Chiral Dynamics with Wilson Fermions

- from the perspective of the Wilson Dirac eigenvalues

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Urs Heller

lattice 2012, Cairns, Australia, June 29, 2012

What The phase structure of lattice QCD with Wilson fermions

Aoki VS Sharpe-Singleton puzzle

What The microscopic eigenvalue density

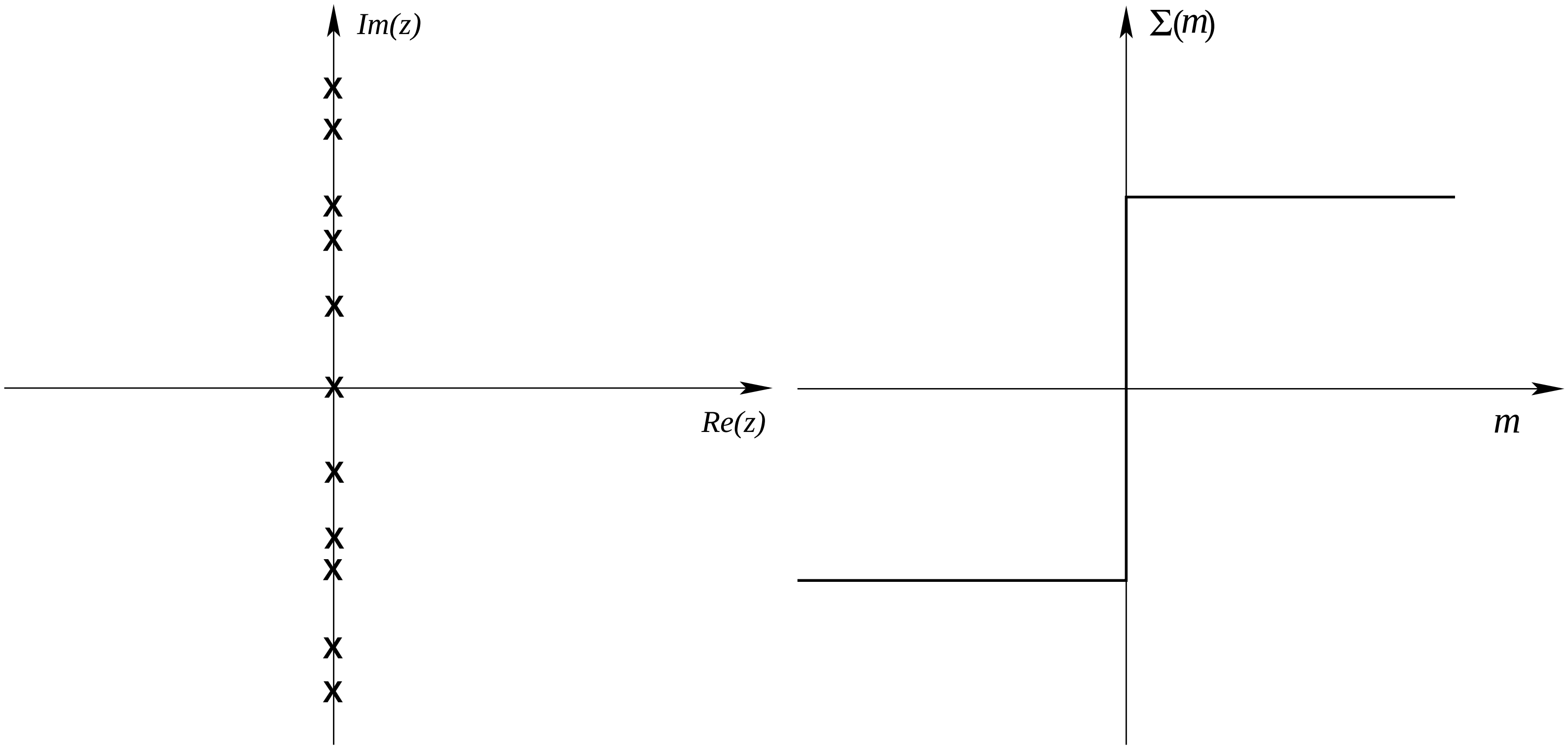
$a/\sqrt{V}$ *scaling*

Why Separate lattice artifacts from continuum physics

How Wilson Chiral Perturbation Theory

$a = 0$

Banks Casher



$$\Sigma = \frac{\pi}{V}\rho(0)$$

Banks Casher NPB 169 (1980) 103

$a \neq 0$

