Chiral symmetry breaking in lattice QED mode with 2+1d fermion brane

Eigo Shintani (RIKEN-BNL)
and Tetsuya Onogi (Osaka)

arXiv:1203.1091 [hep-lat] and progress

Motivation

- Lattice study of effective model of Graphene
  - *Conjecture* of correspondence between Graphene and QED model in which fermion is bounded in 2+1 D brane.

QED model	**Graphene**
Chiral phase transition	Excitonic insulator transition
Energy gap	Band gap
Massless Dirac fermion	Electron-hole
Charn-Simon, magnetic catalysis	Anomalous Hall effect

  - The non-perturbative study of QED mode need:
    - The phase transition occurs in strong region.
    - Coupling constant is enlarged by small fermi-velocity: $\alpha^{\mathrm{QED}} = 1/(4\pi\upsilon) \sim O(1)$ because $\upsilon \sim O(10^{-2})$.
    - Strong dynamics plays important role.

Model prediction

Relativistic QED model

- Gauge invariant, (super-)renormalizable
- Velocity effect
- Non-trivial relation between honeycomb lattice

Chiral transition, renormalization

Graphene: Insulator transition

Chiral transition

Non-relativistic QED model

- Starting from tight-binding model on honeycomb lattice
- Instantaneous Coulomb interaction
- No-scale and unrenormalizable

Perturbation:
Gonzalez et al., NPB 424, 595 (1994).
Kotov et al., arXiv:1012.3484
Monte-Carlo study:
ES and Onogi, arXiv:1203.1091 [hep-lat]

Chiral transion ⇔ Graphene band gap: analogy of spontaneous mass gap in QED_3

Jackiw and Templeton (1981),
Appelquiest and Pisarski (1981)

Large N analysis:
Son, PRB 75, 235423 (2007).
Herbut, PRL 97, 146401 (2006).
Schwinger-Dyson equation:
Gorbar et al., PRD 64, 105028 (2001).
Gamayun, et al., PRB 81, 075429 (2010).
Monte-Carlo study:
Drut and Lahde, PRL 102, 026802 (2009).
Armour et al., PRB 81, 125105 (2010).

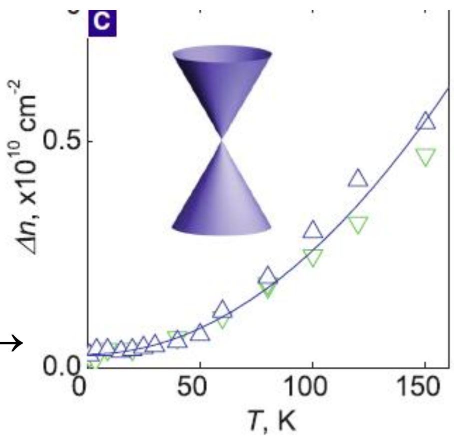
Experimental behavior

- Insulator transition
  - There is no evidence in monolayer Graphene.

Possibly weak coupling $\alpha^{\text{eff}} = \alpha^{\text{QED}}\,\varepsilon_0 \ll 1$ due to effect of substrate of Si.

<u>Monolayer</u>

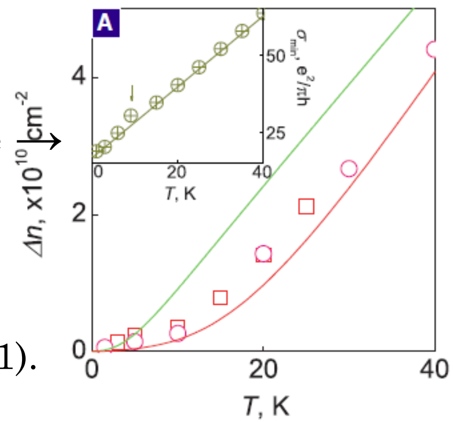
Zero ?→



A.S. Mayorov, et al.,
Science 333, 860 (2011).

<u>Doublelayer</u>

Finite →

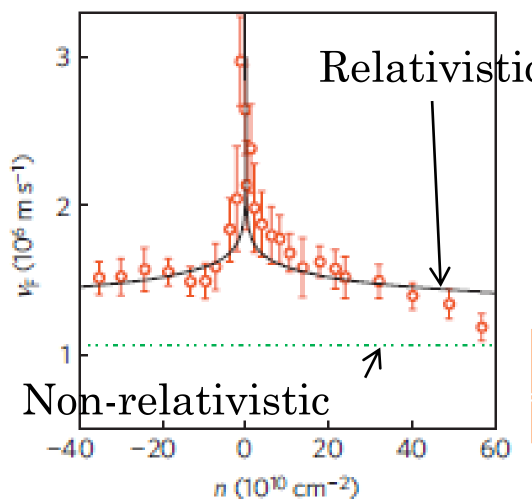


- Scaling of velocity
  - Fermi-velocity has logarithmic scaling:

$$v(\mu) = v\left(1 + \frac{\alpha}{4}\ln\frac{\Lambda}{\mu}\right), \qquad e^2(\mu) \sim \mathcal{O}(\alpha^2)$$

$$\alpha_E(\mu) = e^2(\mu)/(4\pi v_F(\mu))$$

Consistent behavior of perturbation result in relativistic model.



D. C. Elias, et al.,
Nature Phys. 7, 701 (2011).

Our strategy

- Monte Carlo study of relativistic model
  - Generalize to relativistic QED action with velocity

$$S_{QED_{2+1}} = \int d^2x dt \sum_{f=1}^{N_f} \bar{\psi}_f \Big[\gamma_0(\partial_t + A_0) + v_F \sum_{i=1,2} \gamma_i(\partial_i + A_i)\Big] \psi_f$$

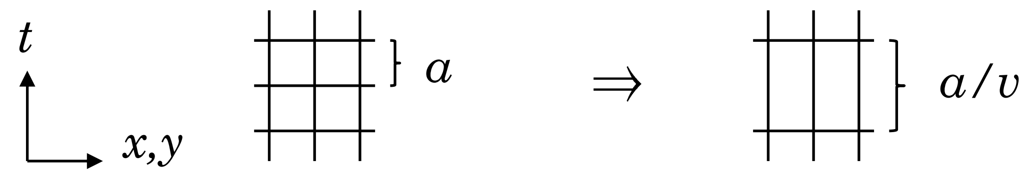
Gauge invariance and including fermi velocity,
which is more realistic than non-relativistic approach.

Drut and Lahde (2009), Armour et al.(2010).

  - Rescaled

$$S = \int dx^2 dt \bar{\psi} [D_0(A_0)\gamma_0 + \vec{D}(A_i)\vec{\gamma}]\psi + \frac{1}{2e^2}\int d^3x dt [vF_{0i}^2 + \frac{1}{v}F_{ij}^2]$$

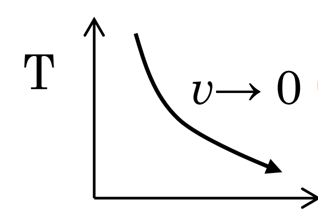
$t \to tv,\ A_0 \to A_0/v$



$$S = \sum_{t,x,y} \chi^\dagger [\eta_0 D_0(\theta_0) + \eta_i D_i(\theta_i) + m]\chi + \sum_{t,x,y,z} \frac{1}{2}[\beta v F_{0i}^2 + \frac{\beta}{v}F_{ij}^2]$$

2+1 dimensional 3+1 dimensional

- “Strong” electric and “weak” magnetic field.
- Photon field at z direction is interaction free.
- Two bare parameters, (v. β)



Lattice simulation

- Dynamical staggered fermion simulation
  - HMC simulation
  - Omelyan integrator
  - Hasenbush mass preconditioning, $m_h = 0.05$
  - Low-mode projection (if CG iteration is over than 10^4)
  - HMC time step $\tau \backsimeq 0.01 - 0.05$, HMC time $N_\tau = 1/\tau$
  - 10000 – 30000 HMC traj after 400 traj therm
  - 20 steps for use of statistics, O(100) total statistics
  - Jackknife error analysis, bin size = 10
  - Spatial periodic, temporal anti-periodic BC
  - Finite temperature, N_t=20 fixed.

