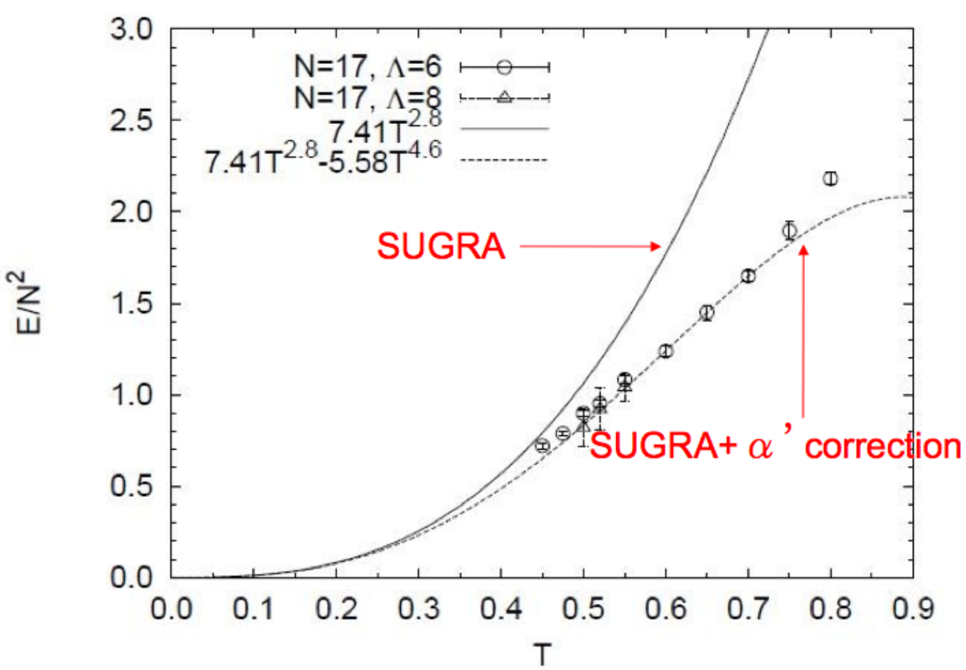
Monte Carlo approach to string/M theory

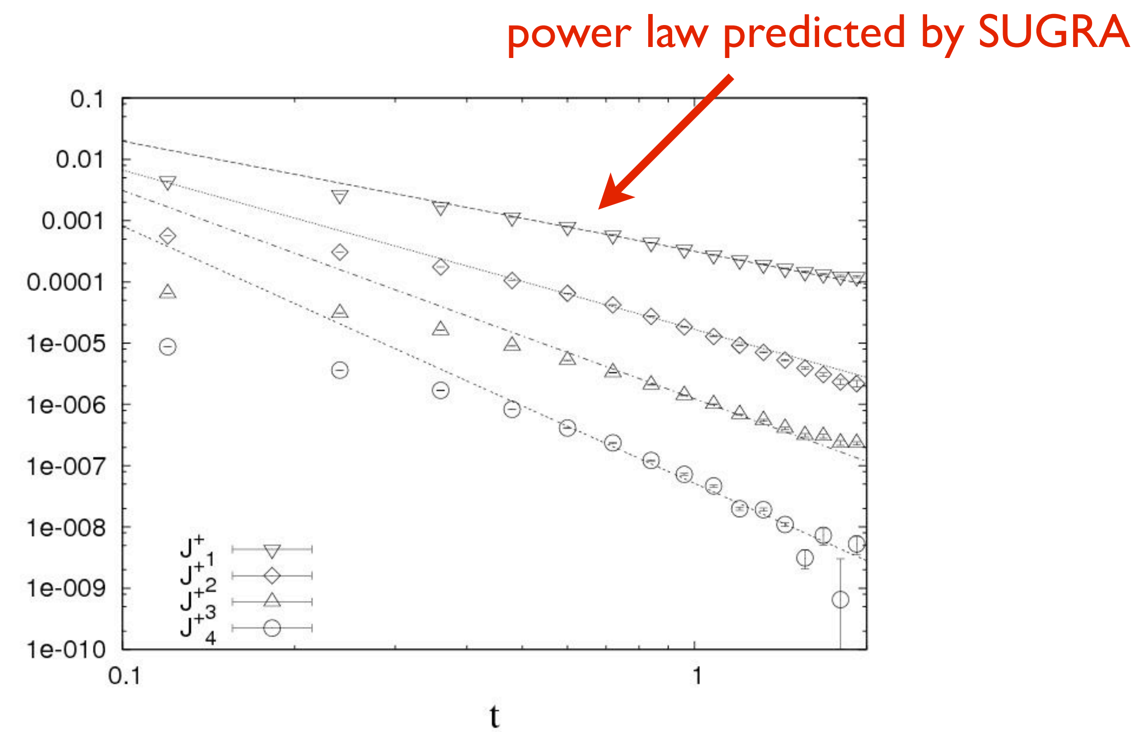
Masanori Hanada
花田政範

KEK Theory Center

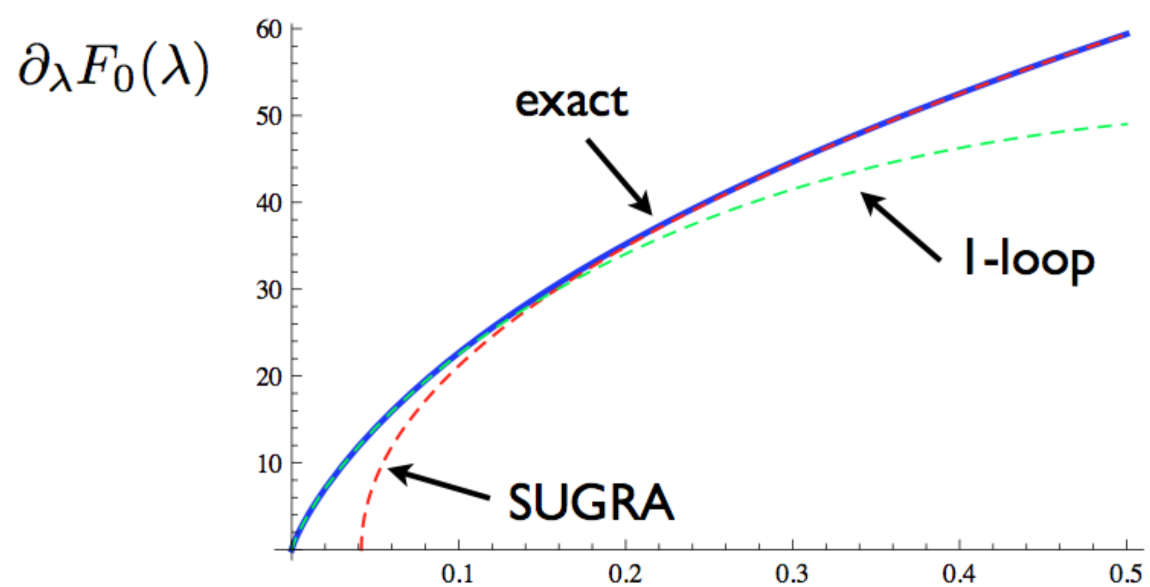
Goal (1) : understand these figures.



energy density of SUSY quantum mechanics



two-point function



Free energy of ABJM model λ

Goal (2)

Convince you that lattice theorists can give very important contributions to string/M-theory, which usual string theorists can never give.

quantum nature of the black hole,
Hawking evapolation,
birth of the universe, inflation,
multiverse,...

Apologies

Because we have only 30 minutes...

- I skip all the technical details, e.g. how to put supersymmetry on computer. I explain only basic ideas about physics.
- I can explain only a few topics.
- My explanation is sometimes too simplified and not very precise.

Plan

(1) Gauge/Gravity duality (AdS/CFT) and Super Yang-Mills

(2) ABJM Theory and M-theory (‘membrane mini revolution’)

(M.Honda’s talk)

(3) Expanding universe out of a matrix model?

(S.-W. Kim’s talk & K.N.Anagnostopoulos’s talk)

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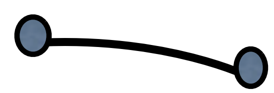
What is string theory?

(Only PERTURBATIVE definition exists)

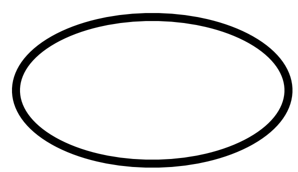
- Point particles are promoted to (1+1)-dimensional 'string'.

- Open string $\rightarrow$ gauge fields and infinitely many massive fields



gauge degrees of freedom
live on the edges

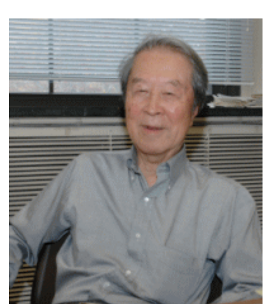
- Closed string $\rightarrow$ graviton, tensors and massive fields

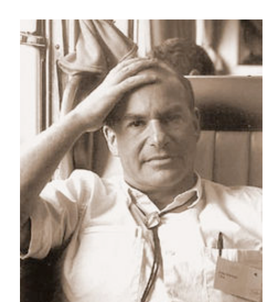


no boundary

- The scattering amplitudes can be calculated.

- Spacetime dimension is 1+9=10.


Supergravity as a low-energy effective theory

- From the scattering amplitudes, one can determine the low-energy effective action in terms of the massless fields, which reproduce the amplitudes.

Graviton $G_{\mu\nu}$, dilaton φ,
NS B-field $B_{\mu\nu}$, R-R fields $C_{\mu\nu\rho\ldots}$

IIA/IIB superstring ⇒ IIA/IIB supergravity

- There are supersymmetric black p-brane solutions coupling to p-form tensor. (Horowitz-Strominger 1991)

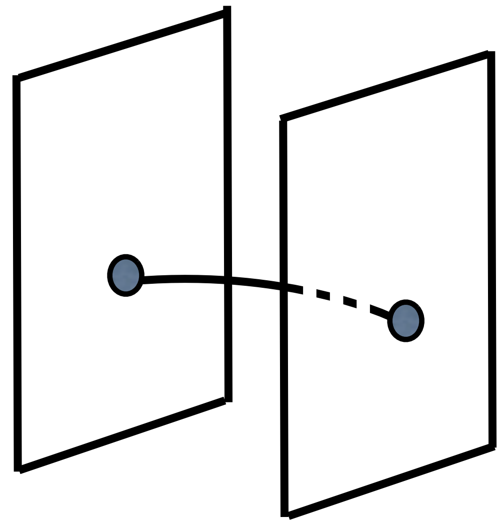
(p+1)-d analogues of the black hole

(p=even for IIA, p=odd for IIB)

Can we understand black p-brane from the perturbative string picture?

black brane = D-brane

- Dp-brane (Dirichlet p-brane) is a (p+1)-d object on which open string can be attached.



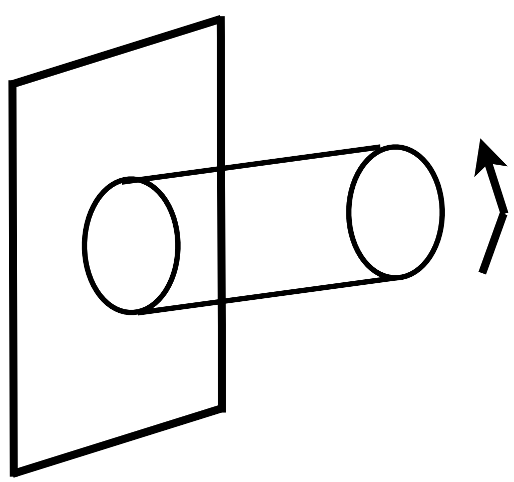
Dirichlet boundary condition along the transverse direction

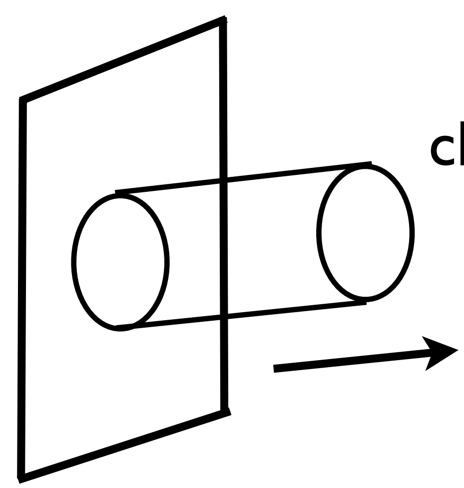
Polchinski

- It has the same charge as the black p-brane.

D-brane

open string picture



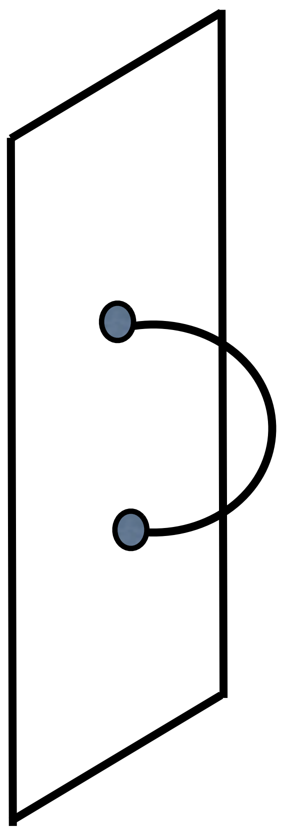


closed string picture

black brane

bunch of many D-branes
= black brane
(large-N → heavy and big → classical gravity)

SYM from D-brane (1)



x_μ (μ=0,1,...,p) D-brane world-volume

y_i (i=p+1,...,9)

→ (p+1)-d SYM

Massless d.o.f.

