sup> <sub>-2.6</sub> )	1.05(1)
MILC 10	2+1	C	○	★	★	○	87.0(4)(5)	1.06(5)
MILC 09A	2+1	C	○	★	★	○	86.8(2)(4)	1.062(1)(3)
MILC 09	2+1	A	○	★	★	○		1.052(2)( <sup>+6</sup> <sub>-3</sub> )
PACS-CS 08	2+1	A	★	■	■	■	89.4(3.3)	1.060(7)
RBC/UKQCD 08	2+1	A	○	■	★	★	81.2(2.9)(5.7)	1.080(8)
Bernardoni 11	2	C	○	■	■	○	79(4)	
TWQCD 11	2	P	★	■	■	★	83.39(35)(38)	
ETM 09C	2	A	○	★	○	★		1.0755(6)( <sup>+8</sup> <sub>-94</sub> )
ETM 08	2	A	○	○	○	★	86.6(7)(7)	1.067(9)(9)
JLQCD/TWQCD 08A	2	A	○	■	■	★	79.0(2.5)(0.7)( <sup>+4.2</sup> <sub>-0.0</sub> )	1.17(4)
ETM 09B	2	C	★	○	■	★	90.2(4.8) <sup>§</sup>	1.02(5)
HHS 08	2	A	★	■	○	★	90(4)	1.02(5)
JLQCD/TWQCD 07	2	A	★	■	■	★	87.3(5.6)	1.06(7)
CD 03							86.2(5)	1.0719(52)