Chiral symmetry breaking

Chiral condensate: σ
Chiral suceptibility: χ_m

Lowlying distribution of Dirac kernel

Hadronic spectrum

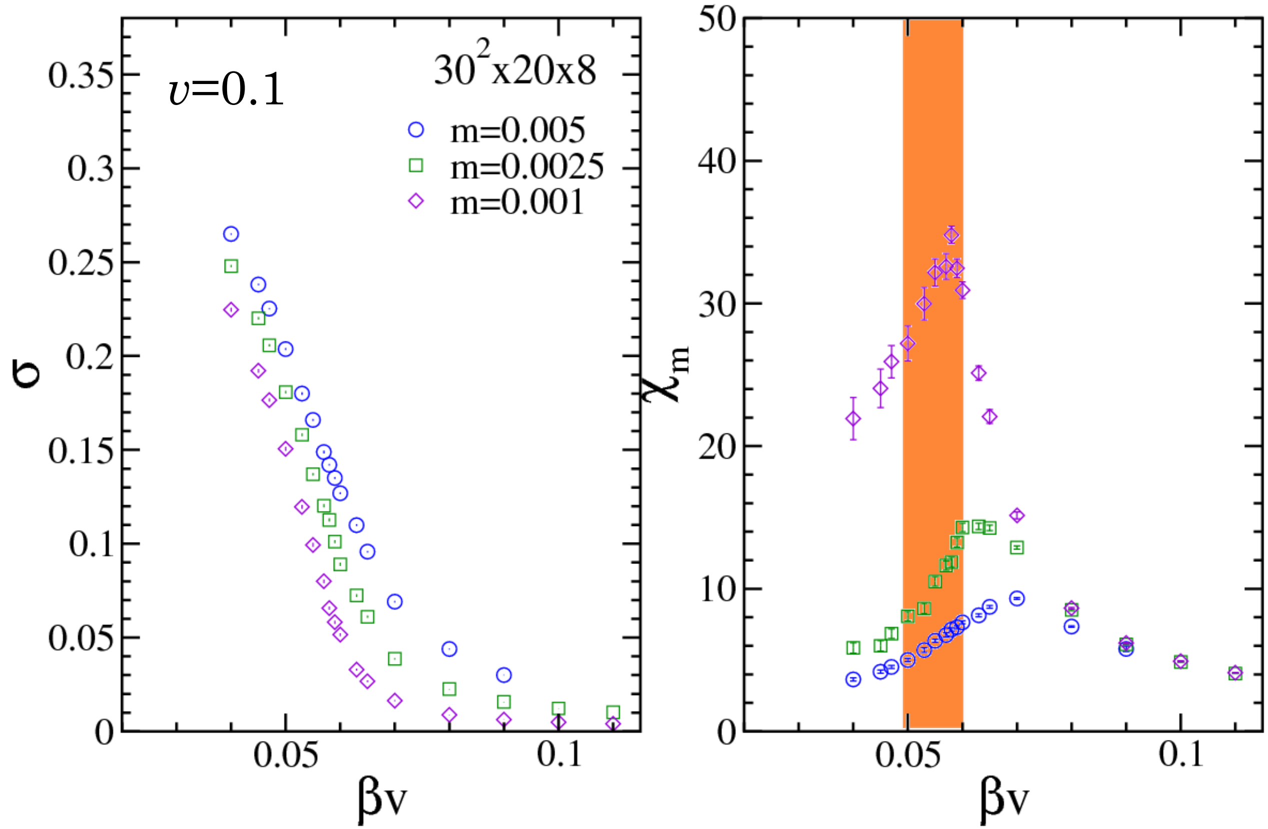
Order parameter
Peak of χ_m
Critical point

Banks-Casher relation
Random matrix theory

Nambu-Goldstone boson
Chiral perturbation
PCAC relation

Chiral symmetry breaking

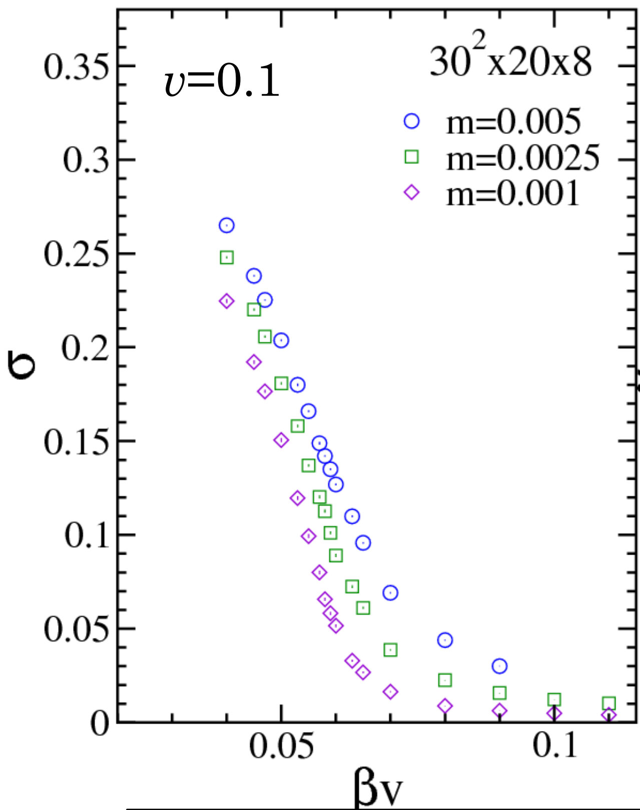
Chiral condensate and susceptibility

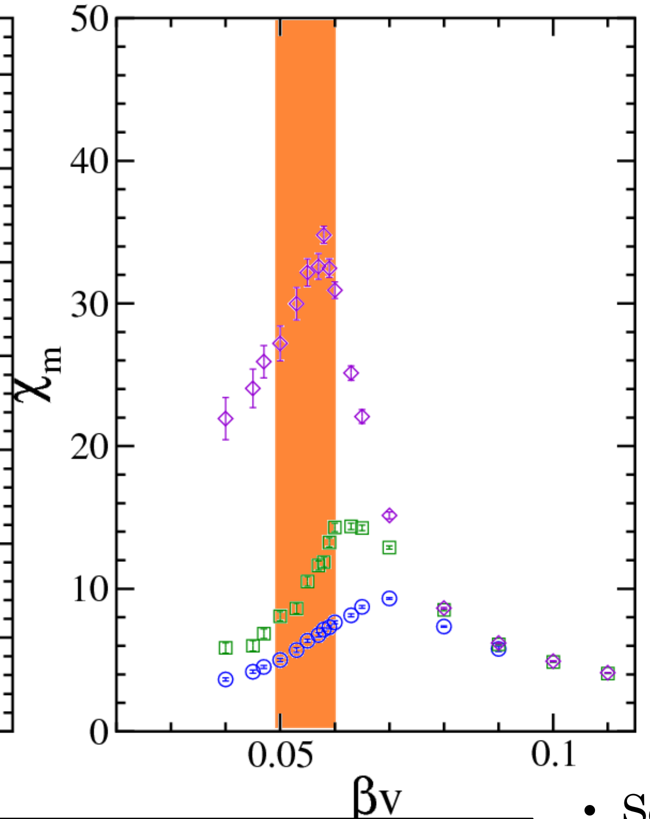


$$\sigma = \langle \mathrm{Tr} D_\chi^{-1} \rangle,$$
$$\chi_m = d\sigma/dm$$

$\alpha = 1/(4\pi\beta v)$

Chiral condensate and susceptibility





$$\sigma = \langle \mathrm{Tr} D_\chi^{-1} \rangle,$$

$$\chi_m = d\sigma/dm$$

$$\alpha = 1/(4\pi\beta v)$$

- Significant mass dependence
- Critical behavior of 2nd order transition
- Clear peak of χ_m
- Critical point:
  $(\beta v)_c \simeq 0.05 - 0.06$, $\alpha_c \simeq 1.3 = O(1)$.