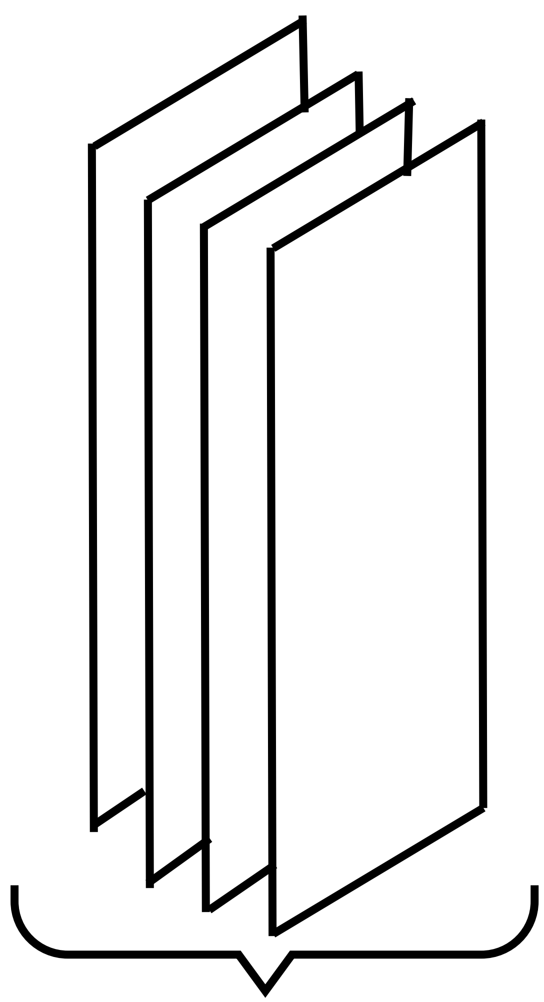
$A_\mu(x_\mu)$: Gauge field

$\Phi(x_\mu)$: Scalar field

(coordinate of the brane)

$\psi(x_\mu)$: fermion

SYM from D-brane (2)



N Dp-branes

A_μ, Φ and ψ become
$N \times N$ matrices

(i,j)-component
= string connecting
i-th and j-th D-branes

→ (p+1)-d U(N) Super Yang-Mills

(more generally, the Dirac-Born-Infeld action)

Maximally supersymmetric Yang-Mills

(Brink-Schwarz-Scherk 1977)

10d $\mathcal{N}$=1 U(N) SYM

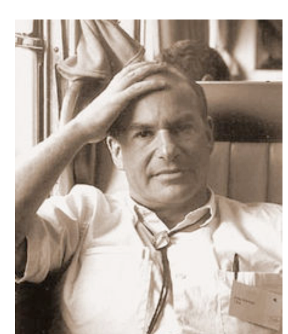
$$S_{10d} = \frac{1}{g_{YM}^2} \int d^{10}x Tr\left(\frac{1}{4}F_{MN}^2 + \frac{1}{2}\bar{\psi}\gamma^M D_M \psi\right)$$

$A_1,\ldots,A_{10}$
$\psi_1,\ldots,\psi_{16}$

dimensional reduction

$$S_{(p+1)d} = \frac{1}{g_{YM}^2} \int d^{p+1}x Tr\Big(\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(D_\mu X_i)^2 + \frac{1}{4}[X_i, X_j]^2$$
$$+\frac{1}{2}\bar{\psi}\gamma^\mu D_\mu \psi - \frac{i}{2}\bar{\psi}\gamma^i [X_i, \psi]\Big)$$

Dp-brane worldvolume


SUSY transf.

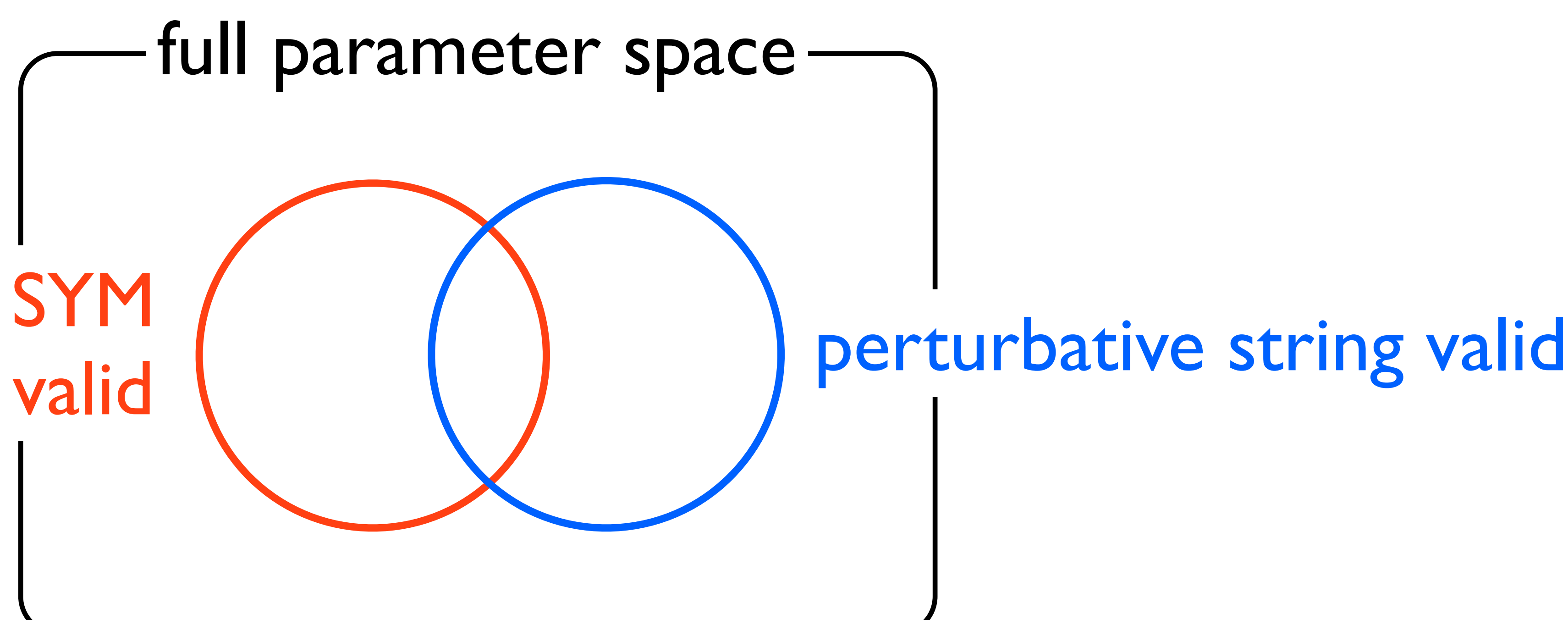
$$\delta A_M \sim \bar{\epsilon}\Gamma_M \psi$$

$$\delta\psi \sim F^{MN}\Gamma_{MN}\epsilon$$

Gauge/gravity duality *conjecture*
(Maldacena 1997)

- In a special limit, both SYM and weakly-coupled string pictures become valid.
- But they are two different descriptions of the same D-brane system. So...

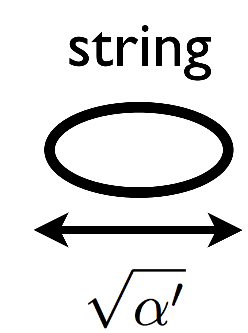
SYM = superstring



The dictionary

Gravity/string	SYM
black hole mass	Energy density
mass of field excitation	scaling dimension

SYM	STRING
large-N, strong coupling	SUGRA
large-N, finite coupling	tree-level string (SUGRA+α')
finite-N, finite coupling	Quantum string (g_{string}>0)



SYM$_{\text{difficult}}$

STRING

large-N,
strong coupling

SUGRA
easier

large-N,
finite coupling

tree-level string
(SUGRA+α')
more difficult

finite-N,
finite coupling

Quantum string
(g_{string}>0)
very difficult

SYM$_{\text{difficult}}$	STRING
large-N, strong coupling	SUGRA easier
large-N, finite coupling	tree-level string (SUGRA+α’) more difficult
finite-N, finite coupling	Quantum string ($g_{string}>0$) very difficult

SYM$_{\text{difficult}}$ STRING

large-N, strong coupling	$\longleftarrow$	SUGRA easier
large-N, finite coupling	$\longrightarrow$	tree-level string (SUGRA+α') more difficult
finite-N, finite coupling	$\longrightarrow$	Quantum string ($g_{\text{string}}>0$) very difficult

The opposite direction of the dictionary can be useful, if we use Monte Carlo !

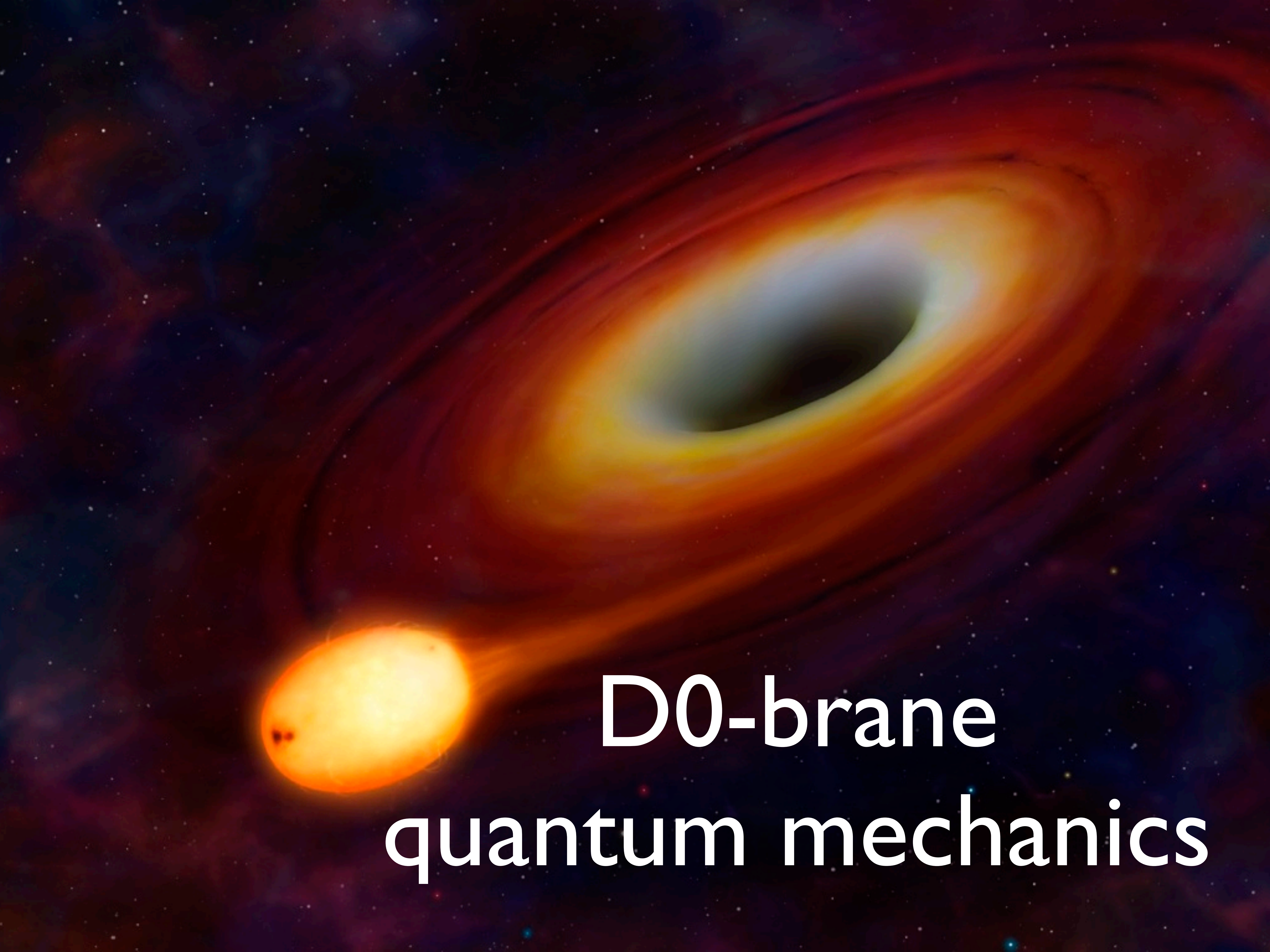
Very important remark

- From the string theory point of view, *SYM theories in less than four spacetime dimensions are as interesting as four dimensional theories!*

(0+1)-d SYM $\Leftrightarrow$ Black hole (0-brane)

(1+1)-d SYM $\Leftrightarrow$ Black 1-brane, black string

(3+1)-d SYM $\Leftrightarrow$ Black 3-brane ($AdS_5 \times S^5$)


D0-brane
quantum mechanics

$$S=\frac{N}{\lambda}\int_{0}^{1/T} dt\ Tr\Big\{\frac{1}{2}(D_tX_i)^2-\frac{1}{4}[X_i,X_j]^2 +\frac{1}{2}\bar{\psi}D_t\psi-\frac{1}{2}\bar{\psi}\gamma^i[X_i,\psi]\Big\}$$

- Dimensional reduction of 4d N=4 (or 10d N=1)

- Strong coupling = low temperature
  and/or long distance

$\boxed{\text{effective temperature (dimensionless) : } \lambda^{-1/3}T}$

- No UV divergence ⇒ no need for lattice

Prediction from gravity side

$$\boxed{\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \qquad (\lambda = 1)}$$