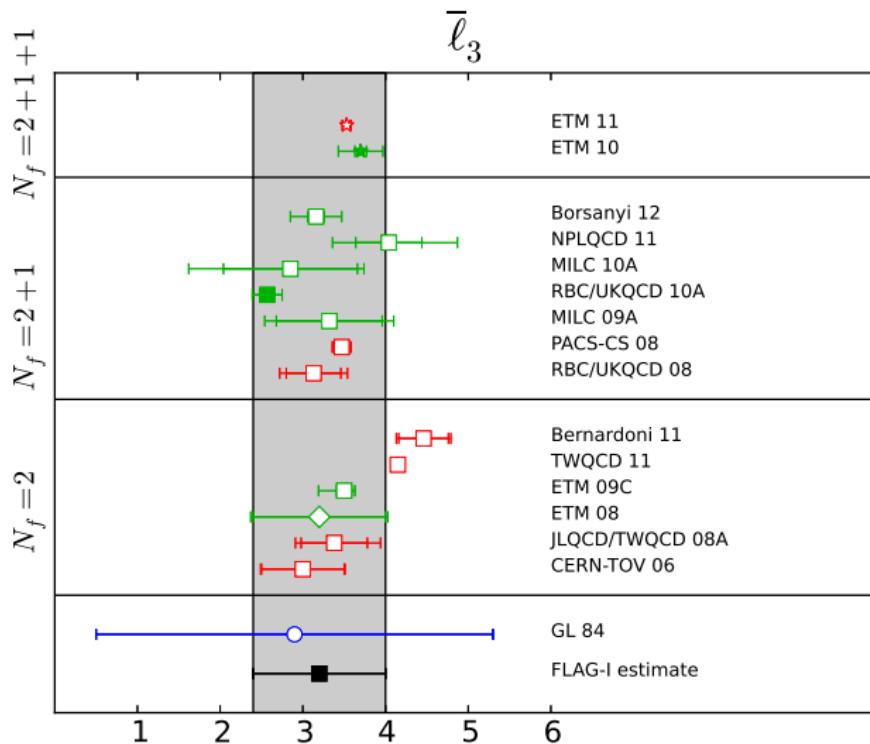
# LEC: $F_\pi$ and $F$



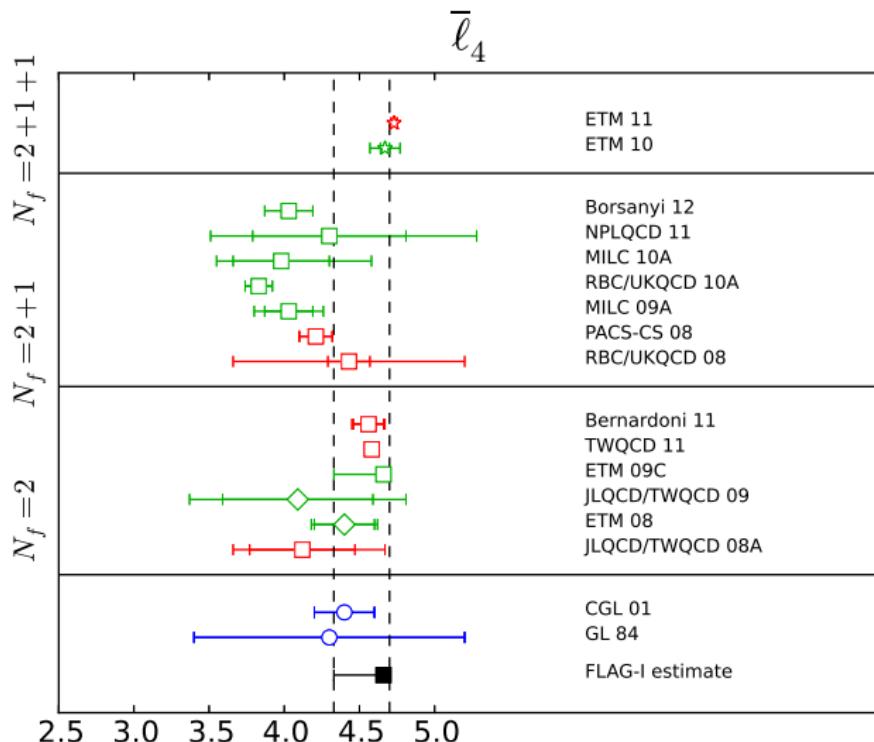
# LEC: $\bar{\ell}_3$ and $\bar{\ell}_4$

Collaboration	$N_f$	publ.	$mud \nearrow 0$	$a \nearrow 0$	$F_V$	$\bar{\ell}_3$	$\bar{\ell}_4$
ETM 11	2+1+1	C	○	★	○	3.53(5)	4.73(2)
ETM 10	2+1+1	A	○	○	○	3.70(7)(26)	4.67(3)(10)
Borsanyi 12	2+1	P	★	★	○	3.16(10)(29)	4.03(03)(16)
NPLQCD 11	2+1	P	★	○	★	4.04(40)( <sup>+73</sup> <sub>-55</sub> )	4.30(51)( <sup>+84</sup> <sub>-60</sub> )
MILC 10A	2+1	C	○	★	★	2.85(81)( <sup>+37</sup> <sub>-92</sub> )	3.98(32)( <sup>+51</sup> <sub>-28</sub> )
MILC 10	2+1	C	○	★	★	3.18(50)(89)	4.29(21)(82)
RBC/UKQCD 10A	2+1	A	○	○	★	2.57(18)	3.83(9)
MILC 09A	2+1	C	○	★	★	3.32(64)(45)	4.03(16)(17)
MILC 09A	2+1	C	○	★	★	3.0(6)( <sup>+9</sup> <sub>-6</sub> )	3.9(2)(3)
PACS-CS 08	2+1	A	★	■	■	3.47(11)	4.21(11)
PACS-CS 08	2+1	A	★	■	■	3.14(23)	4.04(19)
RBC/UKQCD 08	2+1	A	○	■	★	3.13(33)(24)	4.43(14)(77)
Bernardoni 11	2	C	○	■	■	4.46(30)(14)	4.56(10)(4)
TWQCD 11	2	P	★	■	■	4.149(35)(14)	4.582(17)(20)
ETM 09C	2	A	○	★	○	3.50(9)( <sup>+9</sup> <sub>-30</sub> )	4.66(4)( <sup>+4</sup> <sub>-33</sub> )
JLQCD/TWQCD 09	2	A	○	■	■		4.09(50)(52)
ETM 08	2	A	○	○	○	3.2(8)(2)	4.4(2)(1)
JLQCD/TWQCD 08A	2	A	○	■	■	3.38(40)(24)( <sup>+31</sup> <sub>-0</sub> )	4.12(35)(30)( <sup>+31</sup> <sub>-0</sub> )
CERN-TOV 06	2	A	○	○	■	3.0(5)(1)	

