One dimensional Supersymmetric Yang–Mills theory with sixteen supercharges

LATTICE 2012@Cairns
Parallel talk

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D.K. and Syo Kamata in preparation.

1. Motivation

Gauge/Gravity duality $\leadsto$ AdS/CFT

Strongly coupled gauge theory in large N limit

$\simeq$ Classical gravity on a curved space

Many applications:

Holographic QCD, Holographic superconductor…

However, this is a conjecture.

$\Downarrow$ Recent progress of Lattice supersymmetry

Numerical verification of gauge/gravity duality
from lattice gauge theory

=> target: 1D SYM with 16 supercharges.

Numerical simulations of 1D SYM with 16 supercharges

Non-lattice

non-compact gauge field
(completely gauge fixing)
& momentum sharp cutoff

J. Nishimura , M. Hanada et al.

PRL 99 (07) 161602
PRL 100 (08) 021601
PRL 102 (09) 181602
PRL 102 (09) 191602
PRL 102 (09) 111601
…

Good agreement with predictions from gravity side.

Lattice

S. Catterall and T. Wiseman

JHEP 0712 (2007) 104
Phys. Rev. D 78, 041502(2008)
JHEP 1004 (2010) 077
…

Consistent with gravity side .

⇨ Both group used the regularized theories which break full SUSY. In contrast, our lattice formulation holds two supersymmetries even on the lattice. (some advantages in taking continuum limit)

Talk's Plan

1. Motivation

2. Gauge/gravity duality and BH thermodynamics

3. Continuum and lattice action
   of 1D SYM with 16 supercharges

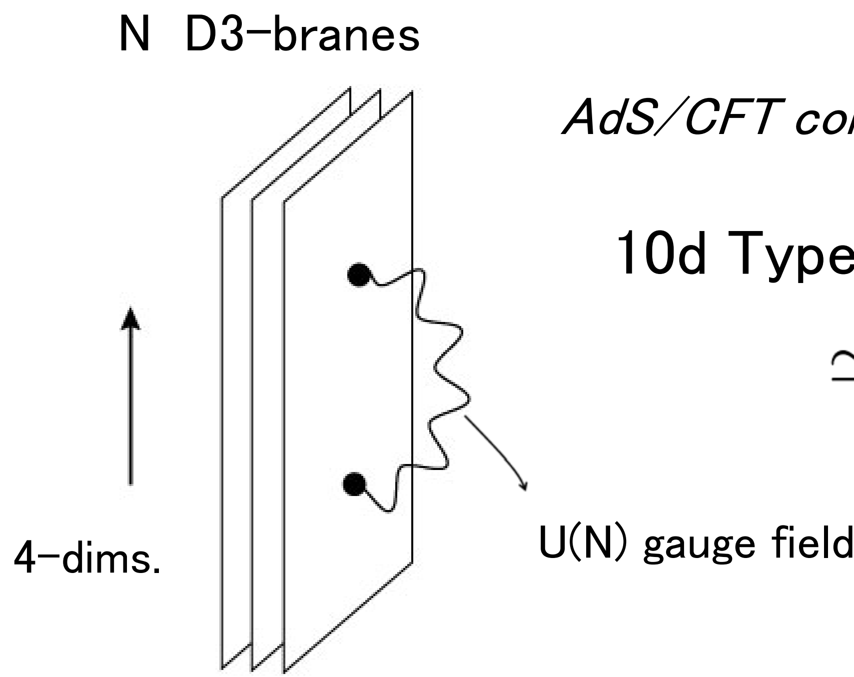
4. Simulation results

   SUSY-WTI, Complex phase problem,
   Internal energy, Wilson loop

5. Summary

2. Gauge/gravity duality

1998 Maldacena



AdS/CFT correspondence

10d Type IIB superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5$

$\simeq$ 4D N=4 U(N) SYM

CFT

$$g_{YM}^2 = (2\pi)\, g_s$$

$$\alpha' \to 0, \qquad U \equiv \frac{r}{\alpha'} = \text{fixed.}$$

$$ds^2 = \alpha' \left[\underbrace{\frac{U^2}{\sqrt{4\pi g_s N}}(-dt^2 + \sum_{i=1}^{2} dx_i^2) + \sqrt{4\pi g_s N}\frac{dU^2}{U^2}}_{\mathrm{AdS}_5} + \underbrace{\sqrt{4\pi g_s N} d\Omega_5^2}_{\mathrm{S}^5} \right]$$