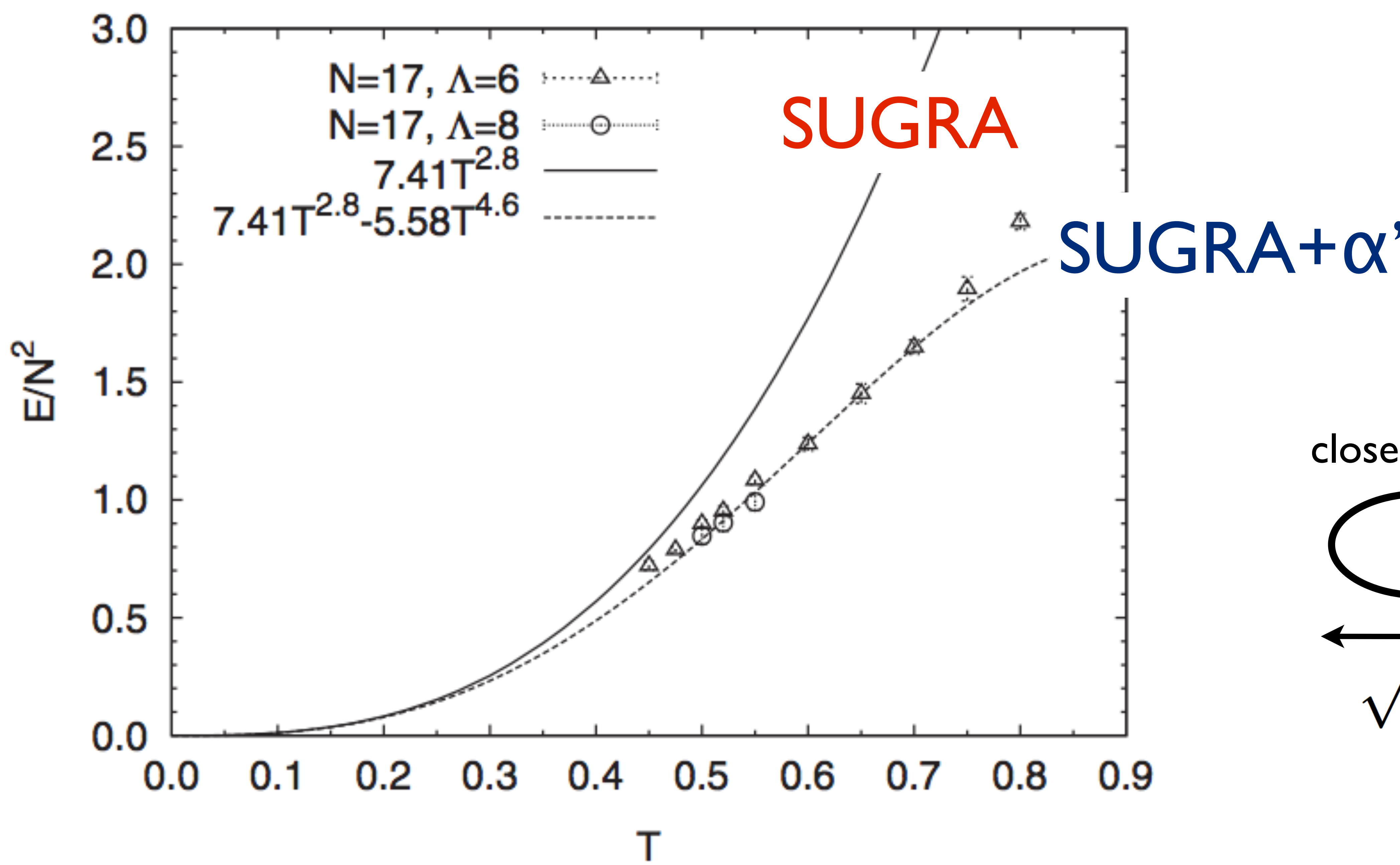
at large-N & low temperature (=strong coupling)

$$\lambda^{-1/3} T \ll 1.$$

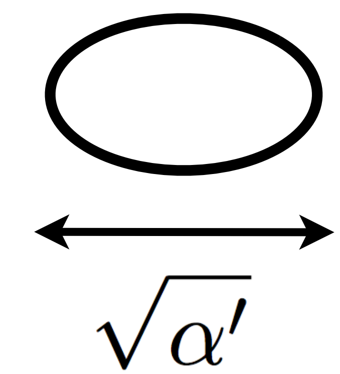
Temperature in YM = Hawking temperature

SYM should give a microscopic understanding of the black hole thermodynamics

black hole mass vs energy density

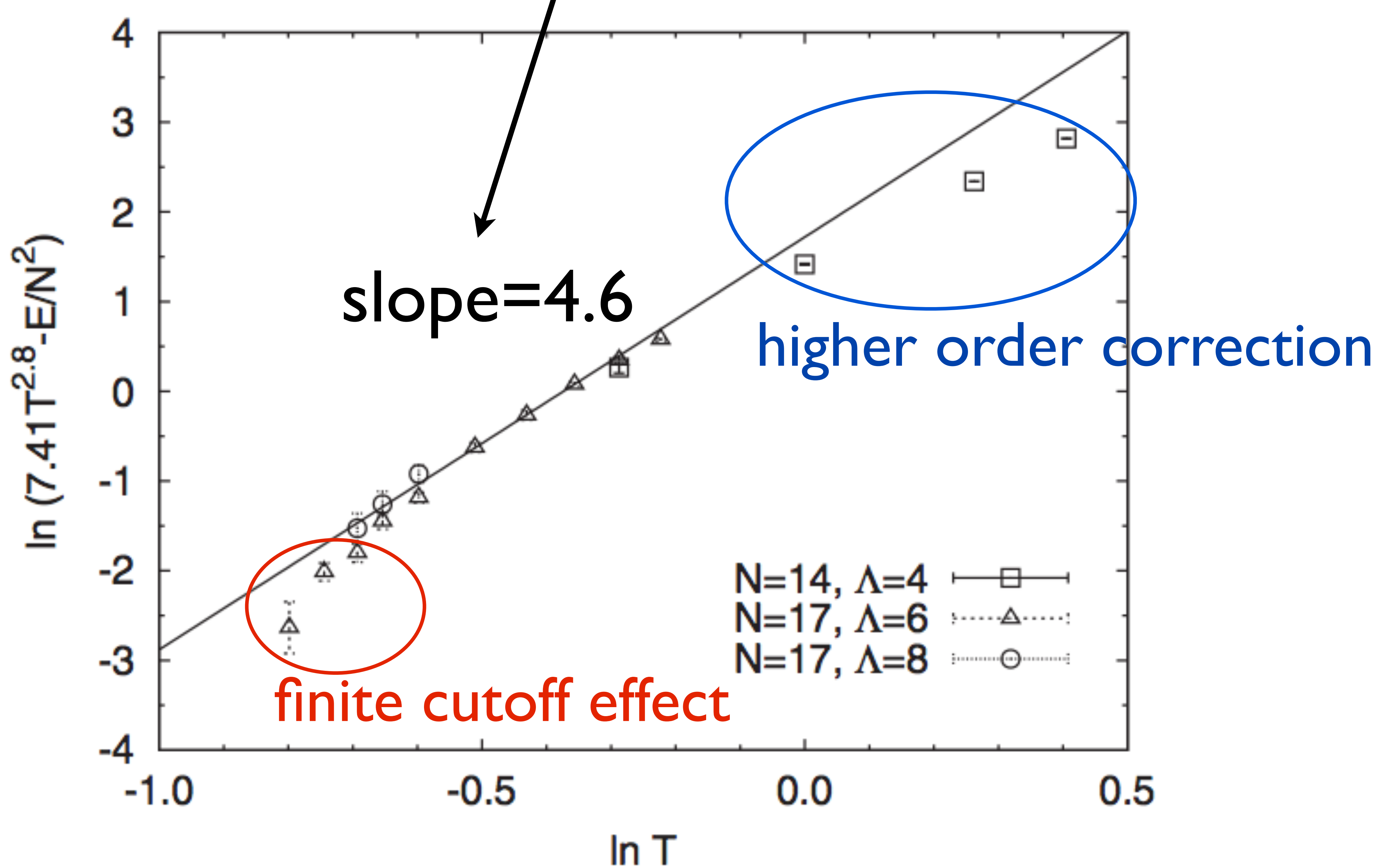


closed string



Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,
M.H.-Hyakutake-Nishimura-Takeuchi 2008

Perfect agreement with string theory!



M.H.-Hyakutake-Nishimura-Takeuchi 2008

Correlation functions
(GKPW relation)



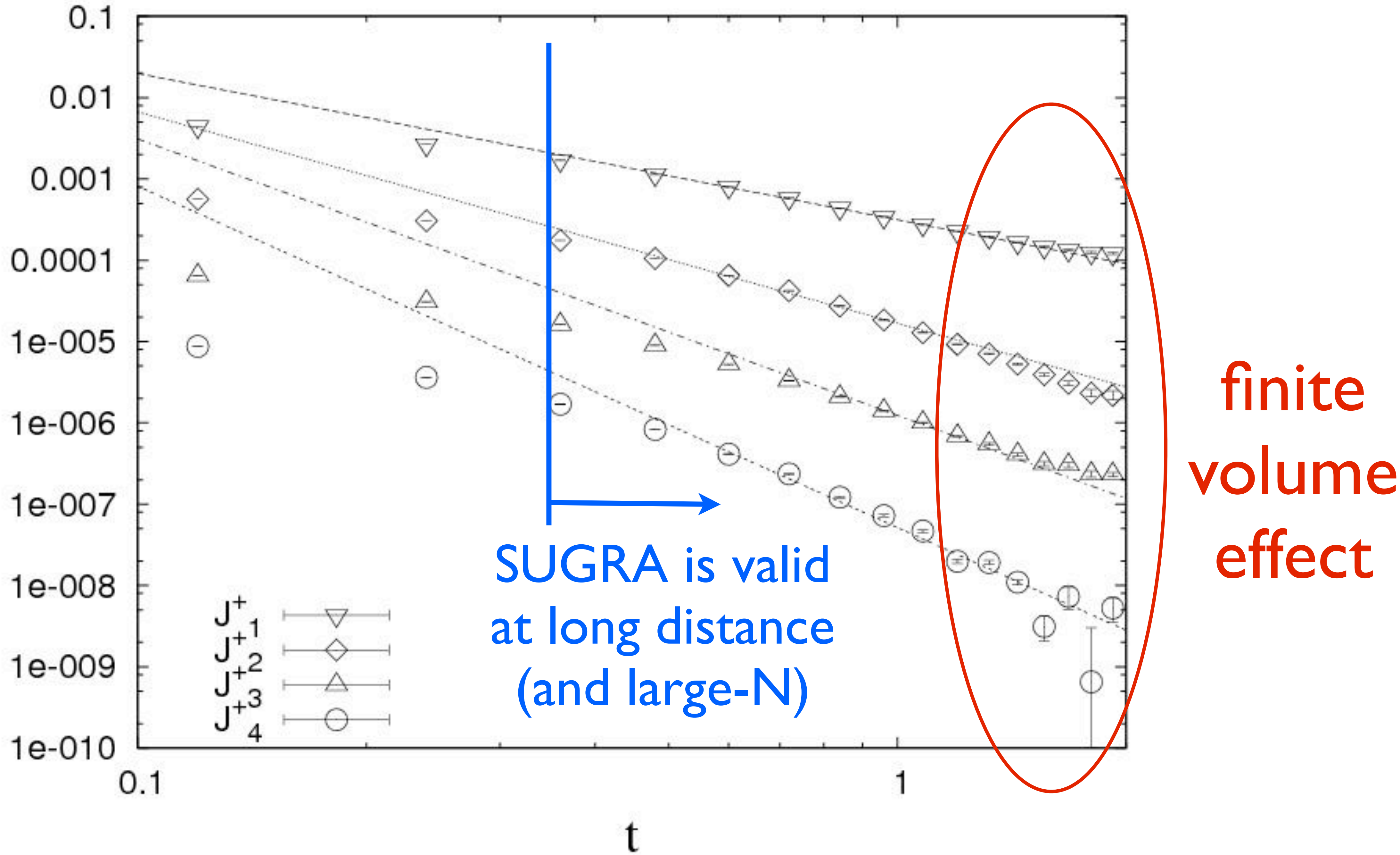

G K P W

- Recipe to calculate the correlation function at large-N and strong coupling from supergravity
  (Gubser-Klebanov-Polyakov 1998, Witten1998)
- Similar relation holds also in D0-brane theory.

(Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

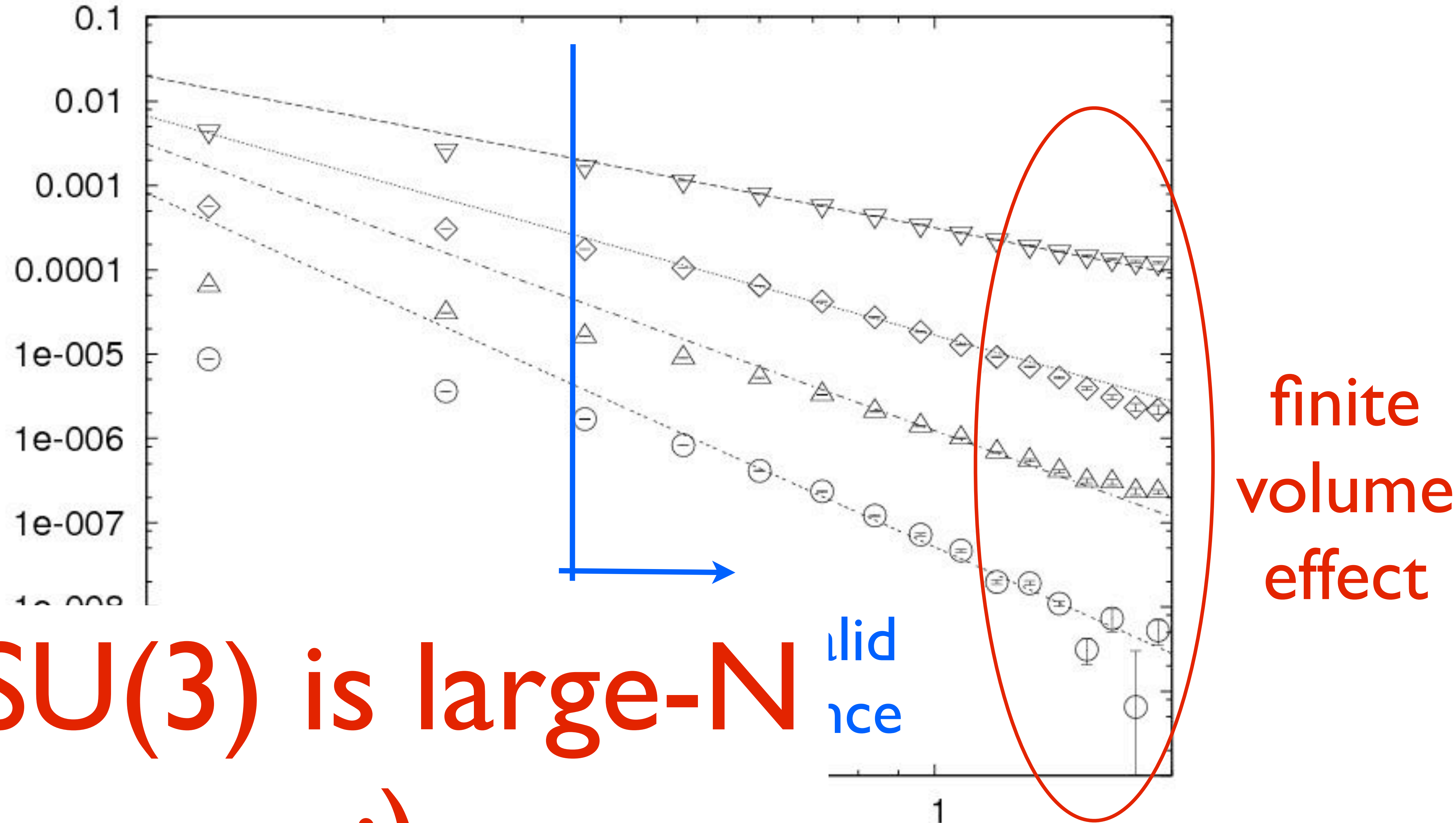
calculable
via SUGRA



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij} X_{i_1} \cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya 2009,2011)