- Schwinger-Dyson model:
  $\alpha_c = 0.92 - 1.13$
  Gamayun, et al. (2010), Khveshchenko (2009)
- Non-relativistic MC
  $\alpha_c = 1.11(6)$
  Drut, Lahde (2009)

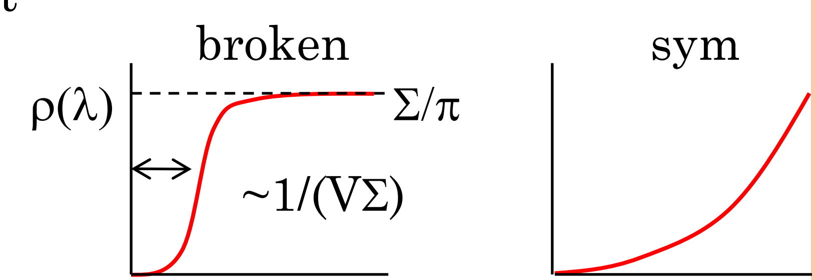
LOWMODE DISTRIBUTION

- Spectral density
  - Lowmode distribution which is related to chiral symmetry breaking
  - Banks-Casher relation:

$$\rho(\lambda) = \left\langle \frac{1}{V} \sum_n \delta(\lambda - \lambda_n) \right\rangle \qquad \lim_{m \to 0} \lim_{V \to \infty} \langle \bar{q} q \rangle = \Sigma = \pi \rho(0)$$

$\Rightarrow$ spectral density provides chiral condensate

  - Gap appears due to finite size effect
  - Scale generation: $1/(V\Sigma)$

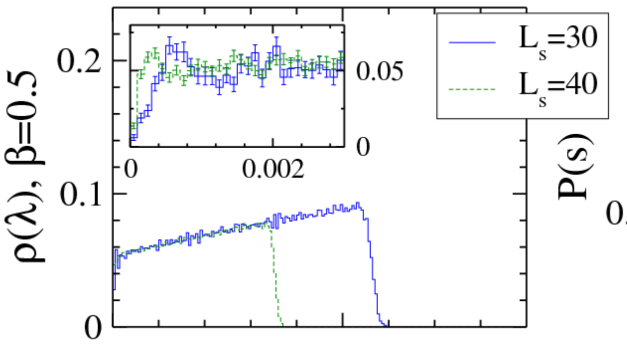


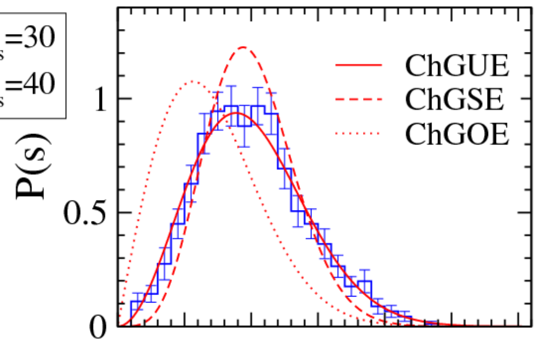
- Level spacing distribution
  - According to random matrix theory, we can distinguish universal distribution of level spacing: $s = (\lambda_2 - \lambda_1)/\langle \lambda_2 - \lambda_1 \rangle$

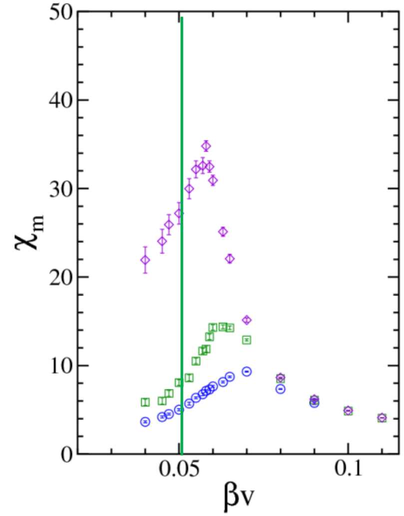
$$P(s) = \begin{cases} \frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi} s^2} & \text{chGUE} \\ \frac{2^{18}}{3^6 \pi^3} s^4 e^{-\frac{64}{9\pi} s^2} & \text{chGSE} \\ \pi s e^{-\frac{\pi}{2} s^2} & \text{chGOE} \end{cases}$$

$\leftarrow$ fundamental rep., U(N)$\times$U(N)/U(N)