Black hole thermodynamics

Black hole: a solution of Einstein' s eq

thermodynamics

Bekenstein–Hawking entropy

$$S_{BH} = \frac{k_B c^3}{4\hbar G_N} A$$

A: area of horizon

$$dS_{BH} \geq 0$$

macroscopic pictures ← thermodynamics

microscopic pictures

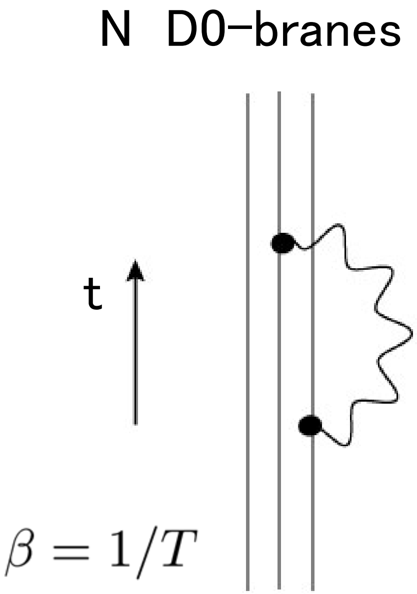
1996 Strominger–Vafa

String theory

⇨ thermodynamical quantities of black hole can be also calculated from gauge side.

N D0-branes system

1998 Itzhaki-Maldacena -Sonnenschein-Yankielowicz



N D0-branes in 10D type IIA superstring

$\simeq$ 1D SYM with 16 supercharges

$$\alpha' \to 0, \qquad U \equiv \frac{r}{\alpha'} = \text{fixed}.$$

$$ds^2 = \alpha' \left[-\frac{U^{\frac{7}{2}}}{\sqrt{d_0 g_{YM}^2 N}} dt^2 + \sqrt{d_0 g_{YM}^2 N} \frac{dU^2}{U^2} + \sqrt{d_0 g_{YM}^2 N} \frac{dU^2}{U^2} d\Omega_8^2 \right]$$

BH thermodynamics

1996 Klebanov -Tseytlin

$$\frac{1}{N^2} \frac{E}{\lambda^{\frac{1}{3}}} = c \left(\frac{T}{\lambda^{\frac{1}{3}}} \right)^{2.8}, \qquad c = \frac{9}{14} \left(4^{13} 15^2 \left(\frac{\pi}{7} \right)^{14} \right)^{\frac{1}{5}} = 7.41..$$

➡ We calculate the internal energy from gauge side to check gauge/gravity duality.

3. 1 D SYM with 16 supercharges

Continuum action

$$S=\frac{N}{\lambda}\int_0^\beta \mathrm{d}t\,\mathrm{tr}\left\{\frac{1}{2}\left(D_1\varphi_j(t)\right)^2-\frac{1}{4}[\varphi_i(t),\varphi_j(t)]^2\right.$$
$$\left.+\frac{1}{2}\bar{\psi}(t)\Gamma_1 D_1\psi(t)+\frac{1}{2}\bar{\psi}(t)i\Gamma_i[\varphi_i(t),\psi(t)]\right\}$$

$$i,j=2,\cdots,10$$

A_1 : gauge field

$\varphi_2\sim\varphi_{10}$: adjoint scalar fields(9 real components)

ψ : adjoint fermions(16 real components)

$\lambda=g_{YM}^2 N_c$ ‘t Hooft coupling

c.f. Dimensional reduction of D=10 N=1 SYM $\mu,\nu=1,\cdots,10$

$$S=\frac{N}{\lambda_{10}}\int \mathrm{d}^{10}x\,\mathrm{tr}\left\{\frac{1}{4}\left(F_{\mu\nu}(x)\right)^2+\frac{1}{2}\bar{\lambda}(x)\Gamma_\mu D_\mu\lambda(x)\right\}$$

$$A_\mu(t,x_2,...,x_{10})\to A_\mu(t),\quad \lambda(t,x_2,...,x_{10})\to\lambda(t)$$