SU(3) is large-N :)

two-point functions, SU(3), pbc

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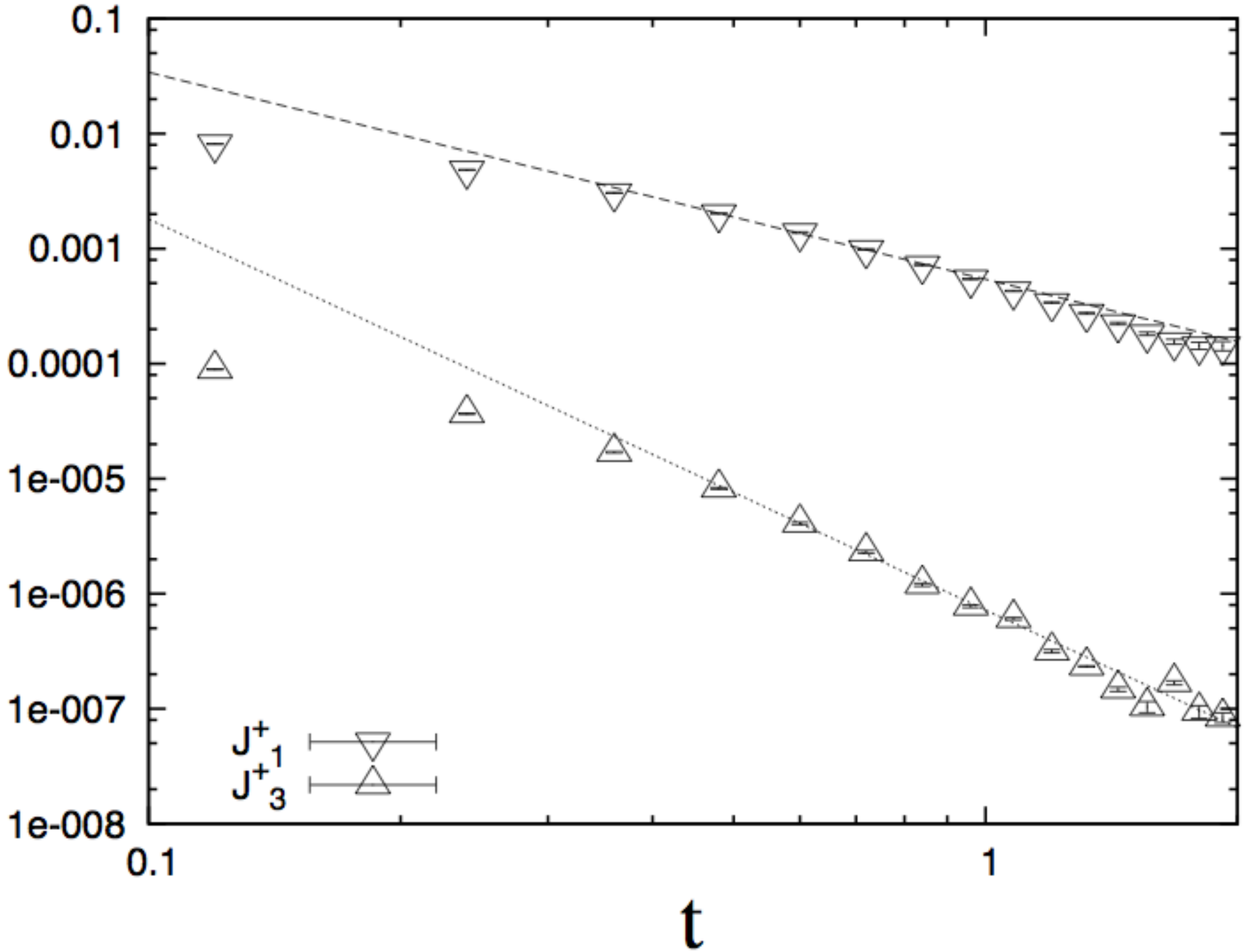
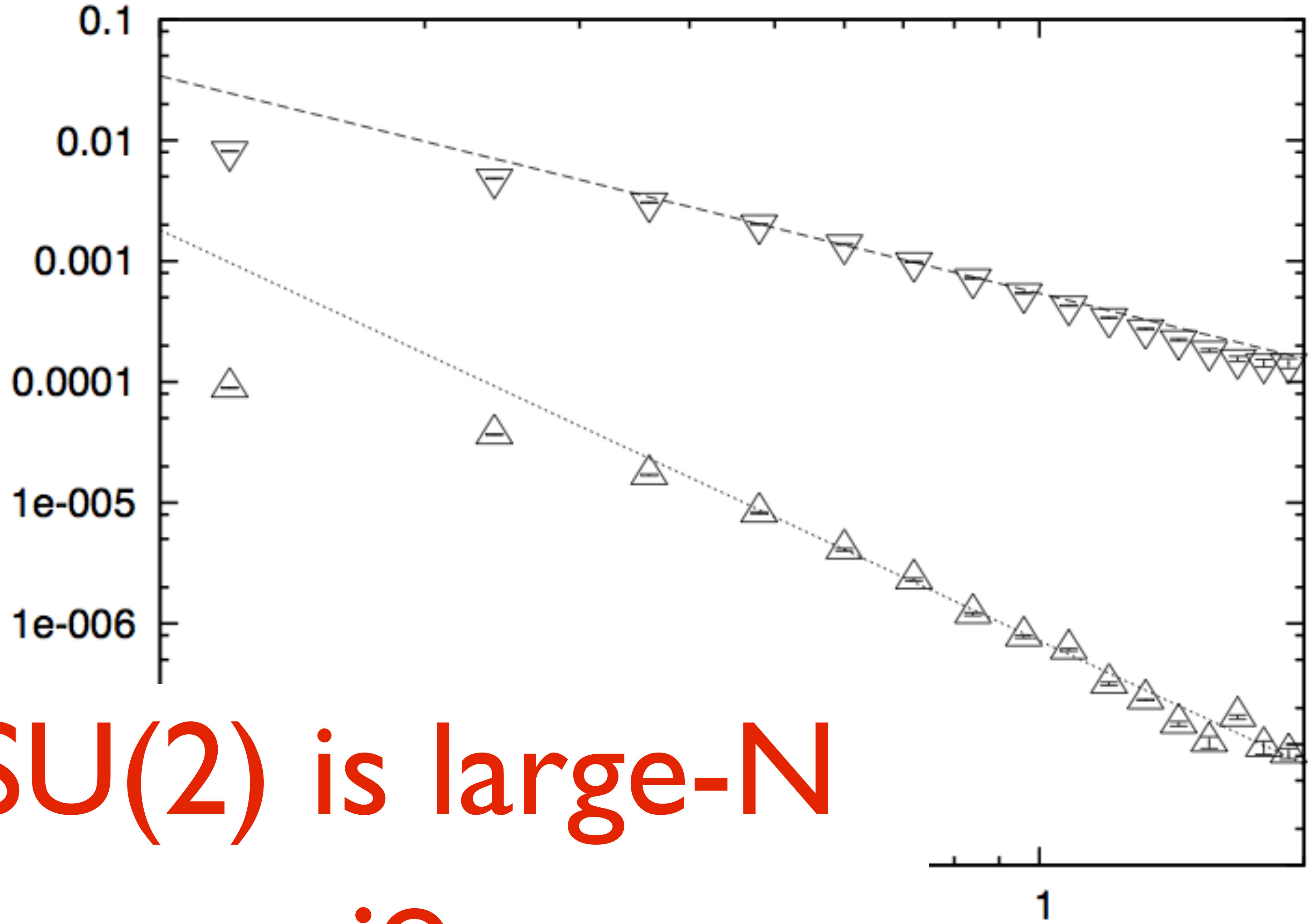


two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya 2011)



SU(2) is large-N

:o

two-point functions, SU(2), pbc

(M.H.-Nishimura-Sekino-Yoneya 2011)

(1+1)-d SYM and black hole/black string transition

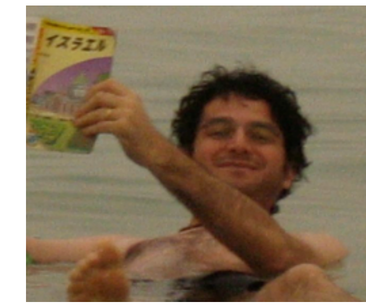
Susskind, Barbon-Kogan-Rabinovici,
Li-Martinec-Sahakian,
Aharony-Marsano-Minwalla-Wiseman,…

SYM simulation : Catterall-Wiseman, 2010

Several lattice theories exist (from around 2002-2005)

- Cohen, Kaplan, Katz, Unsal
- Sugino
- Catterall
- D'Adda, Kanamori, Kawamoto, Nagata

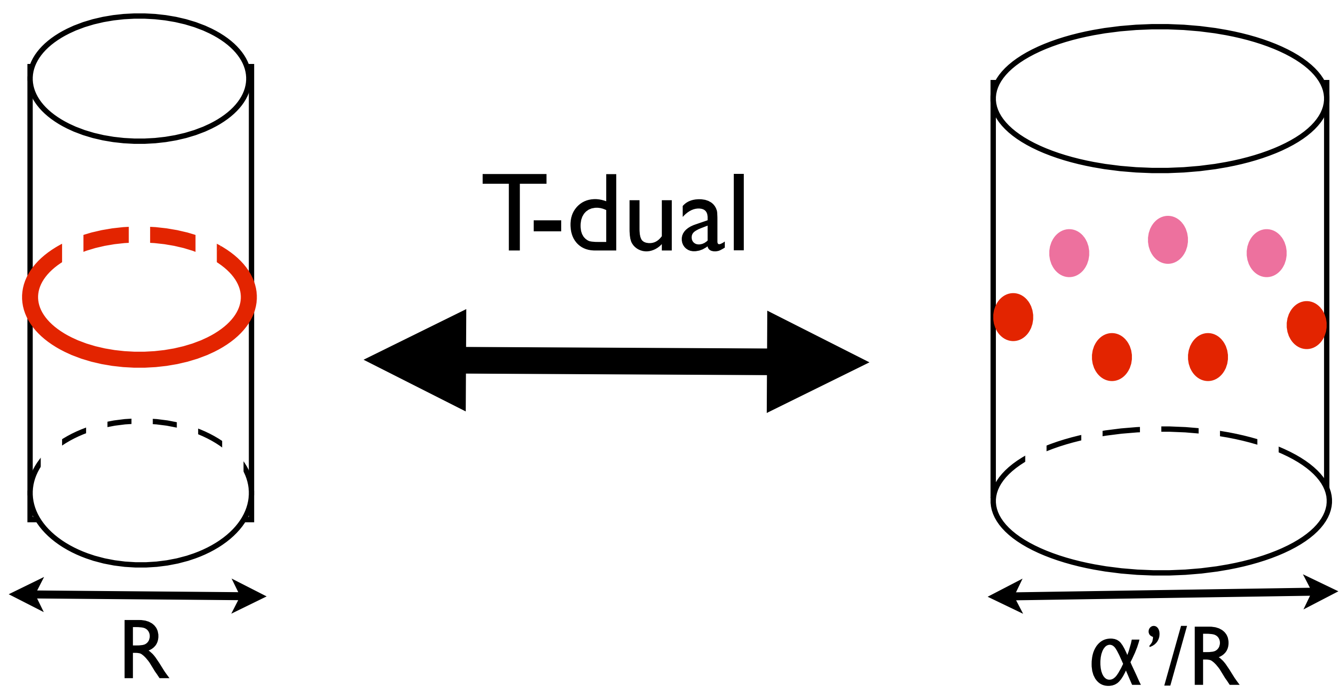




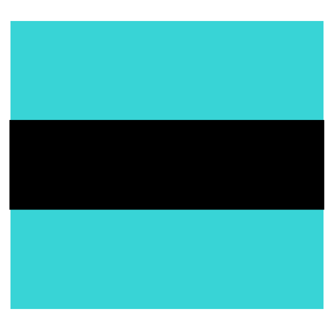
- No fine tuning to all order in perturbation.
  (Kaplan-Katz-Unsal 2002)

- Numerical simulation supprt the absence of the fine tuning at nonperturbative level.
  (Suzuki 2007, Kanamori-Suziki 2008, M.H.-Kanamori 2009, 2010)

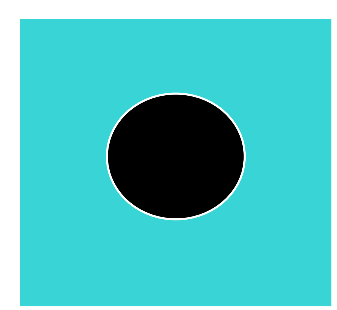

- Consider 2d U(N) SYM on a spatial circle. It describes N D1-branes in $R^{1,8}\times S^1$, winding on S^1.
- T-dual picture : N D0-branes in $R^{1,8}\times S^1$.