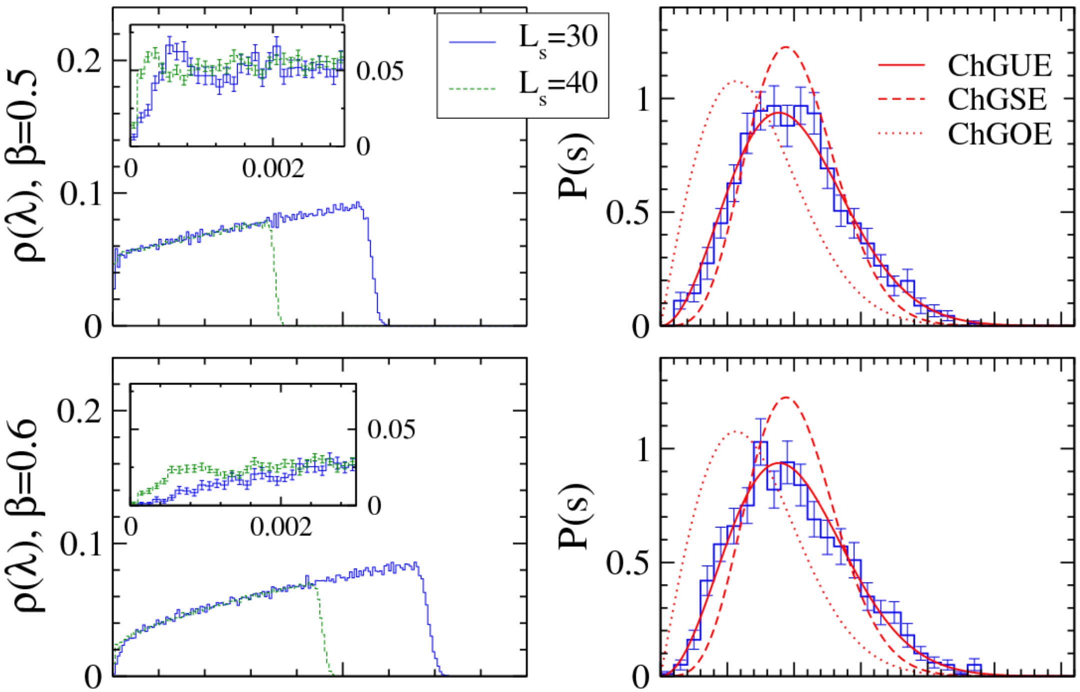
LOWMODE DISTRIBUTION

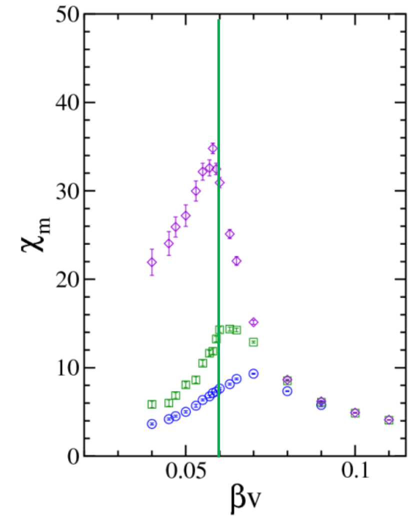




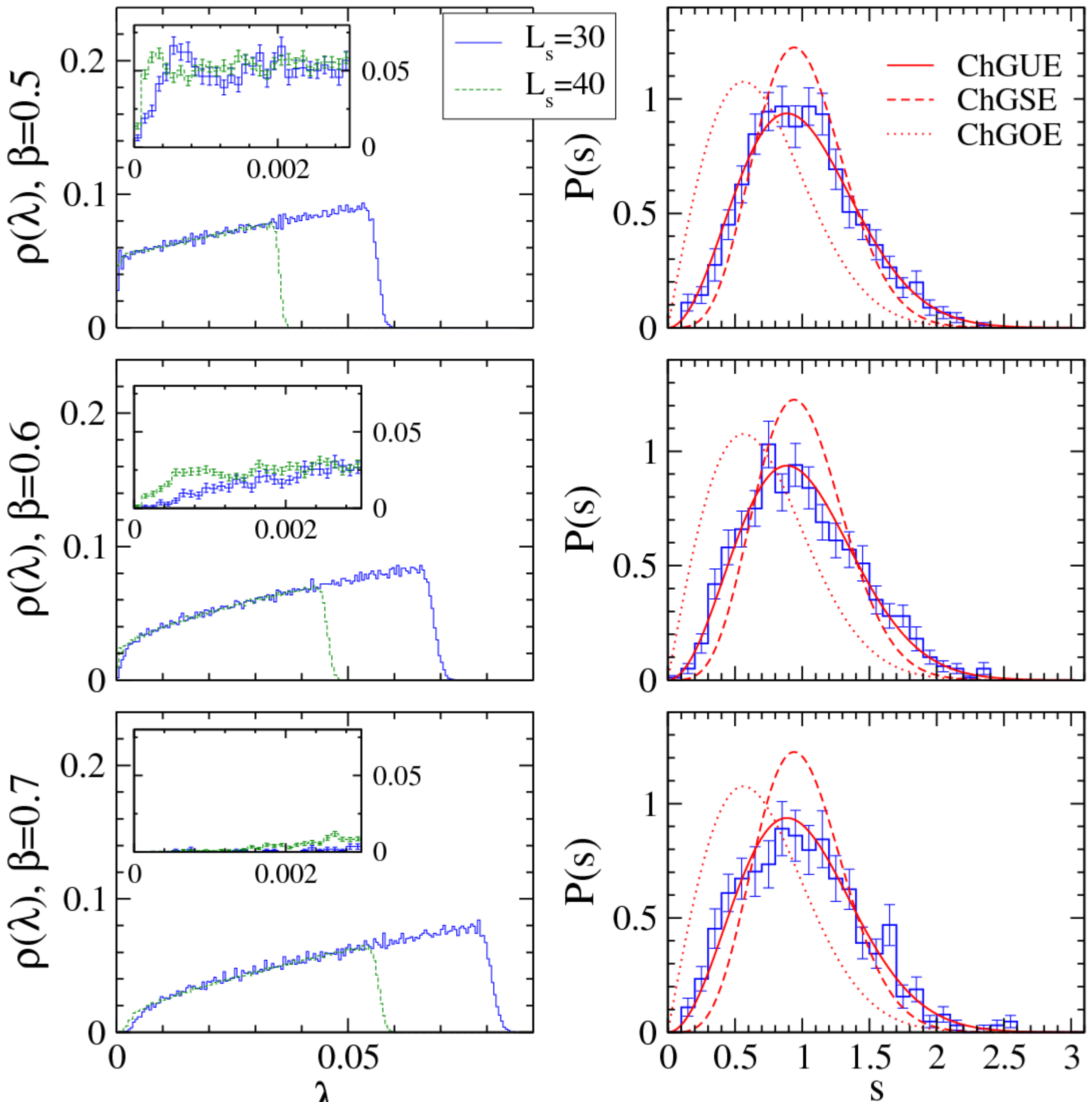


Lowmode distribution





LOWMODE DISTRIBUTION

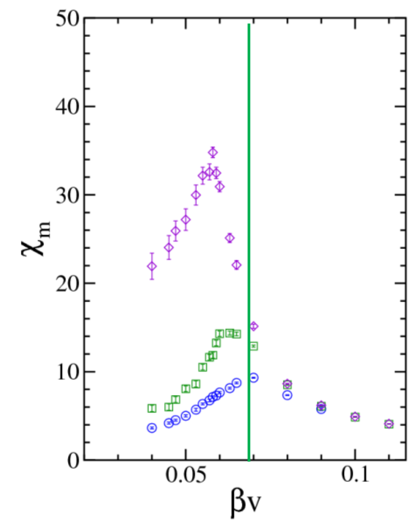


- Correspondence between critical point and behavior of spectral density
- β = 0.5, "hard edge"
  β > 0.6, perturbative

$$\rho(\lambda) \sim \lambda^d$$

cf. Damgaard, Heller, et al. (2000)

- Level spacing distribution is consistent with RMT



Hadronic spectrum

- Goldstone boson spectrum

  Correlation function of NG boson: $P_{\rm GS} = \bar{\psi}(\gamma_5 \otimes \gamma_5)\psi$

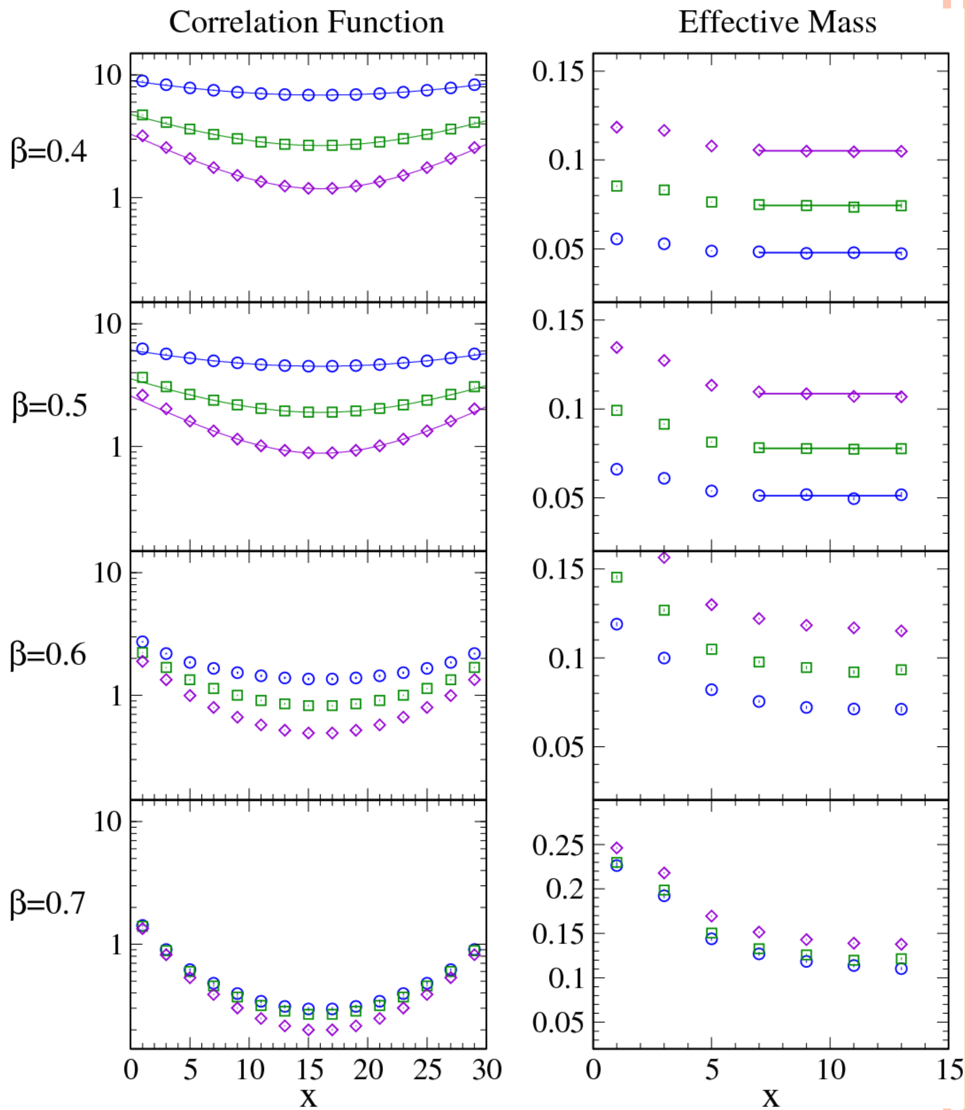
  $$G(x) = \sum_{y,t} \langle P_{GS}(x,y,t) P_{GS}(0) \rangle$$

  Effective (screening) mass m_π :

  $$\lim_{x \gg 1} G(x) =$$

  $$Z\left[e^{-m_{\rm NG}x} + e^{-m_{\rm NG}(N_x - x)}\right]$$

  <u>Existing asymptotic state of NG boson</u>



β= 0.4 – 0.5 : χ^2 fitting works well.
β= 0.6: seems to be plateau
β= 0.7: do not observe plateau.