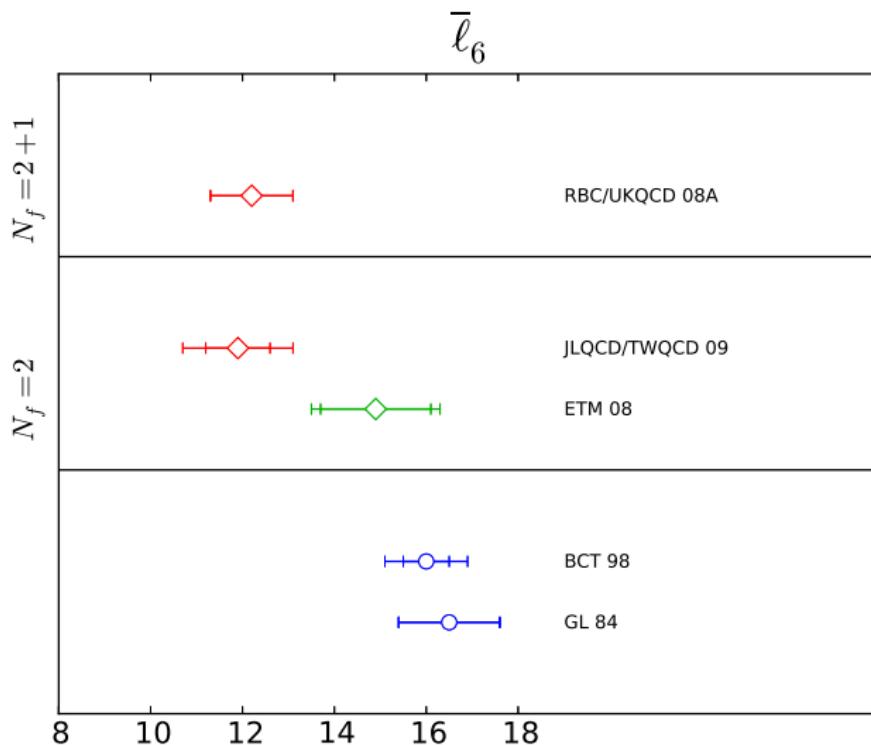
# LEC: $\bar{\ell}_3$ and $\bar{\ell}_4$