- Wilson line phase = position of D0

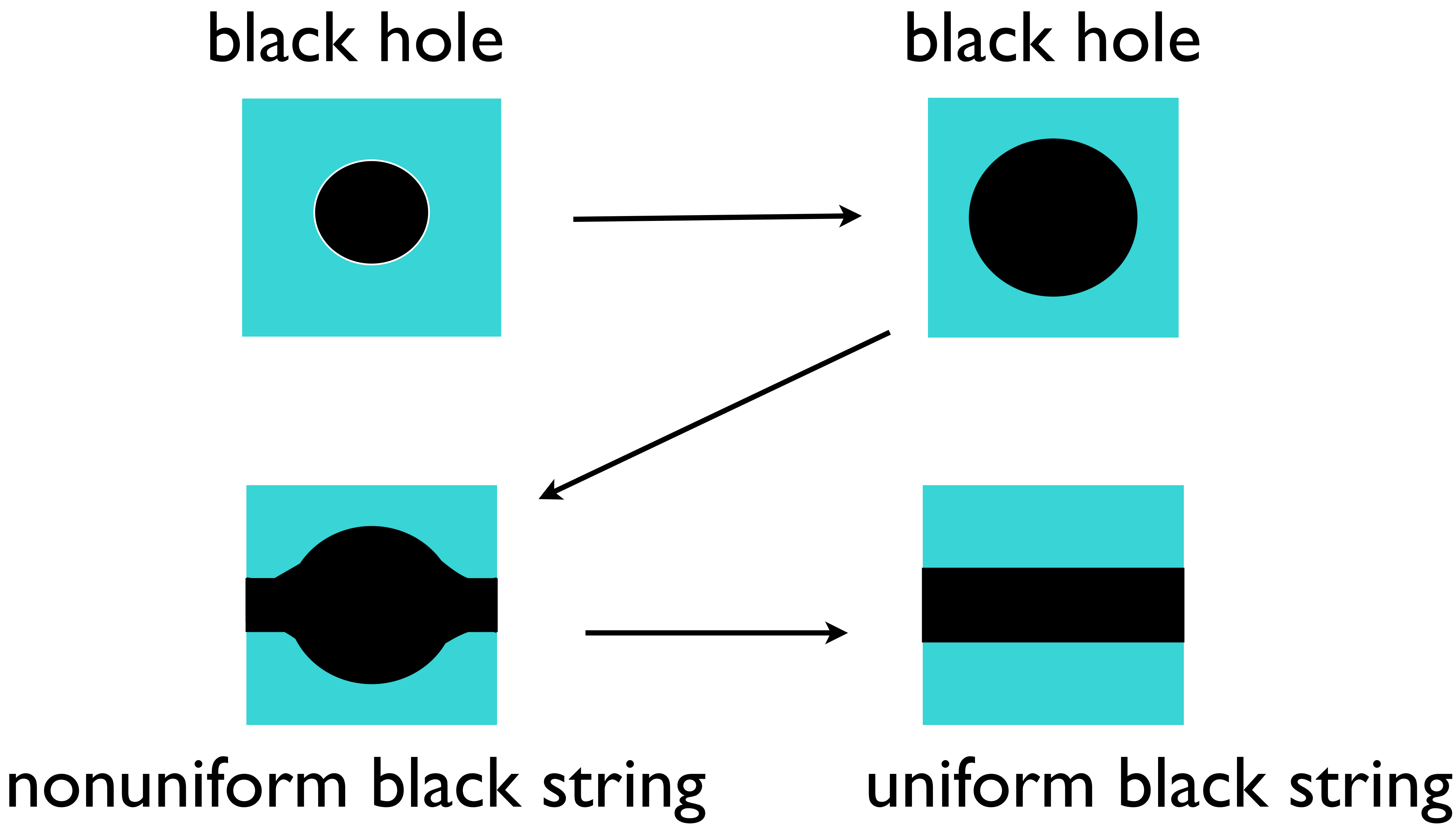


uniform distribution
= 'black string'



localized distribution
= 'black hole'

If the black hole becomes larger, then...



Counterpart in SYM
= center symmetry breakdown

- Wilson line phase = position of D0

$$W = diag(e^{i\theta_1}, \cdots, e^{i\theta_N})$$

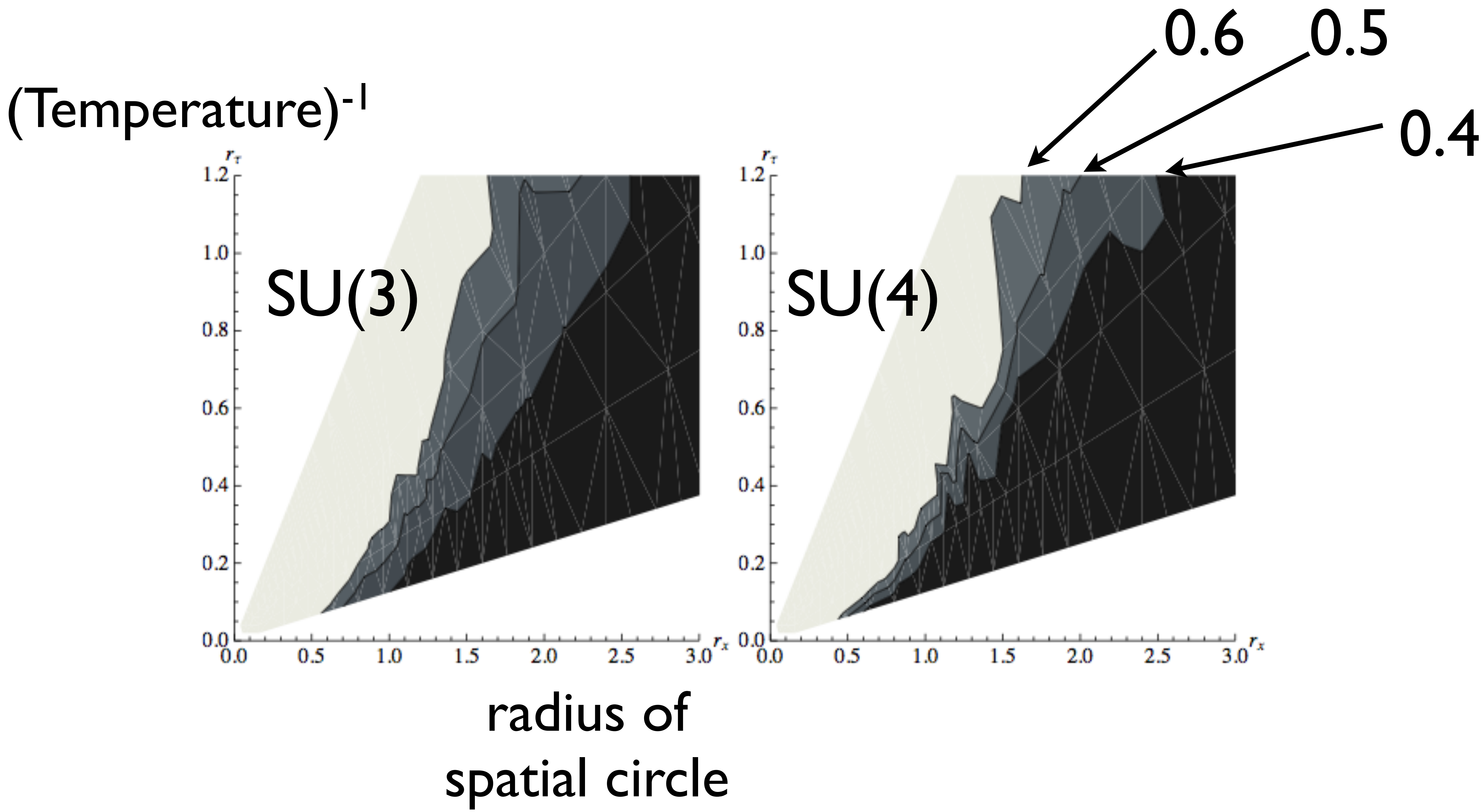
- U(1) center symmetry

$$\theta_i \to \theta_i + const.$$

Uniform = center unbroken $\left\langle \frac{1}{N} TrW \right\rangle = 0$

Non-uniform = center broken $\left\langle \frac{1}{N} TrW \right\rangle \neq 0$

Value of spatial Wilson loop



(Catterall-Wiseman, 2010)

Plan

(1) Gauge/Gravity duality (AdS/CFT) and Super Yang-Mills

(2) ABJM Theory and M-theory (‘membrane mini revolution’)

(M.Honda’s talk)

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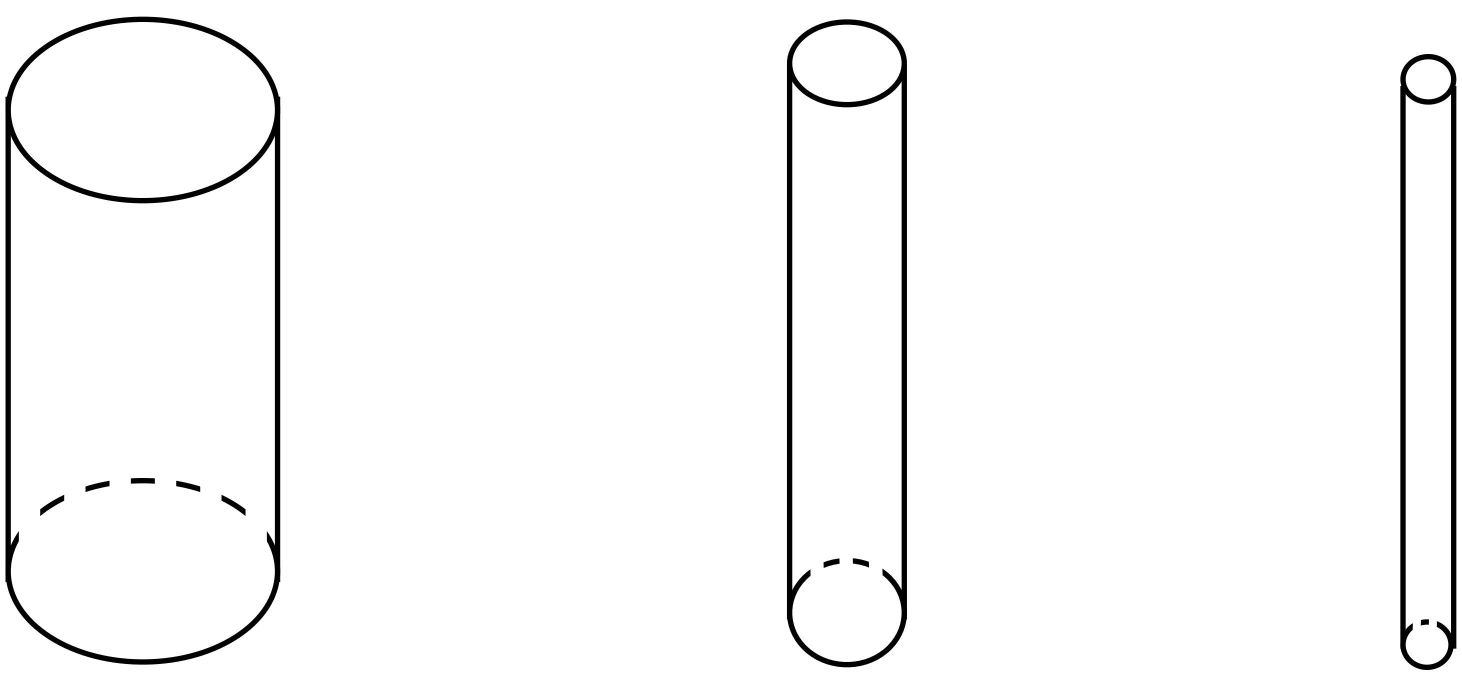
(S.-W. Kim’s talk & K.N.Anagnostopoulos’s talk)

M-theory (conjecture!)

- Strong coupling limit of type IIA superstring.
- Theory of membrane in 11d spacetime.
- string = membrane winding on 11-th dimension.
- Low-energy effective theory is 11d supergravity.



AdS/CFT correspondence can hold also in **M-theory**.

D3-brane in IIB string → $AdS_5 \times S^5$

M5-brane in M-theory → $AdS_7 \times S^4$

M2-brane in M-theory → $AdS_4 \times S^7$

$$\begin{tikzcd} \text{D2-brane} \arrow[r, "\text{RG flow}"] \arrow[d, leftrightarrow, "\text{dual}"'] & \text{M2-brane} \arrow[d, leftrightarrow, "\text{dual}"'] \\ \text{3d maximal SYM} \arrow[r] & \text{3d CFT} \end{tikzcd}$$

what is it?

CFT, maximal SUSY, correct moduli space, ...