Lattice action

c.f. 2004 F.Sugino

$$S_{LAT} = Q_+Q_-\frac{N}{\lambda_0}\sum_t -\frac{1}{2}\mathrm{tr}\left\{-iB_i\Phi_i - \frac{1}{3}\epsilon_{ijk}B_i[B_j,B_k]\right.$$
$$\left. -\psi_{+\mu}\psi_{-\mu} - \chi_{+i}\chi_{-i} - \frac{1}{4}\eta_+\eta_- \right\}$$

some combinations of original scalars/fermions

- Exact gauge invariance

$$U_1(t) \to \Lambda(t)U_1(t)\Lambda(t+1)^{-1}$$
$$f(t) \to \Lambda(t)f(t)\Lambda(t)^{-1}$$

$U_1 \cdots$ compact gauge field

$f \cdots$ scalars/fermions

- Two exact supercharges, $Q_\pm$

c.f. Vafa–Witten twist

$$Q_\pm U_1 = i\psi_{1\pm}U_1, \cdots\cdots$$

$$\Rightarrow \quad Q_+^2 = \delta_\phi, \quad Q_-^2 = \delta_{\bar\phi}, \quad \{Q_+,Q_-\} = \delta_C$$

$$Q_\pm S_{\mathrm{LAT}} = Q_\pm^2(\cdots) = 0.$$

- Continuum limit

$$\lambda_0 = \lambda a^3 \to 0, \qquad \lambda : \text{fixed}$$

SUSY Ward-Takahashi identity

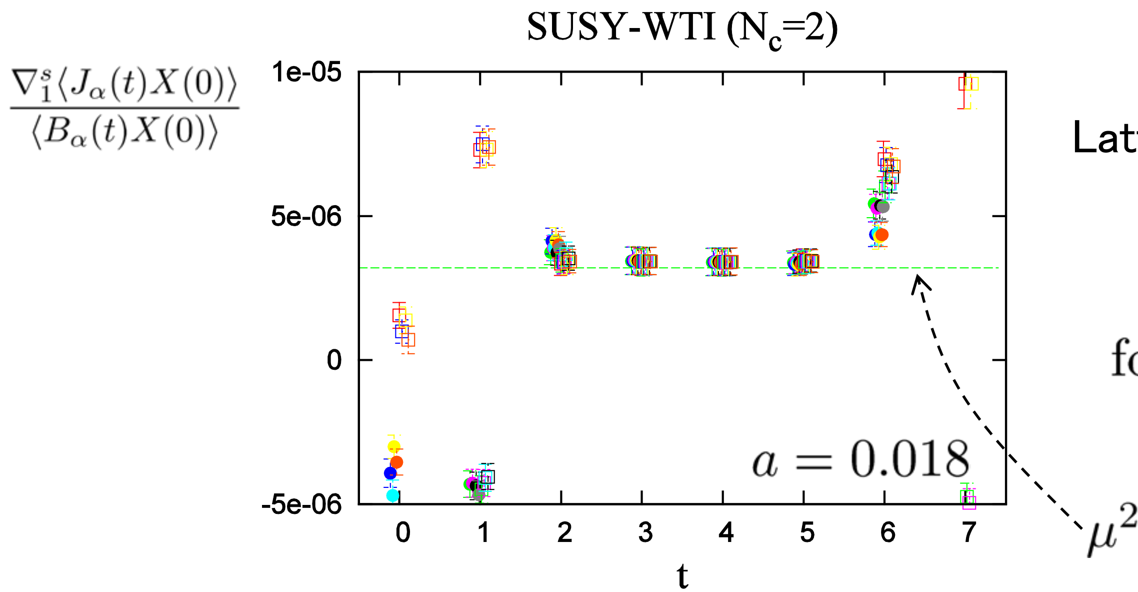
c.f. 2007, Kanamori-Suzuki,
for 2D N=(2,2) SYM