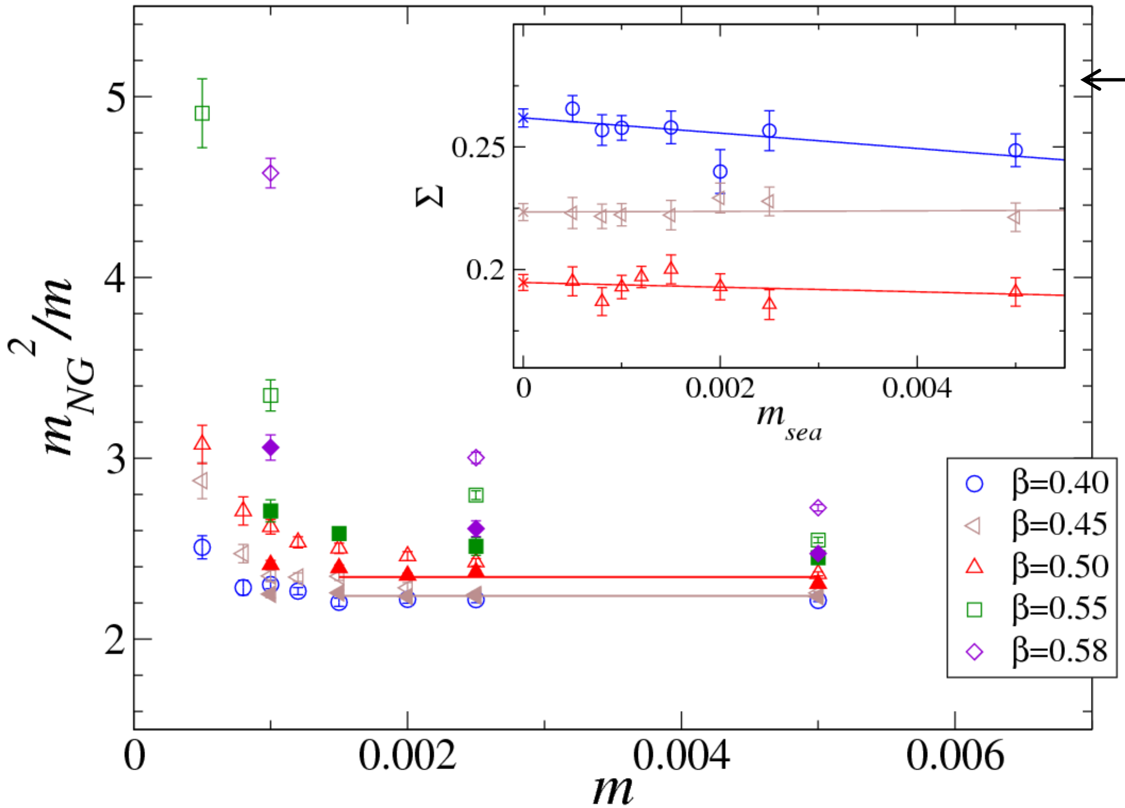
NG boson state appear in strong broken phase

Hadronic spectrum

- Comparison with ChPT and PCAC

NG boson mass and decay constant have to be satisfied with ChPT and PCAC relation

$$m_{\rm NG}^2 = 2Bm$$



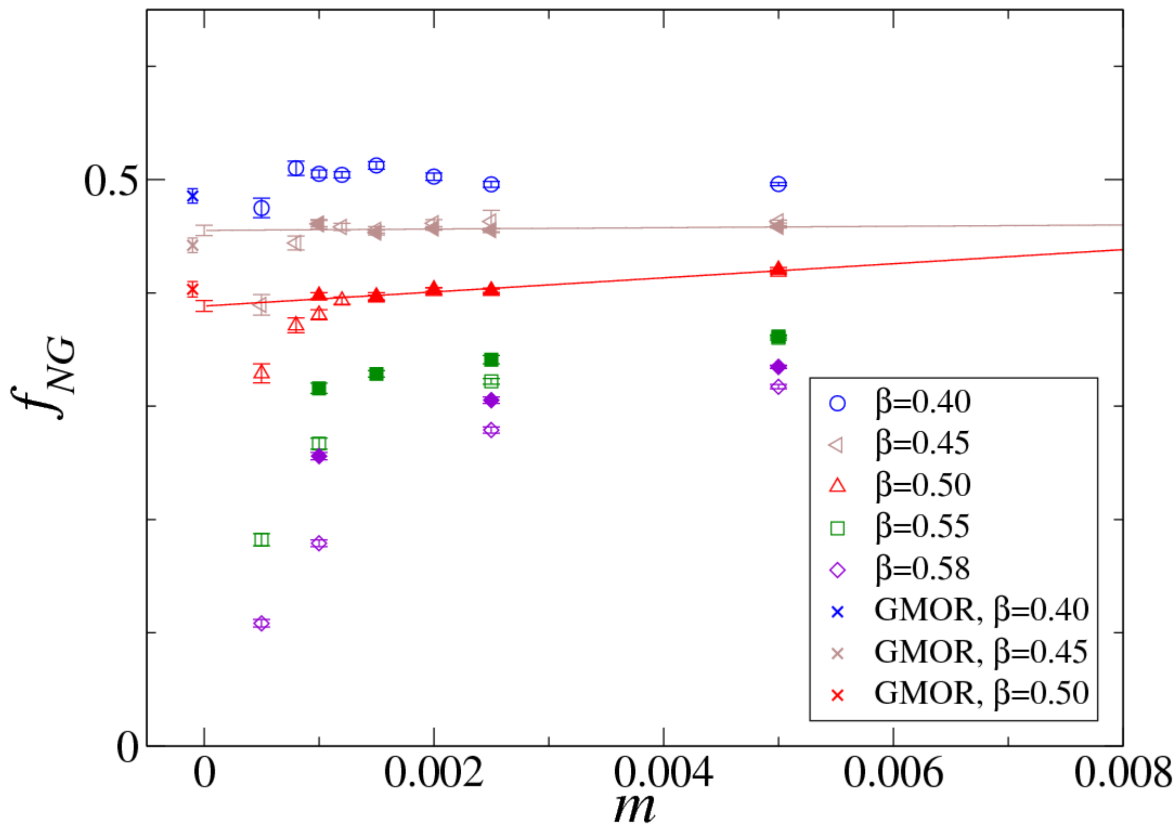
Linear sea mass fitting of Σ obtained by spectral density.

- Plateau appears in [0.0015, 0.005]
- obtain B factor
- non-linear behavior at β > 0.55

HADRONIC SPECTRUM

- Comparison with ChPT and PCAC

$$\langle 0|A_\mu|\pi\rangle = ip_\mu f_{\rm NG} \qquad f_{\rm NG} = \sqrt{\frac{\Sigma}{B}} : \text{ GMOR relation}$$



- GMOR relation:
  Σ: Banks-Casher rel.
  B: constant fit
- β > 0.55, $f_{NG} \to 0$ linearly.
- Consistent value with GMOR using linear chiral extrapolation at β ≦0.5.