ABJM theory

(Aharony-Bergman-Jafferis-Maldacena, 2008)

A

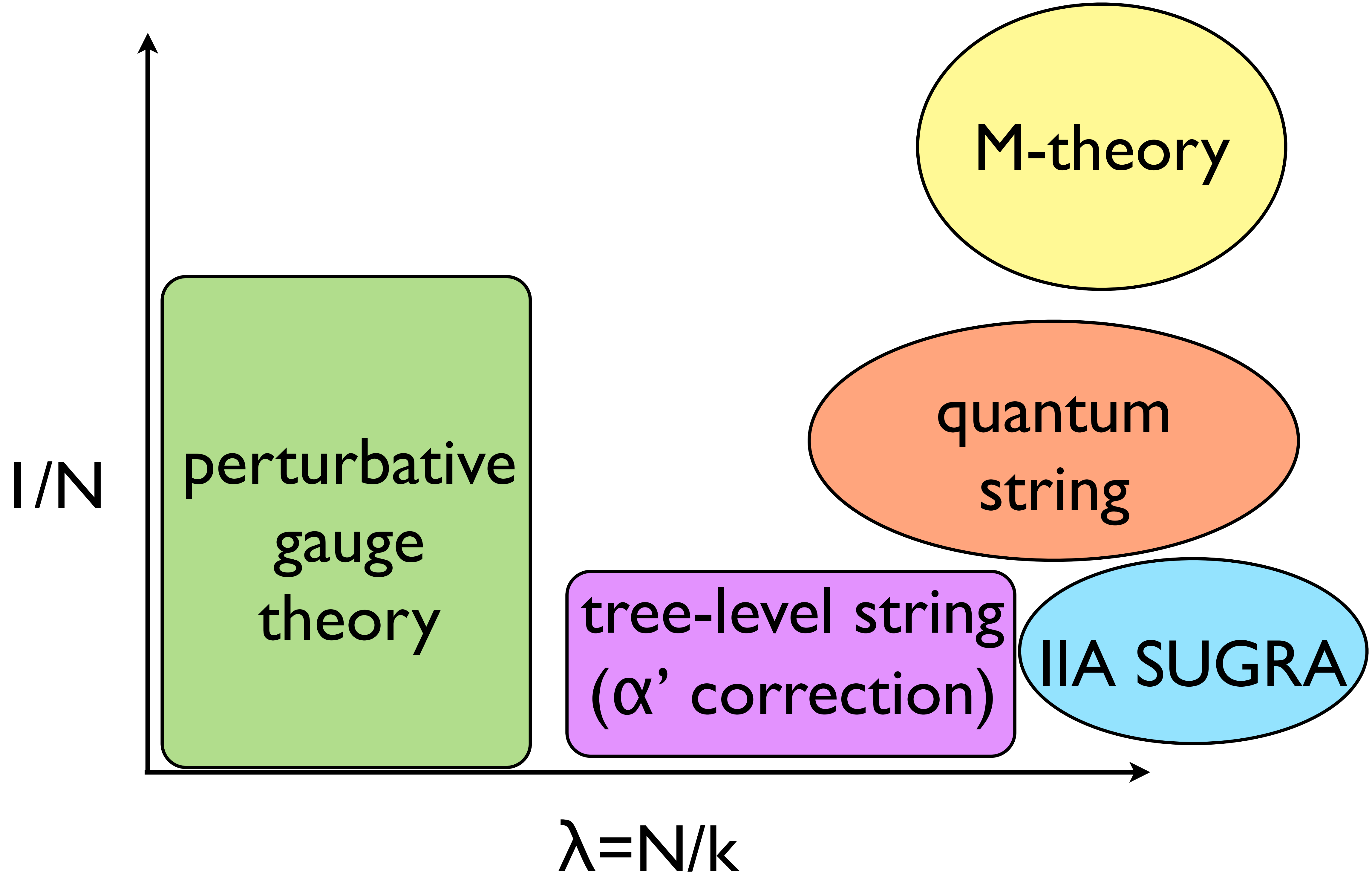
B

J

M

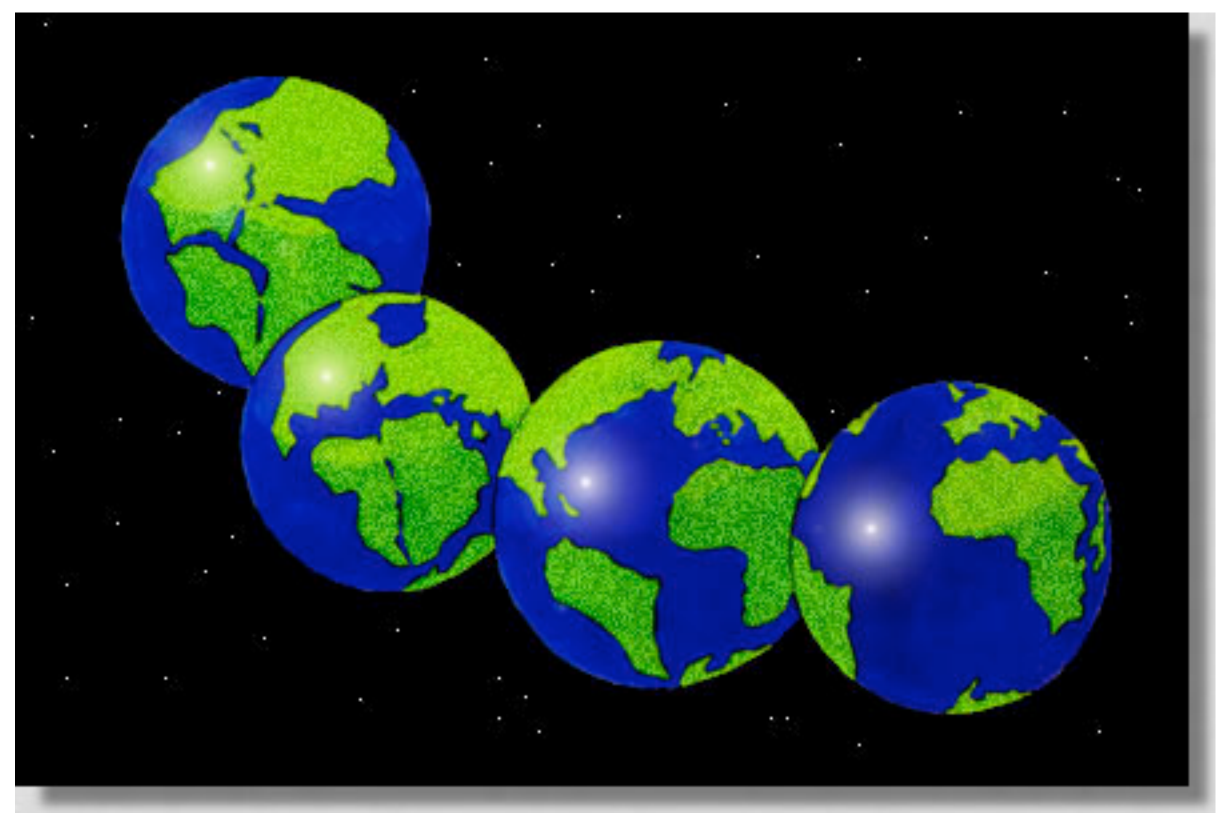
$$
\begin{aligned}
k\,Tr\Bigg\{ & \frac{\epsilon^{\mu\nu\rho}}{2}\left(-A_\mu\partial_\nu A_\rho - \frac{2}{3}A_\mu A_\nu A_\rho + \tilde{A}_\mu\partial_\nu\tilde{A}_\rho + \frac{2}{3}\tilde{A}_\mu\tilde{A}_\nu\tilde{A}_\rho\right) \\
& + \left(-D_\mu\bar{\Phi}^\alpha D^\mu\Phi_\alpha + i\bar{\Psi}^\alpha \not{D}\Psi_\alpha\right) - i\epsilon^{\alpha\beta\gamma\delta}\Phi_\alpha\bar{\Psi}_\beta\Phi_\gamma\bar{\Psi}_\delta + i\epsilon_{\alpha\beta\gamma\delta}\bar{\Phi}^\alpha\Psi^\beta\bar{\Phi}^\gamma\Psi_\delta \\
& + i\left(-\bar{\Psi}_\beta\Phi_\alpha\bar{\Phi}^\alpha\Psi^\beta + \Psi_\beta\bar{\Phi}_\alpha\Phi^\alpha\bar{\Psi}^\beta + 2\bar{\Psi}_\alpha\Phi_\beta\bar{\Phi}^\alpha\Psi^\beta - 2\Psi^\beta\bar{\Phi}^\alpha\Phi_\beta\bar{\Psi}_\alpha\right) \\
& + \frac{1}{3}\left(\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\bar{\Phi}^\gamma\Phi_\gamma\bar{\Phi}^\alpha + \Phi_\alpha\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\gamma\bar{\Phi}^\gamma + 4\Phi_\beta\bar{\Phi}^\alpha\Phi_\gamma\bar{\Phi}^\beta\Phi_\alpha\bar{\Phi}^\gamma - 6\Phi_\gamma\bar{\Phi}^\gamma\Phi_\beta\bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\right)
\end{aligned}
$$

3d $U(N)_k \times U(N)_{-k}$ Superconformal Chern-Simons-Matter theory



By solving ABJM we can understand string and M in a unified manner.

localization



- Deform the theory, keeping the expectation value of a SUSY-invariant operator unchanged.

'localization technique'

$$S \to S + tQV \qquad QS = 0, \qquad Q\mathcal{O} = 0$$

supercharge

$$\begin{aligned} \frac{d}{dt}\int [dX]\mathcal{O}e^{-S-tQV} &= -\int [dX]\mathcal{O}\cdot(QV)e^{-S-tQV} \\ &= -\int [dX]Q\left(\mathcal{O}\cdot Ve^{-tQV}\right)e^{-S} \\ &= Z_{t=0}\times\langle Q\text{-exact}\rangle = 0 \end{aligned}$$

- At t=∞, the path integral 'localizes' to a matrix model.

localization in ABJM

- The partition function of ABJM on S^3 reduces to 'ABJM matrix model'

(Kapustin-Willett-Yaakov 2009)

$$Z_{ABJM}$$
$$= \frac{1}{N!^2}\int \frac{d^N\mu}{(2\pi)^N}\frac{d^N\nu}{(2\pi)^N}\frac{\prod_{i<j}\left[2\sinh\left(\frac{\mu_i-\mu_j}{2}\right)\right]^2\left[2\sinh\left(\frac{\nu_i-\nu_j}{2}\right)\right]^2}{\prod_{i,j}\left[2\cosh\left(\frac{\mu_i-\nu_j}{2}\right)\right]^2}\exp\left[\frac{ik}{4\pi}\sum_{i=1}^{N}(\mu_i^2-\nu_i^2)\right]$$

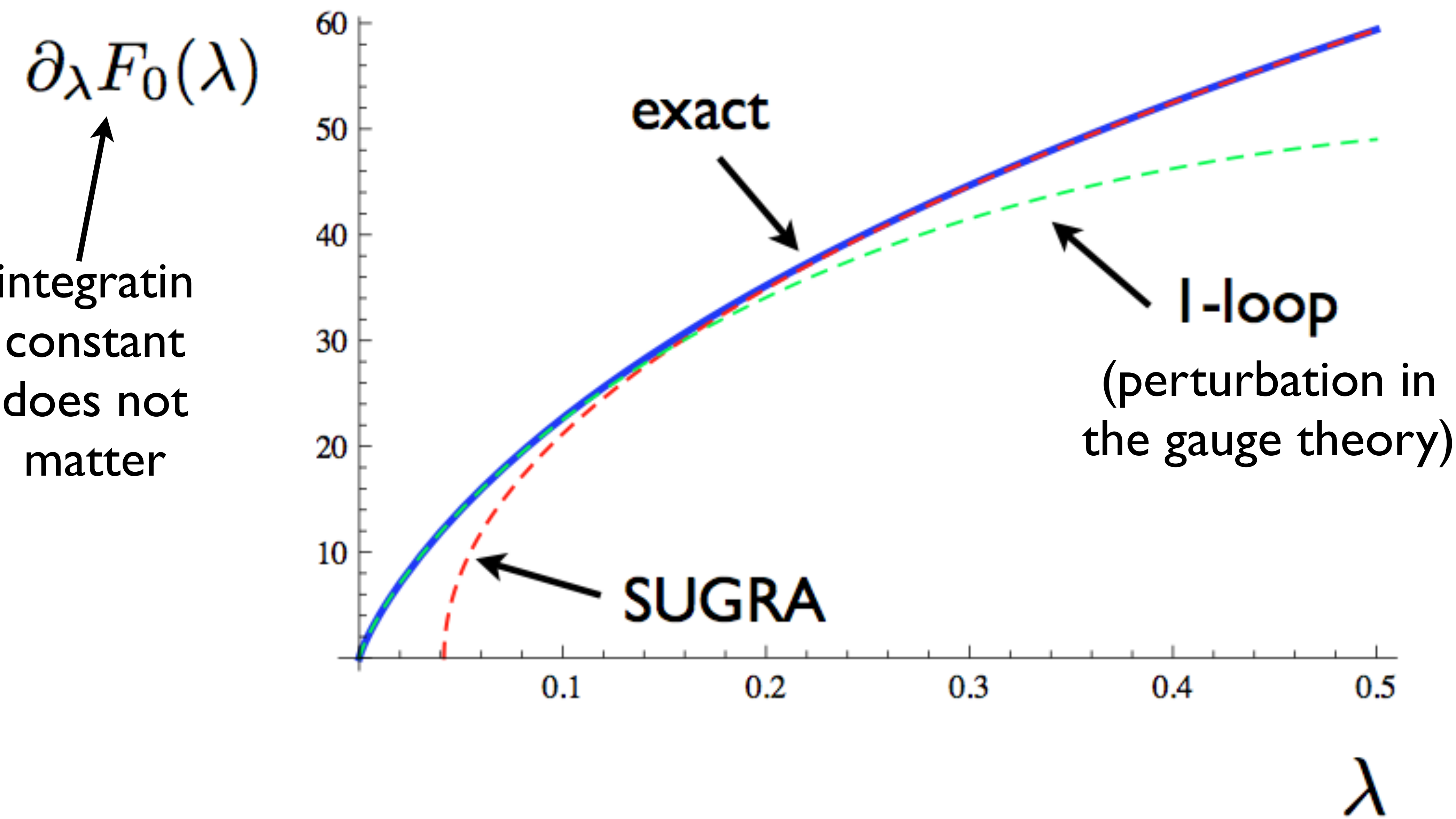
- Perturbative part (in string language) can be studied analytically even at strong coupling.

(Drukke-Marino-Putrov 2009-2011, Fuji-Hirano-Moriyama2011)

- Monte-Carlo gives the full answer at any N and any coupling.