SUSY-WTI with SUSY breaking term (partially conserved susy current)

$$S \to S + \mu^2 \int \mathrm{d}t\ \mathrm{tr}\left[\phi_i^2(t)\right], \quad \text{mass term(SUSY breaking term)}$$

$$\partial_1 \langle J_\alpha(t) X(s)\rangle = \mu^2 \langle B_\alpha(t) X(s)\rangle + \delta(t-s) < \delta X(s) >$$

$$J(t) = \frac{i}{2}\Sigma_{\mu\nu}\Gamma_1 \mathrm{tr}\left(F_{\mu\nu}(t)\psi(t)\right), \qquad B(t) = \mathrm{tr}\left(A_\mu(t)\Gamma_\mu \psi(t)\right)$$



Lattice date shows that

$$\frac{\nabla_1^s \langle J_\alpha(t) X(0)\rangle}{\langle B_\alpha(t) X(0)\rangle} = \mu^2,$$

$$\text{for} \quad t \neq s,$$

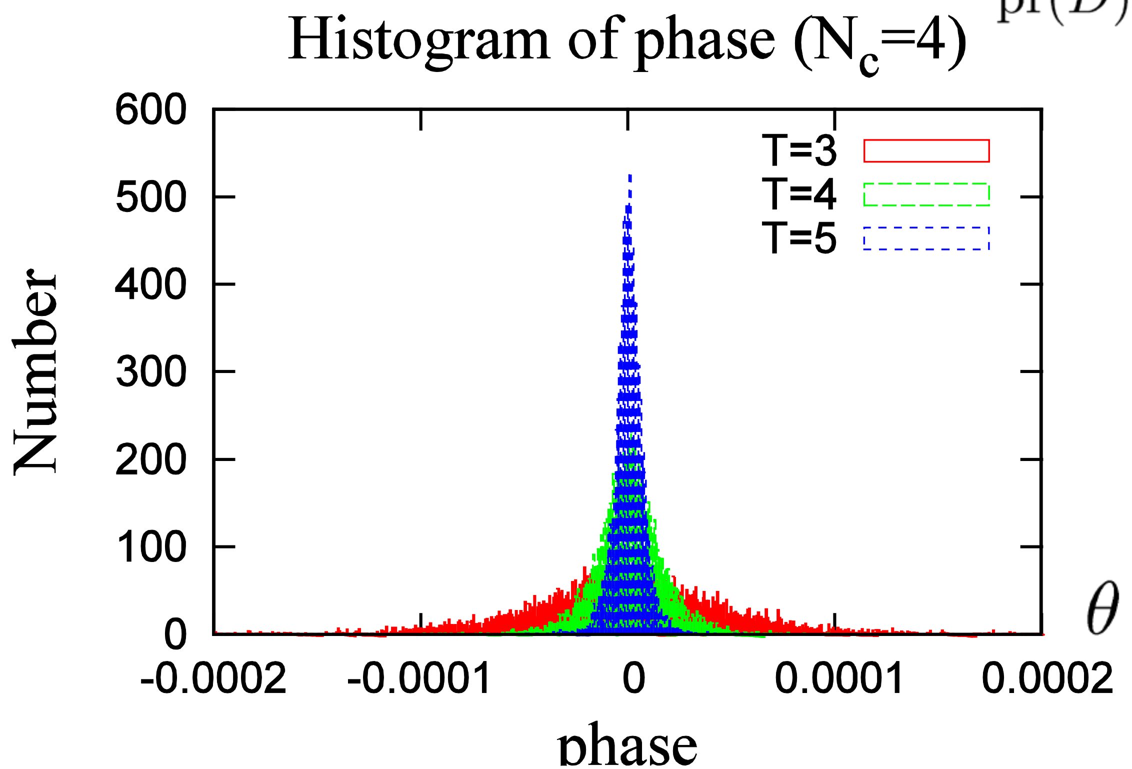
$$\alpha = 1, \cdots, 16.$$

$$\mu^2 \to 0 \Rightarrow \nabla_1 J_{1\alpha}(t) = 0, \quad (\alpha = 1, \cdots, 16).$$

All supersymmetries are restored in the continuum limit.

Complex phase problem

$$\mathrm{pf}(D) = |\mathrm{pf}(D)|\, \mathrm{e}^{i\theta}$$



Phase fluctuation becomes larger as temperature is decreasing.