Summary

Lattice QED model with fermion brane

Relativistic definition:
2+1 D fermion + 4D gauge,
gauge invariance, velocity scaling

Chiral symmetry breaking

1[st] step

Continuum limit ?
Phase diagram ?
Renormalization ?

Graphene model

- Metal-insulator transition
- Hall effect

What is NG boson ?

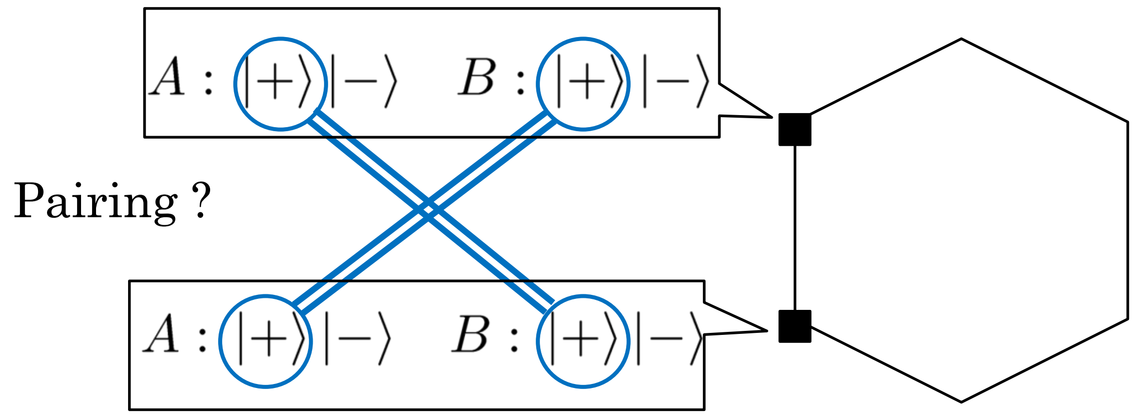
PS operator:

$$\bar{\psi}\gamma_5\tau\psi = \int\sum_{\sigma,\sigma'}\Big[-A^\dagger_\sigma(k+K)\tau_{\sigma\sigma'}B_{\sigma'}(k-K) - B^\dagger_\sigma(k+K)\tau_{\sigma\sigma'}A_{\sigma'}(k-K)$$
$$+B^\dagger_\sigma(k-K)\tau_{\sigma\sigma'}A_{\sigma'}(k+K) + A^\dagger_\sigma(k-K)\tau_{\sigma\sigma'}B_{\sigma'}(k+K)\Big]$$

NG boson: bound state of electron-hole with different valley and sublattice

Considering 4 NG boson states as spin degenerate

$$|++\rangle,\ |--\rangle,\ \frac{1}{2}(|+-\rangle + |-+\rangle),\ \frac{1}{2}(|+-\rangle - |-+\rangle)$$



Backup slide

Graphene lattice and Hamiltonian

- Tight binding approximation

$$
\begin{aligned}
H &= t \sum_{x,\sigma=\pm} \sum_{i=1}^{3} \left[A_\sigma^\dagger(x) B_\sigma(x+b_i) + h.c. \right] \\
&+ t' \sum_{x,\sigma=\pm} \sum_{i=1}^{6} \left[A_\sigma^\dagger(x) A_\sigma(x+a_i) + B_\sigma^\dagger(x) B_\sigma(x+a_i) \right]
\end{aligned}
$$

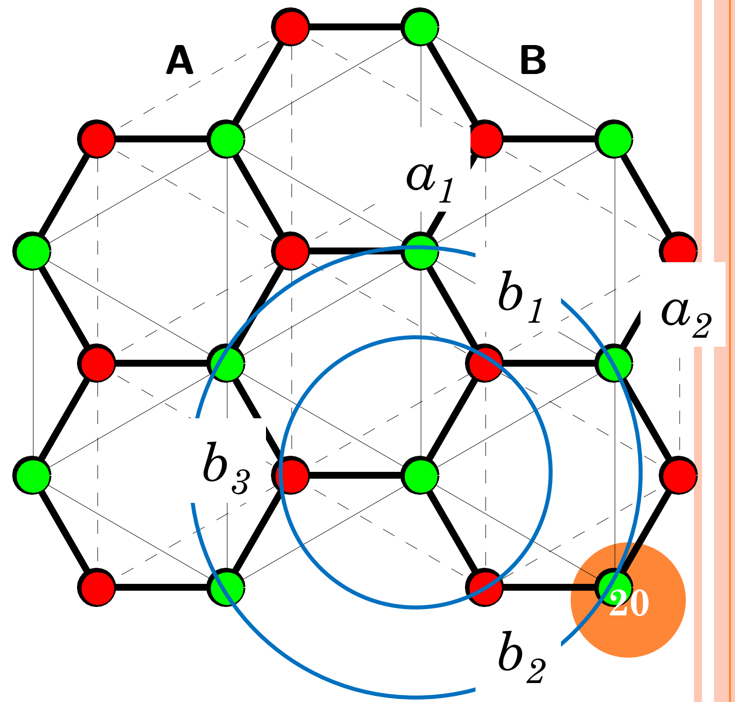
$$
b_1 = \left(\frac{a_g}{2\sqrt{3}}, \frac{a_g}{2} \right), \qquad b_2 = \left(\frac{a_g}{2\sqrt{3}}, -\frac{a_g}{2} \right),
$$

$$
b_3 = \left(-\frac{a_g}{\sqrt{3}}, 0 \right) \qquad a_g = 0.142\,\mathrm{nm}
$$

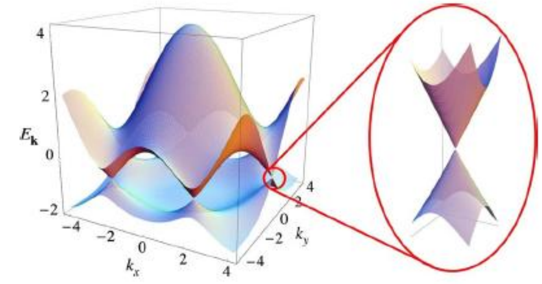
Experimental estimate :

$t = 2.8$ eV, $t' = 0.1$ eV Reich, et al., (2002)

- The second term is next-to-leading order
- Setting to $t' = 0$ is good approximation



Low energy approximation



- Expansion near Dirac point
  - Dirac (K) point : hexagonal points
  - Fourier transformation of H

$$H \simeq t \sum_{\sigma} \int_{\Omega_B} \frac{d^2k}{(2\pi)^2} \begin{pmatrix} \tilde{A}^\dagger_\sigma(k) & \tilde{B}^\dagger_\sigma(k) \end{pmatrix} \begin{pmatrix} 0 & E_k \\ E_k^\dagger & 0 \end{pmatrix} \begin{pmatrix} \tilde{A}_\sigma(k) \\ \tilde{B}_\sigma(k) \end{pmatrix}$$

with $E_k = \sum_{i=1}^{3} e^{-ikb_i}$

  - $|E_k| = 0 \Rightarrow \; k = \{K_1, K_2\},\; K_1 = \left(0, \pm\frac{4\pi}{3a}\right),\; K_2 = \left(\pm\frac{2\pi}{\sqrt{3}a}, \pm\frac{2\pi}{3a}\right)$

$$\begin{aligned} H_0 &= v_F \sum_\sigma \int_{DC} \frac{d^2k}{(2\pi)^2} \psi^\dagger_\sigma(k) \begin{pmatrix} 0 & k_x - ik_y & 0 & 0 \\ k_x + ik_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_x + ik_y \\ 0 & 0 & -k_x - ik_y & 0 \end{pmatrix} \psi_\sigma(k) \\ &= \sum_\sigma \int_{DC} \bar{\psi}_\sigma(k) v_F k \cdot \gamma \psi_\sigma(k) \end{aligned}$$