(M.H.-Honda-Honma-Nishimura -Shiba-Yoshida 2012)

Planar limit (λ=N/k fixed)

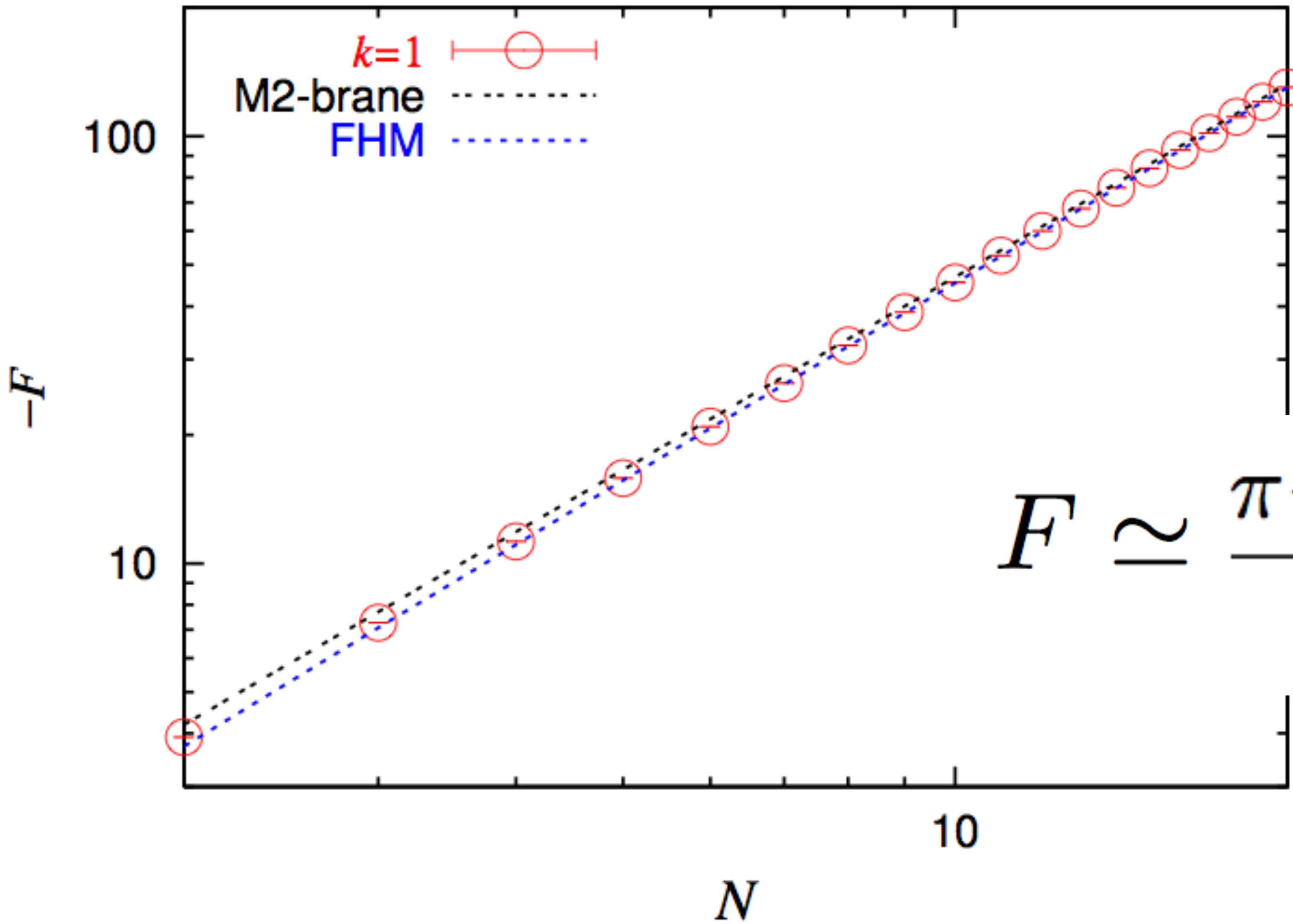


(from M. Marino's talk in strings2011)

10d IIA SUGRA has been reproduced!

Large-N, fixed k

<u>11d</u> SUGRA has been reproduced!



$$F \simeq \frac{\pi\sqrt{2}}{3} k^{1/2} N^{3/2}$$

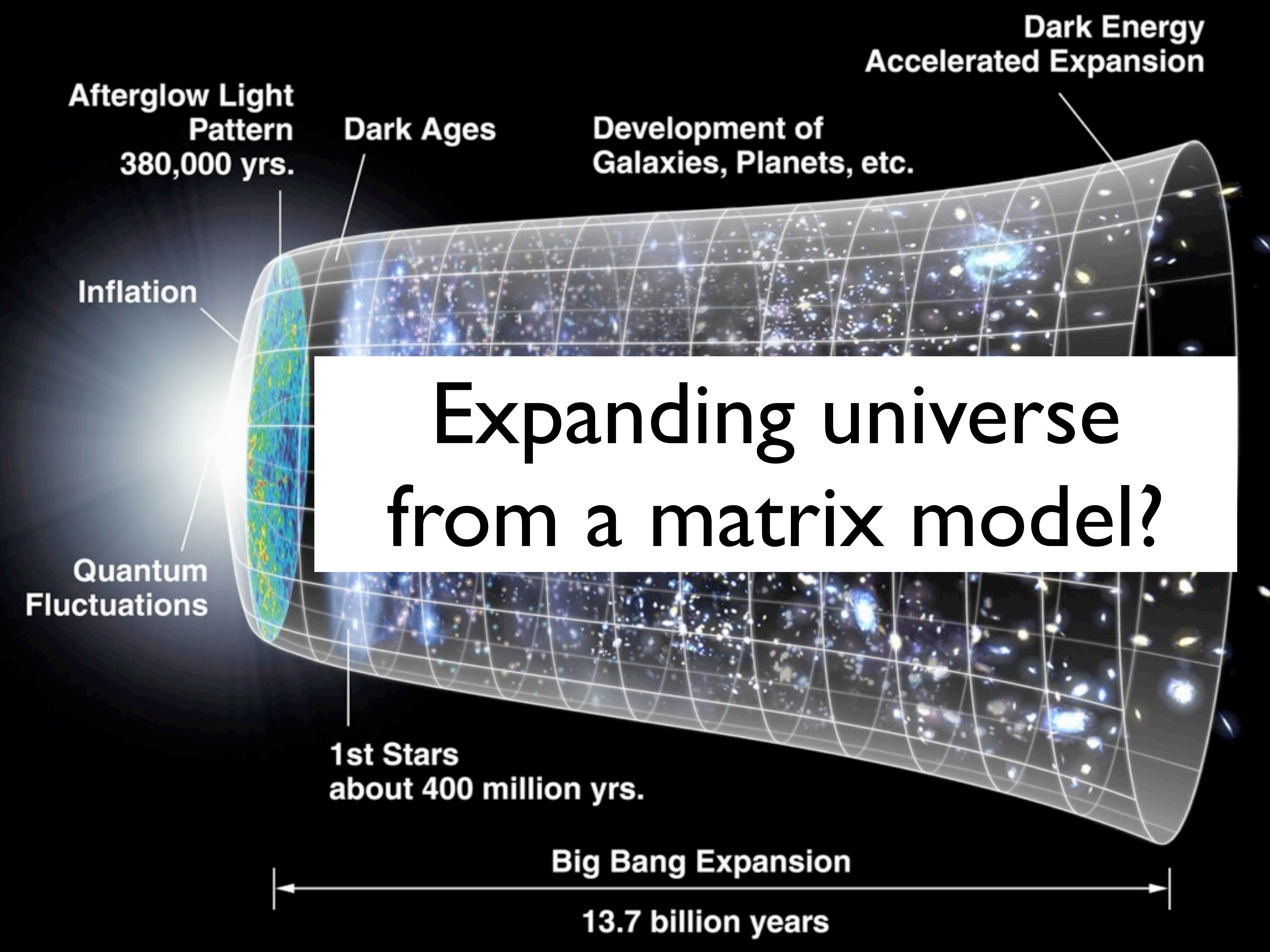
strong evidence that ABJM gives the definition of M-theory

Current status

- By carefully analyzing the Monte Carlo results, analytic solution at finite-N and finite coupling, which is applicable to both string and M-regions, have been obtained.

- The 1/N correction seems to *disagree* with string prediction. →made a miscalculation in the string side? have to modify the dictionary? Or gauge/gravity duality fails at quantum string level?

Expanding universe from a matrix model?

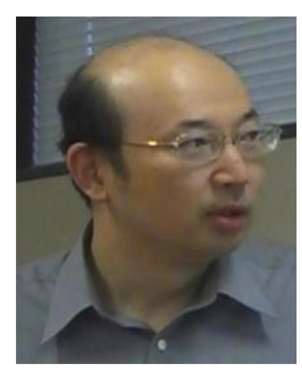


IIB matrix model

(Ishibashi-Kawai-Kitazawa-Tsuchiya 1996)

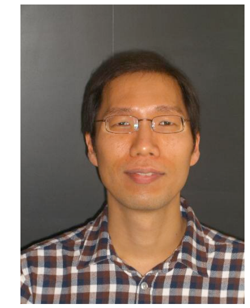
- (0+0)-d SYM
- Matrix regularization of IIB superstring
- Eigenvalue distribution = spacetime ?

$$S = Tr\left(-\frac{1}{4}[X_\mu, X_\nu]^2 - \frac{1}{2}\bar{\psi}\gamma^\mu[X_\mu, \psi]\right)$$

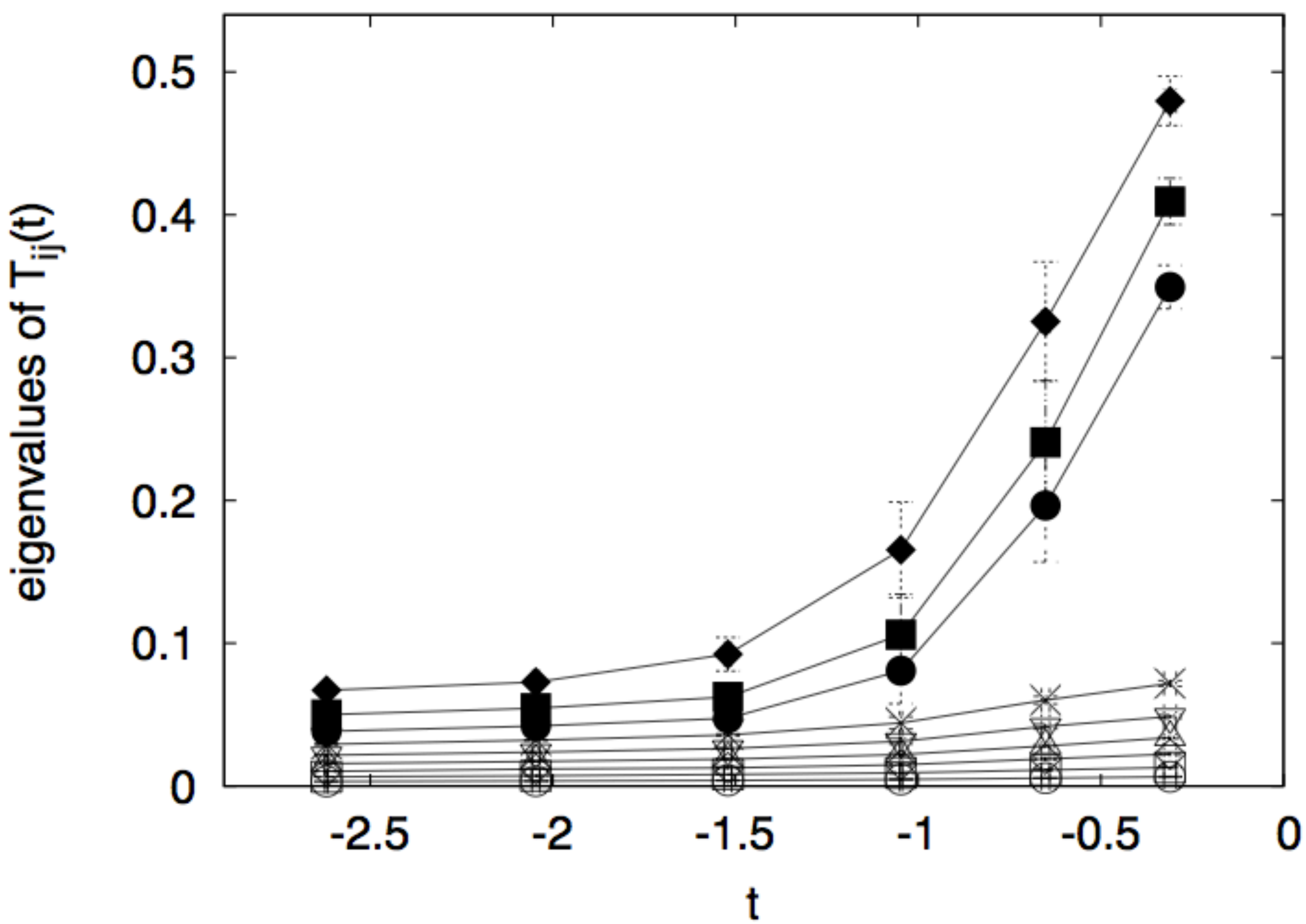
   

Expanding universe!

(Kim-Nishimura-Tsuchiya 2011)



- eigenvalue of X_0 = 'time'
- plot the extent of $X_1,\ldots, X_9$ at each 'time'.



expanding 3d space

small 6d space

Summary

- SUSY can be studied on computer, by combining lattice and non-lattice methods(e.g. Matrix model, localization,..).
- Simulation of the quantum gravity $\Rightarrow$ black hole thermodymanics, inflation, birth of the universe, multiverse, ...
- Not string theorists, but lattice theorists, can study such exciting topics.

THE END

How to put Super Yang-Mills on computer

‘No-Go’ for lattice SYM

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.

- So it is impossible to keep all supercharges exactly on lattice. Then SUSY breaking radiative corrections appear in general.

- Still it is possible to preserve a part of supercharges. (subalgebra which does not contain ∂)

Basic ideas

(Kaplan-Katz-Unsal 2002)

- Keep a few supercharges exact on lattice.
- Use it (and other discrete symmetries) to forbid SUSY breaking radiative corrections.
- Only "extended" SUSY can be realized for a technical reason.
- In (0+1)-d and (1+1)-d, no fine tuning to all order in perturbation.

(0+1)-d SYM

- Matrix quantum mechanics is UV finite.

No fine tuning!

(4d N=4 is also UV finite, but that relies on cancellations of the divergences...)

- We don't have to use lattice. Just fix the gauge & introduce momentum cutoff!
(M.H.-Nishimura-Takeuchi, 2007)

- Lattice can also work, of course.