If we assume that $\quad \mathrm{width} = \alpha T^{-\beta},$

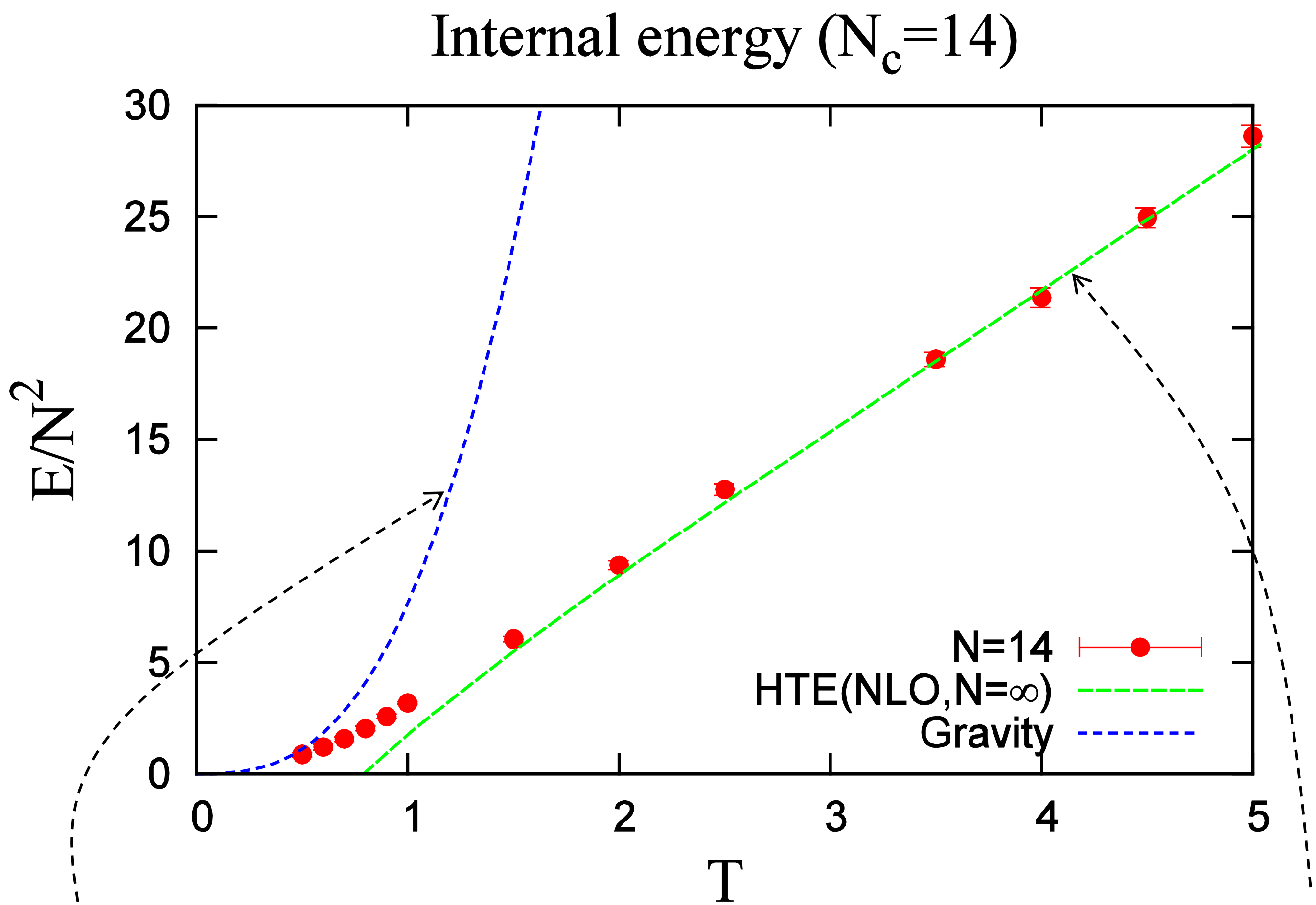
$$\alpha \sim 0.003, \quad \beta \sim 3 \quad (N_c = 2, 3, 4, 6)$$

$\Rightarrow$ phase quench(or phase reweighting) works for $\quad T \gg 0.1.$

$(\mathrm{width} \sim \pi \text{ for } \; T \sim 0.1)$

Internal Energy

$$< E >= -\frac{\partial}{\partial\beta}\log Z$$



Gravity side *(1996 Klebanov –Tseytlin)*

$$\frac{1}{N^2}\frac{E}{\lambda^{\frac{1}{3}}} = c\left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{2.8}, \qquad c = 7.41..$$