# LEC: $\bar{\ell}_3$ and $\bar{\ell}_4$

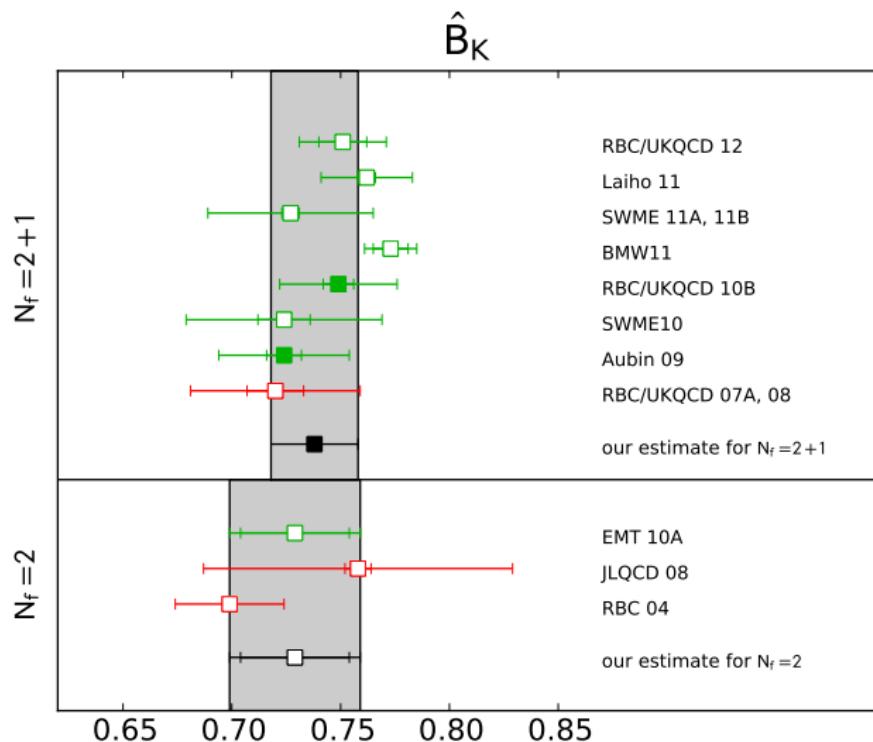


LEC:  $\bar{l}_6$

LEC:  $\bar{\ell}_6$ 

$B_K$ 

Collaboration	$N_f$	publ.	$a \rightarrow 0$	$mud \rightarrow 0$	$F_V$	renom.	running	$B_K$	$\hat{B}_K$
RBC/UKQCD 12	2+1	C	○	★	★	★	a	0.549(5)(26)	0.751(11)(17)
Laiho 11	2+1	C	★	○	○	★	—	0.5572(28)(150)	0.7655(38)(207)
SWME 11A, 11B	2+1	A	★	○	○	○ <sup>‡</sup>	—	0.531(3)(27)	0.727(4)(38)
BMW 11	2+1	A	★	★	★	★	b	0.5644(59)(58)	0.7727(81)(84)
RBC/UKQCD 10B	2+1	A	○	○	★	★	a	0.549(5)(26)	0.749(7)(26)
SWME 10	2+1	A	★	○	○	○	—	0.529(9)(32)	0.724(12)(43)
Aubin 09	2+1	A	○	○	○	★	—	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 07A, 08	2+1	A	■	■	★	★	—	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	■*	★	■	—	0.618(18)(135)	0.83(18)
ETM 10A	2	A	★	○	○	★	c	0.516(18)(12)	0.729(25)(17)
JLQCD 08	2	A	■	○	■	★	—	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■ <sup>†</sup>	★	—	0.495(18)	0.699(25)
UKQCD 04	2	A	■	■	■ <sup>†</sup>	■	—	0.49(13)	0.69(18)

$B_K$ 

## New sections

The new quantities considered require different quality criteria  
exact definition in progress

- ▶ Heavy quarks:
  - ▶ finite volume: requirements may be relaxed
  - ▶ heavy quark treatment: very important, will be tagged with yes/no
  - ▶ weak operator treatment: renormalization, order of improvement and method for matching to the continuum ( $\rightarrow$  yes/no)
- ▶  $\alpha_s$ :
  - ▶ scale where perturbation theory is used and which PT (bare vs. renormalized or lattice vs. continuum)
  - ▶ agreement with perturbative running
  - ▶ continuum limit for large  $\mu$  requires  $a\mu \ll 1$

# Conclusions

- ▶ it is a responsibility of the lattice community to provide experimentalists and non-lattice theorists with a review of phenomenologically relevant lattice results
- ▶ FLAG has now started its phase 2 with a larger group and broader scope
- ▶ we hope that this initiative continues to gain momentum and the support of the whole lattice community
- ▶ I have reviewed the current status also as far as the physics is concerned:
  - ▶ current updates of light-quark related quantities
  - ▶ issues in defining quality criteria for heavy quarks and  $\alpha_s$