(2+1)-d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\ Tr\left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc}X_a X_b X_c + \frac{\mu^2}{18}X_a^2 + \frac{\mu^2}{72}X_i^2\right)$$

+(fermions)

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

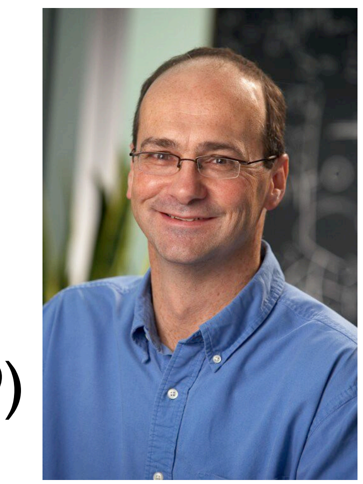
Fuzzy sphere

$$-[X_b,[X_a,X_b]]+i\mu\epsilon^{abc}X_bX_c+\frac{\mu^2}{9}X_a=0$$

$$\rightarrow \quad X_a=\frac{\mu}{3}J_a,\quad [J_a,J_b]=i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains (2+1)-d SYM on *noncommutative space*.

  Fuzzy D2 brane out of N D0 branes (R. C. Myers 1999)



- 'Lattice' is embedded in matrices.

  Large-N = continuum limit

- With maximal SUSY, commutative limit of the noncommutative space is smooth. (no UV/IR mixing)
  (Matusis-Susskind-Toumbas '00)

(3+1)-d SYM

- 4d N=1 *pure* SYM : lattice chiral fermion assures SUSY
  (Kaplan 1984, Curci-Veneziano 1986)

- 4d N=4 :

  • again "Hybrid" formulation: Lattice + fuzzy sphere
    (M.H.-Matsuura-Sugino 2010, M.H. 2010)

  •Large-N Eguchi-Kawai reduction(Ishii-Ishiki-Shimasaki-Tsuchiya, 2008)

  •Another Matrix model approach(Heckmann-Verlinde, 2011)

  •recent analysis of 4d lattice:
  Fine tuning is needed, but only for 3 bare lattice couplings.
  (Catterall-Dzienkowski-Giedt-Joseph-Wells, 2011)

SIGN PROBLEM?

NO PROBLEM!

- SYM has the sign problem (execept for 4d N=1 pure YM)
- Use the 'phase-quenched' effective action

$$S_{eff}[A] = S_B[A] - \log|\det D[A]|$$

- Phase can be taken into account by the 'phase reweighting' in principle, but usually it's hopelessly hard.

$$\begin{aligned}\langle \mathcal{O} \rangle &= \frac{\int [dA] \det D \cdot e^{-S_B} \cdot \mathcal{O}}{\int [dA] \det D \cdot e^{-S_B}} \\ &= \frac{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} \cdot \mathcal{O} / \int [dA] |\det D| \cdot e^{-S_B}}{\int [dA](phase) \cdot |\det D| \cdot e^{-S_B} / \int [dA] |\det D| \cdot e^{-S_B}} \\ &= \frac{\langle (phase) \cdot \mathcal{O} \rangle_{phase\ quench}}{\langle (phase) \rangle_{phase\ quench}} \quad \sim 0/0\end{aligned}$$

Miracles happen in SYM!

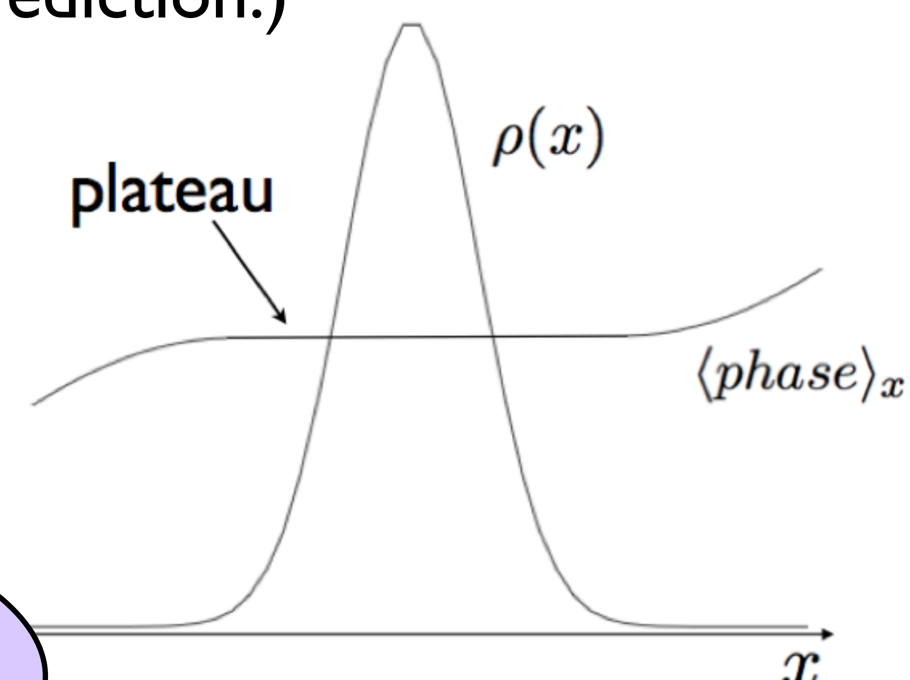
(Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007, Catterall-Wiseman 2008, Catterall et al 2011, Buchoff-M.H.-Matsuura, in progress.)

- Almost no phase except for very low temperature.
- SU(2) is almost sign-free.
- Even when the phase fluctuates, phase quench (without reweight) gives right answer.

  ('right' in the sense it reproduces gravity prediction.)

- Can be justified numerically.

  (M.H.-Nishimura-Sekino-Yoneya 2011)



This is the only theory in which we can believe in any miracle.

(D.B.Kaplan 2010, private communication.)

- Matrix quantum mechanics is UV finite.

  No fine tuning!

- We don't have to use lattice. Just fix the gauge & introduce momentum cutoff!
  (M.H.-Nishimura-Takeuchi, 2007)

(1+1)-d SYM

4 SUSY model (dimensional redcution of 4d N=1; sign-free) has been studied extensively.

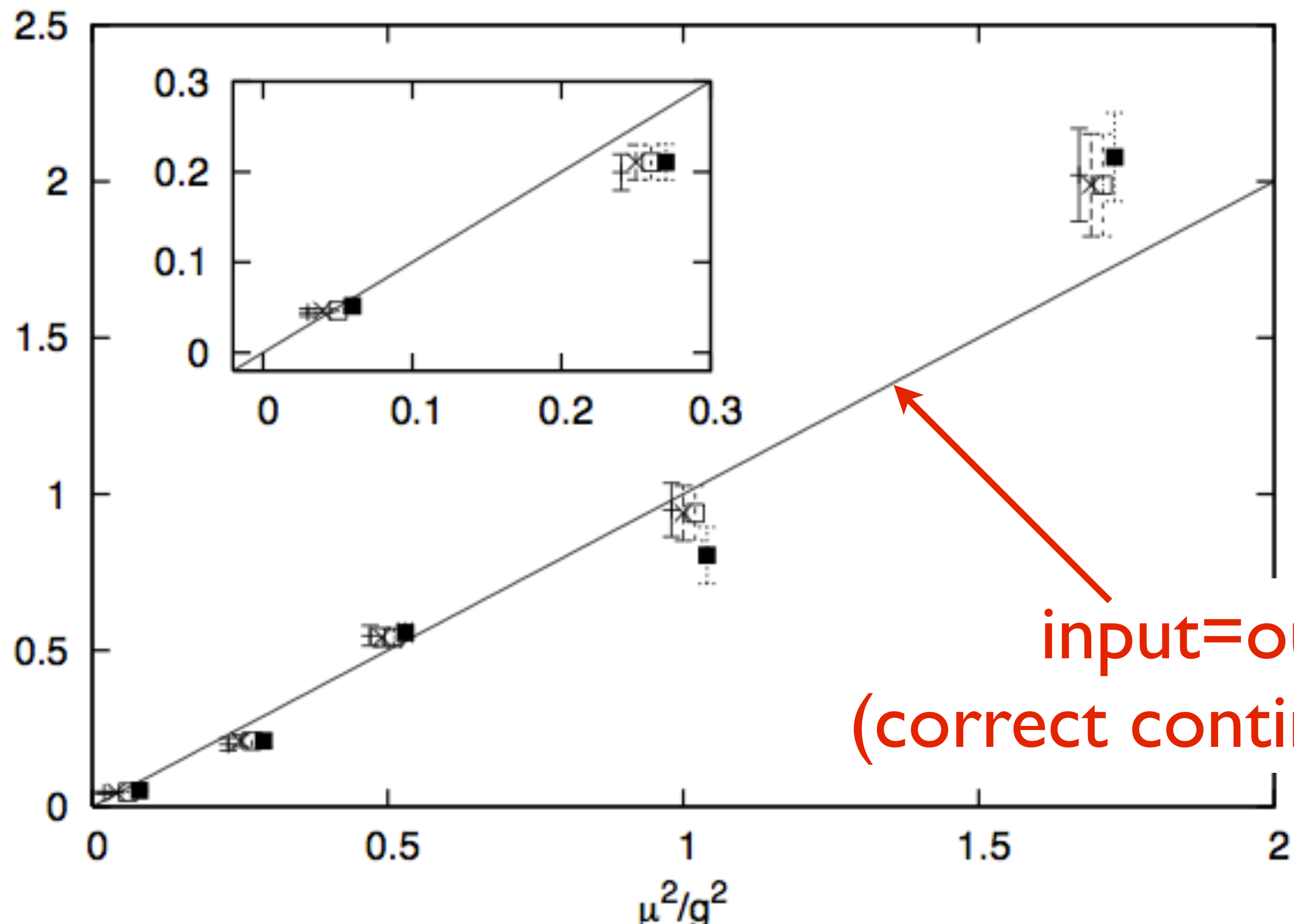
- Conservation of the supercurrents. (Sugino model) (Suzuki 2007, Kanamori-Suzuki 2008, Kadoh-Suzuki 2009)
- Comparison with analytic results at small volume & large-N behaviors. (Sugino model) (M.H.-Kanamori 2009)
- Agreement between Sugino model and Cohen-Kaplan-Katz-Unsal model. (M.H.-Kanamori 2010)

All results supports the emergence of the correct continuum limit without fine tuning.

Supercurrent conservation in the SU(2) Sugino model

soft SUSY breaking mass (output)

$\sim \partial_\mu J^\mu$

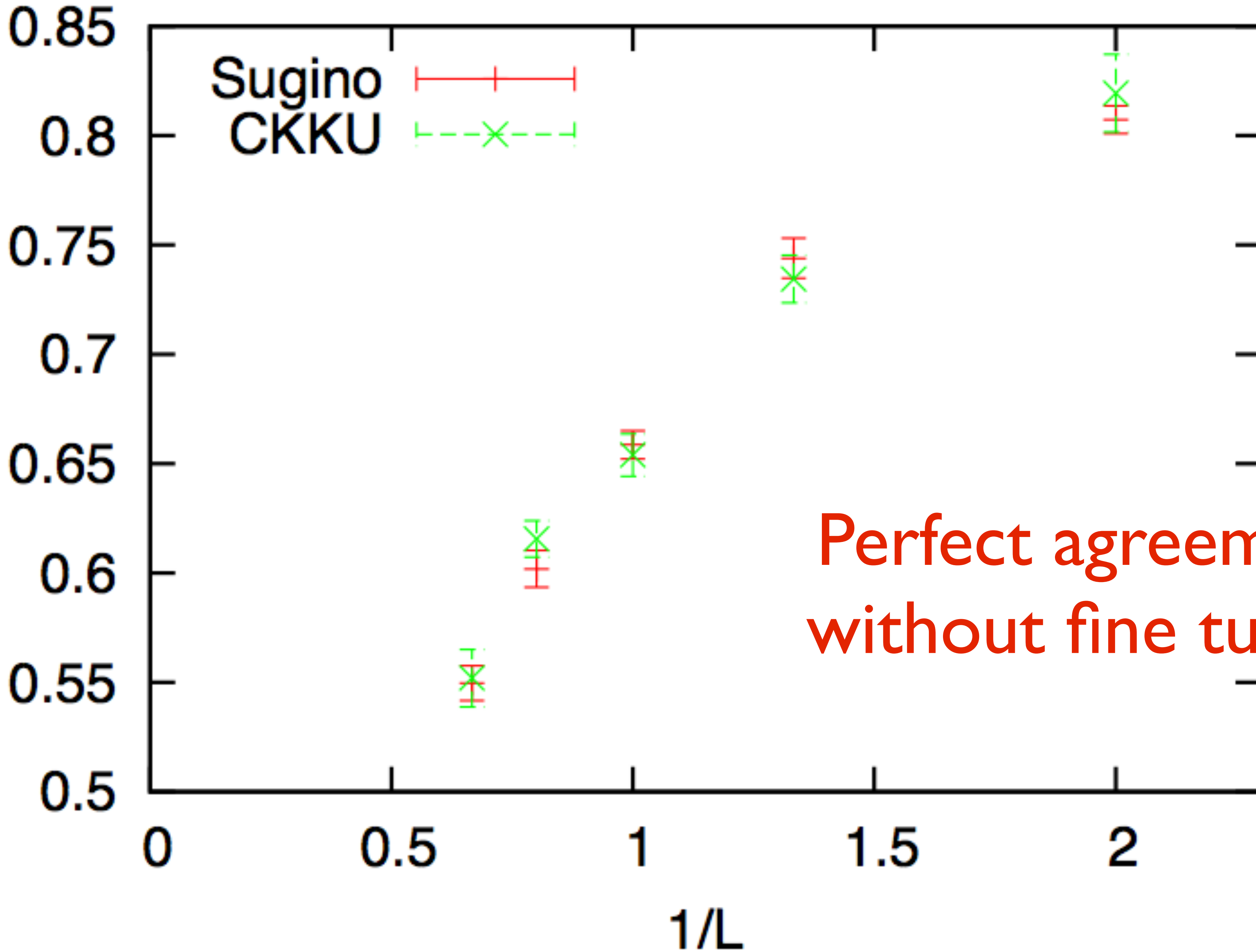


input=output (correct continuum limit)

soft SUSY breaking mass (input)

(Kanamori-Suzuki 2008)

(see also Kadoh-Suzuki 2009)



Perfect agreement without fine tuning

Polyakov loop vs compactification radius
SU(2), periodic b.c. (M.H.-Kanamori 2010)