Wallance (1947), G. W. Semenoff, (1984)

$$\psi_\sigma(k) = \begin{pmatrix} A_\sigma(k+K) \\ B_\sigma(k+K) \\ B_\sigma(k-K) \\ A_\sigma(k-K) \end{pmatrix},\; v_F = \frac{3ta_g}{2} \sim c/100,\; \gamma_i = \tilde{\tau}_i \otimes \tau_i$$

Dirac point

A, B sublattice

Low energy Graphene model

- Effective QED model
  - H_0 (tight-binding) $\Rightarrow$ 2+1D QED with fermi velocity
  - Instantaneous interaction

$$\int d^2x dt\, \bar{\psi}\left[i\gamma_0\partial_t + iv_F\gamma\cdot\partial\right]\psi + \int d^2x dt d^2x' dt'\, \bar{\psi}\gamma_0\psi(t,x)\frac{e^2\delta(t-t')}{8\pi|x-x'|}\bar{\psi}\gamma_0\psi(x',t')$$

$$\to S_{\rm NR} = \frac{1}{2e^2}\int dt d^3x \vec{E}^2 + \int d^2x dt\, \bar{\psi}\left[i\gamma_0(\partial_t+\phi) + iv_F\gamma\cdot\partial\right]\psi$$

- U(4) global symmetry

  In monolayer case, 16 generators $\underbrace{\{1, \gamma_5, i\gamma_3, [\gamma_5,\gamma_3]/2\}}_{\text{valley, sublattice}} \otimes \underbrace{\sigma_{i=0,1,2,3}}_{\text{spin}}$
- Parity can be defined as $P = \gamma_0$
- N_f = spin(=2) $\times$ layer, N_f =2 : monolayer, N_f =4 : double layer
- There were many studies with Schwinger-Dyson, large-N, Monte-Carlo

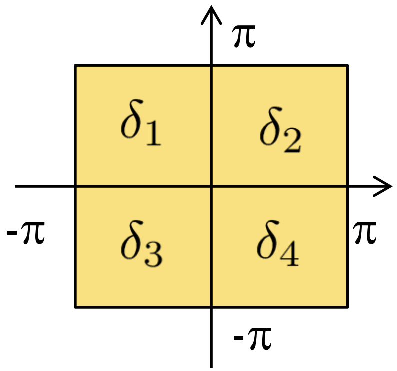
Gorbar (2001), Son (2005), Drut (2009)

Concering issue:

- Velocity free after rescaling $t \to t/v_F,\ \phi \to \phi v_F$
- Gauge variant action → difficult scaling study

Lattice QED model with fermion brane

- Staggered fermion in 2+1 dimension



$$\int_{-\pi}^{\pi} dp^3 M(p) \sim \sum_{f=1}^{8} \int_{-\pi/2}^{\pi/2} d^3p M(p+\delta_f)$$

$$\delta_f = \{(0,0,0)\,, (\pi,0,0), \ (0,\pi,0), \cdots, (\pi,\pi,\pi)\}$$

$$S = \sum_{n,m} \bar{\psi}(n)\Big[\frac{1}{2}\sum_{\mu}\Big\{\nabla_\mu(\gamma_\mu \otimes 1) + \nabla_\mu^2(\gamma_5 \otimes \tau_\mu\tau_3)\Big\} + m(1 \otimes 1)\Big]\psi(m)$$

  - Chiral symmetry

    $$\psi' = \exp[i\alpha(\gamma_5 \otimes \tau_3)]\psi, \ \bar{\psi}' = \bar{\psi}\exp[i\alpha(\gamma_5 \otimes \tau_3)]$$

    which is rotation of spinor and flavor simultaneously.
  - Restore the global U(4) in the continuum limit, without root trick

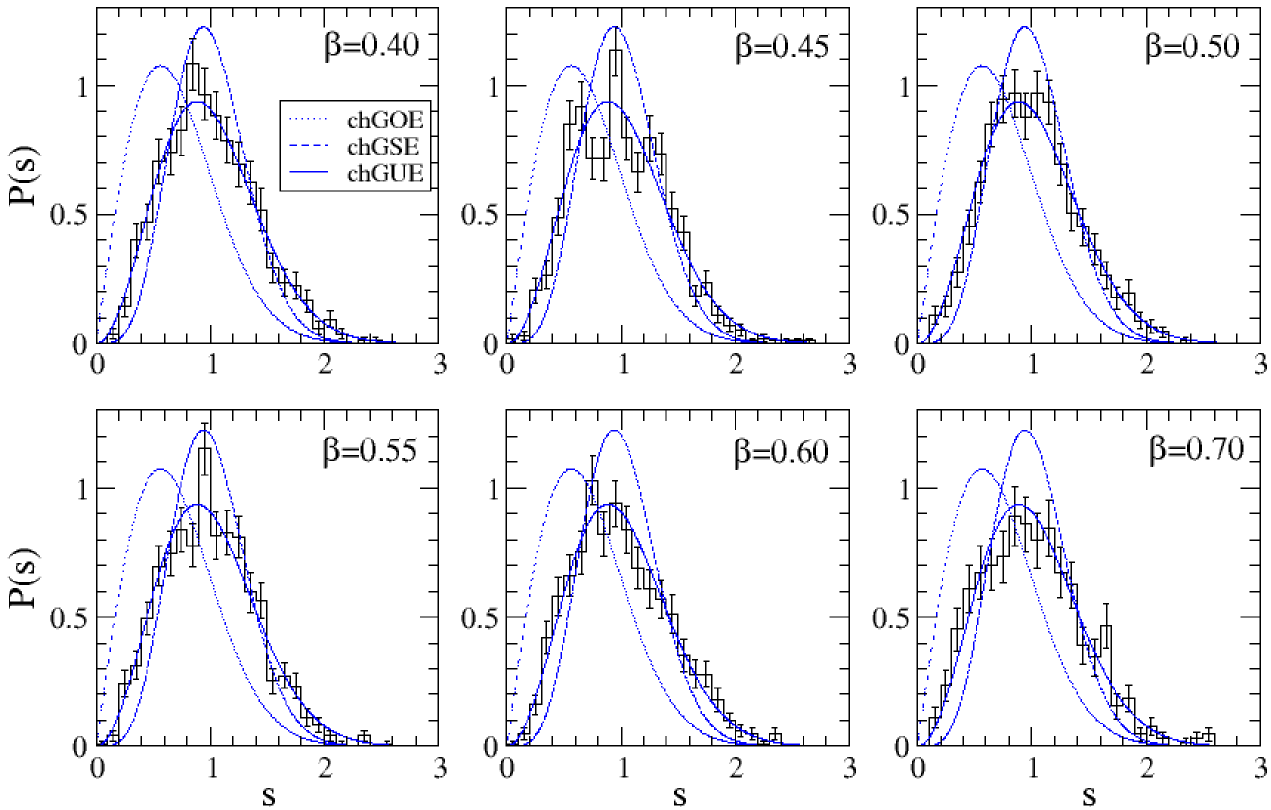
- Non-Compact QED

$$S_g = \sum_{n=x,y,z,t} \frac{1}{2}\Big[\frac{\beta}{v}\sum_{i,,j}(\theta_i(n+\hat{j}) - \theta_i(n))^2 + \beta v \sum_{i}(\theta_i(n+\hat{4}) - \theta_i(n))^2\Big]$$

There were many lattice studies in 3D and 4D QED for chiral SB and scaling study.