NLO HTE in Nc=∞

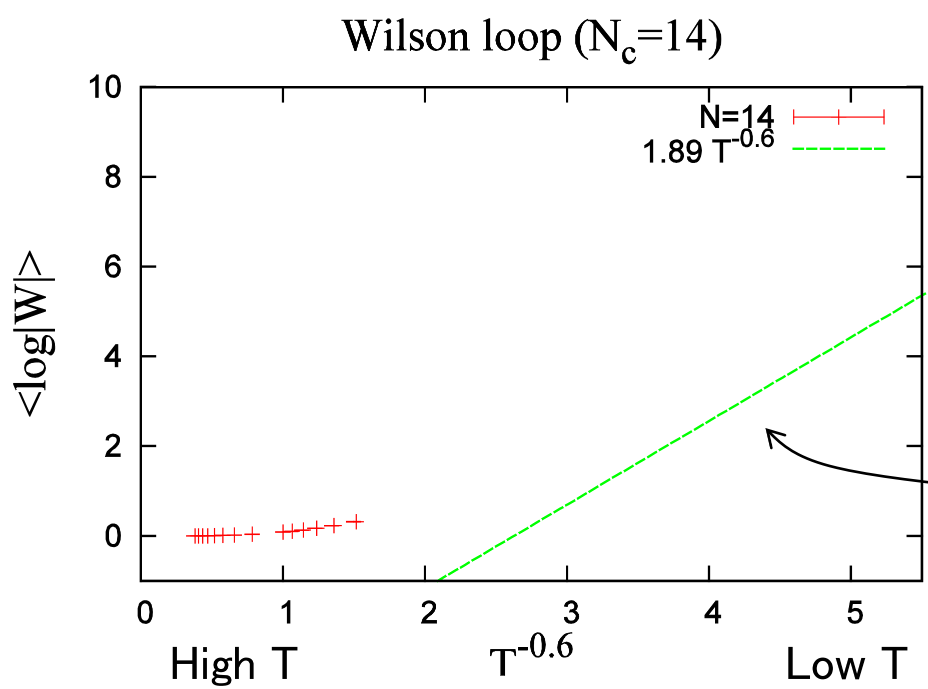
Kawahara–Nishimura –Takeuchi

Wilson loop

Continuum: $$W = \frac{1}{N}\mathrm{tr}\,\mathcal{P}\left\{ e^{i\int dt A_1(t) + \phi_i(t)\theta_i} \right\}, \quad \sum_{i=1}^{9} \theta_i^2 = 1$$

Lattice: $$W = \underbrace{\frac{1}{18}\sum_{k=2}^{10}\sum_{s=\pm}\frac{1}{N}}_{}\mathrm{tr}\left[\prod_t V_k^s(t) U_1(t)\right], \qquad V_k^{\pm}(t) = e^{\pm A_k(t)}$$

Averaging over (internal) directions



Gravity side

Schwarzschild radius

$$\ln W = \frac{\beta R_{sh}}{2\pi\alpha'} = \kappa \left(\frac{T}{\lambda^{1/3}}\right)^{-3/5},$$

$$\kappa = \frac{1}{2\pi}\left(\frac{16\sqrt{15}\pi^{7/2}}{7}\right) = 1.89...$$

Nishimura et. al.

$$\ln W = 1.89 \left(\frac{T}{\lambda^{1/3}}\right)^{-3/5} - 4.58$$

"constant shift"

5. Summary

- Quantitative verification of gauge/gravity duality

=> target : 1D SYM with 16 supercharges.

$\simeq$ N D0-branes in 10D type IIA superstring

- Internal energy

Lattice data is close to the line expected from gravity side.

gravity side: $p = 2.8. \quad c = 7.41..$

$$\frac{1}{N^2}\frac{E}{\lambda^{\frac{1}{3}}} = c\left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^p,$$

gauge side (lattice calculation)

$$p = ?(?)(?), \quad c = ?(?)(?)$$

systematic errors

statistical error

- Wilson loop → Schwarzschild radius

Low temperature simulations are needed.

Thank you.