[Hands, Kogut, et al.(1989--2010)] [QCDSF (1990--1998)]
[Drut, Lahde (2009--2010)]

LOW-LYING MODE DISTRIBUTION



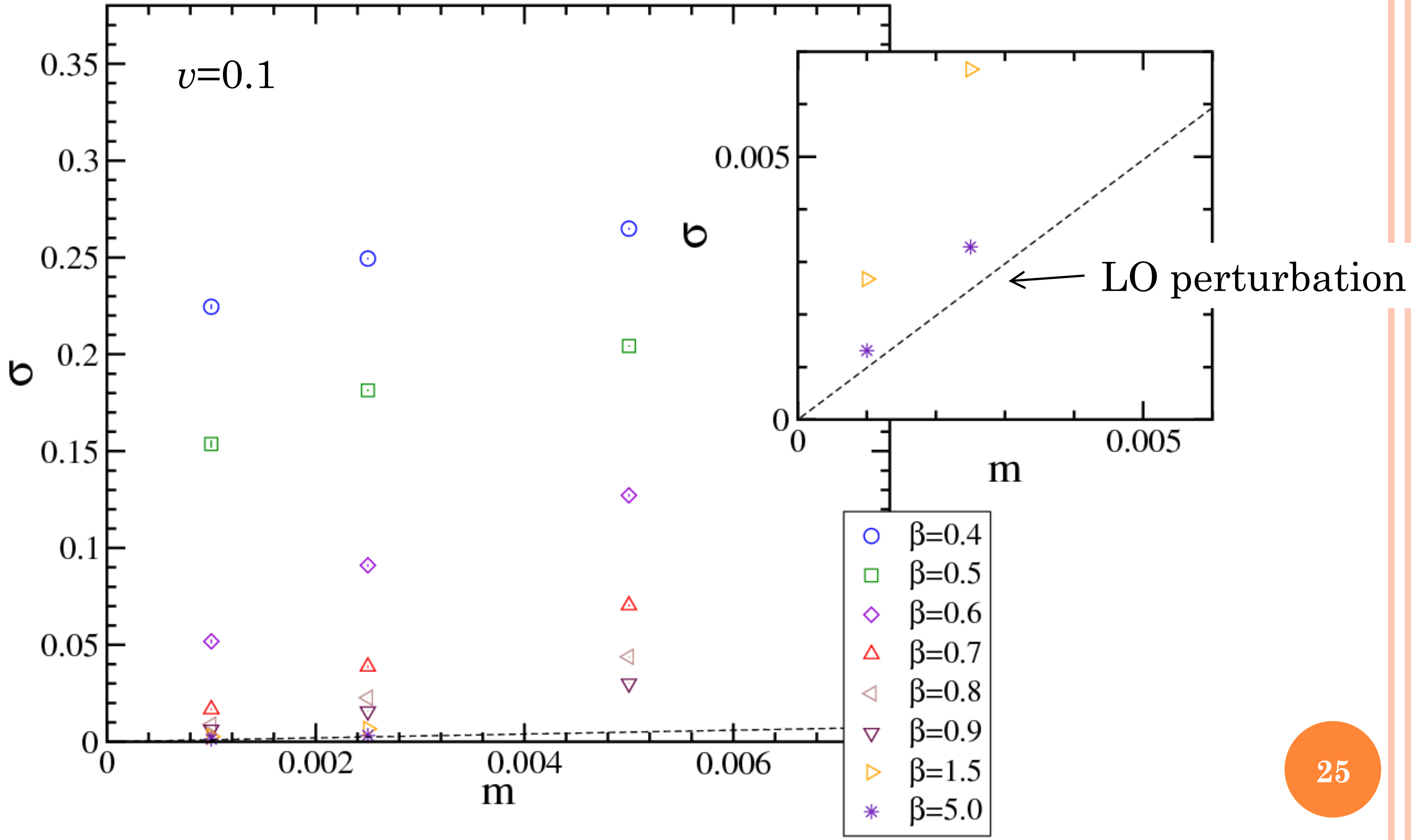
U(N$_f$)×U(N$_f$)/U(N$_f$)

$$P(s) = \begin{cases} \boxed{\frac{32}{\pi^2} s^2 e^{-\frac{4}{\pi}s^2} \quad \text{chGUE}} \\ \frac{2^{18}}{3^6\pi^3} s^4 e^{-\frac{64}{9\pi}s^2} \quad \text{chGSE} \\ \pi s e^{-\frac{\pi}{2}s^2} \quad \text{chGOE} \end{cases}$$

$$s = (\lambda_2 - \lambda_1)/\langle \lambda_2 - \lambda_1 \rangle$$

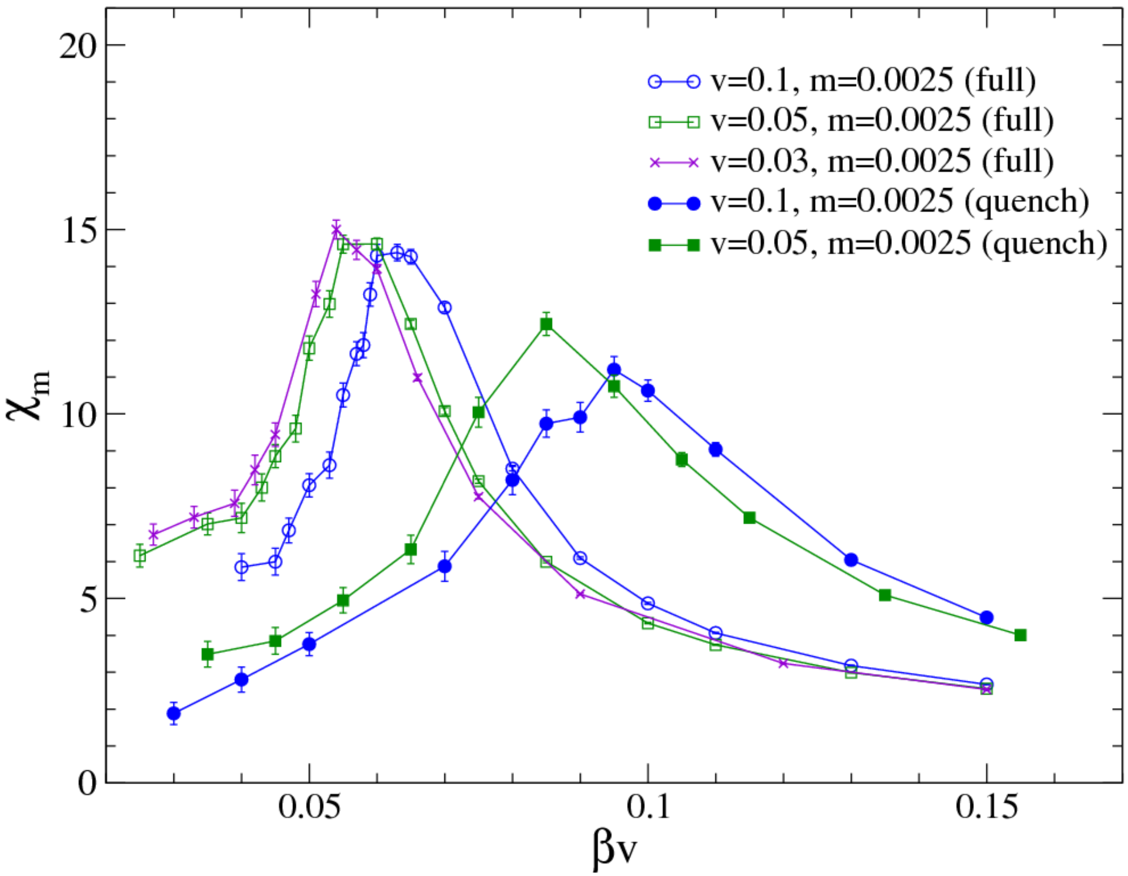
Mass dependence

- Chiral condensate



Vacuum polarization effect

- Chiral susceptibility



- Peak becomes sharp compared with quench results
- Dynamical effects
  - $v = 0.1$
    $(\beta v)_c \simeq 0.1$ (quench)
    $\rightarrow 0.062$ (full)
  - v=0.05
    $(\beta v)_c \simeq 0.09$ (quench)
    $\rightarrow 0.055$ (full)

  for α_c : 50--60% increase

cf. Gap eq. prediction
(quench) : α_c =0.5
(1-loop) : α_c =0.92
$\Rightarrow$ $\simeq$50% increase

Gamayun, et al. (2010)

[MC: α_c= 1.08 in Drut (2009)]

Dynamical effect is significant !
(and also similar to gap equation.)