Wilson fermions break chiral symmetry

$$\gamma_5 D_W \neq -D_W \gamma_5$$
$$D_W^\dagger \neq -D_W$$

γ_5-Hermiticity

$$D_W^\dagger = \gamma_5 D_W \gamma_5$$

Wilson, Phys. Rev. D10 (1974) 2445

Itoh, Iwasaki, Yoshie, PRD 36 (1987) 527

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γ_5-Hermiticity

$$D_W^\dagger = \gamma_5 D_W \gamma_5$$

Eigenvalues, z, of D_W

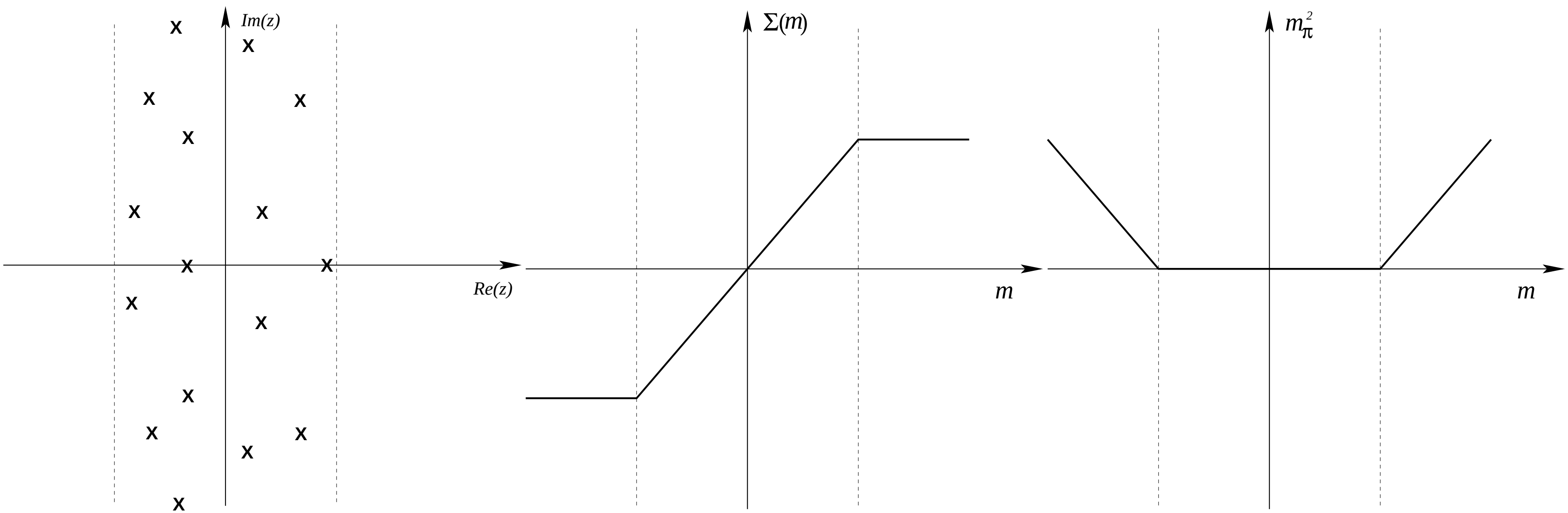
- complex conjugate pairs (z, z^*)

- exact real eigenvalues

Wilson, Phys. Rev. D10 (1974) 2445

Itoh, Iwasaki, Yoshie, PRD 36 (1987) 527

$a \neq 0$ Aoki phase (parity broken phase)



Electrostatic analogy:

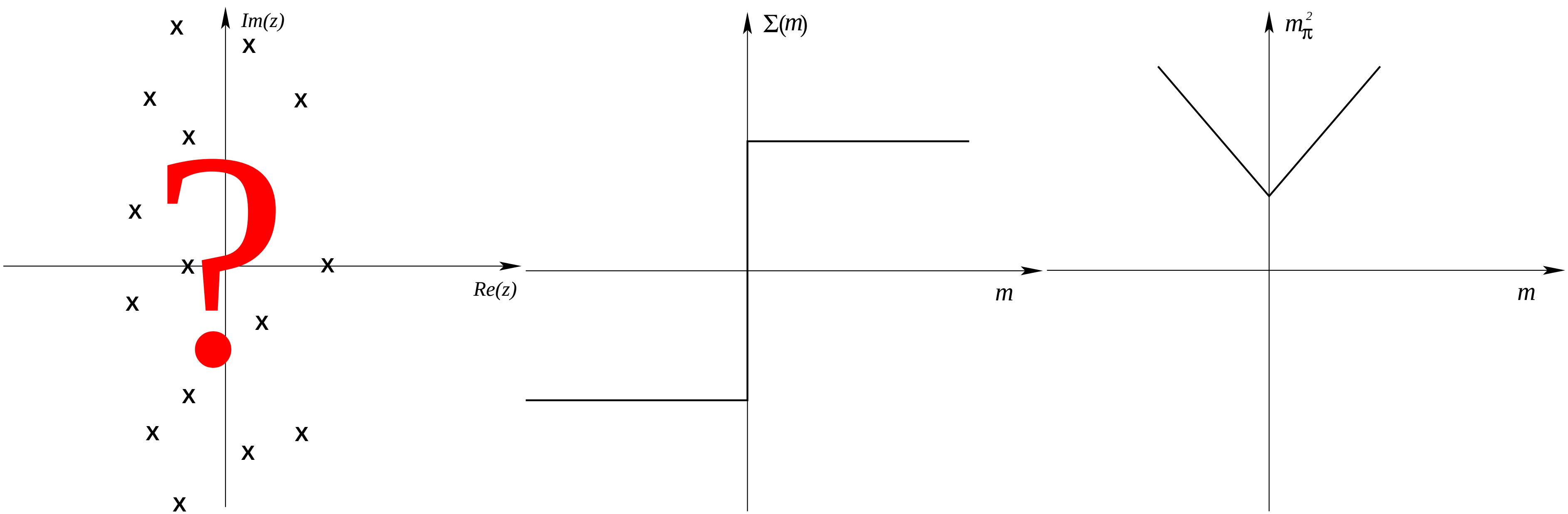
Eigenvalues = charges, quark mass = test charge

Aoki PRD **30** 2653 (1984)

Barbour et al. NPB 275 (1986) 296 (nonzero μ)

Sharpe Singleton PRD 58 (1998) 074501

$a \neq 0$ Sharpe Singleton scenario (1st order)



Creutz hep-lat/9608024

Sharpe Singleton PRD 58 (1998) 074501

Both observed on the lattice

Aoki phase

Aoki Gocksch PRD **45**, 3845 (1992)
Aoki Gocksch PLB **231** (1989) 449
Aoki Gocksch PLB **243**, 409 (1990)
Jansen *et al.* [XLF Collaboration] PLB **624**, 334 (2005)
Aoki Ukawa Umemura PRL **76**, 873 (1996)
Aoki Nucl.Phys.Proc.Suppl. **60A**, 206 (1998)
Ilgenfritz *et al.* PRD **69**, 074511 (2004)

Del Debbio Giusti Luscher Petronzio Tantalo JHEP **0602**, 011 (2006)
Del Debbio Giusti Luscher Petronzio Tantalo JHEP **0702**, 082 (2007)
S. Aoki *et al.* (PACS-CS) PRD **81** (2010) 074503
Ishikawa *et al.* (JLQCD) PRD **78** (2008) 011502
Borsanyi *et al.* arXiv:1205.0440 [hep-lat]
Bali *et al.* (QCDSF) to appear

Bernardoni Bulava Sommer arXiv:1111.4351
Aoki *et al.* [JLQCD Collaboration] PRD **72**, 054510 (2005)

Farchioni et al. Eur.Phys.J.C39:421 (2005)
Farchioni et al. Eur.Phys.J.C42:73 (2005)
Farchioni et al. PLB **624**, 324 (2005)
Farchioni et al. Eur.Phys.J.C47:453,2006

Baron et al. (ETM collab) JHEP08(2010)097

Sharpe-Singleton scenario

Puzzle: Quenched only observes Aoki

Aoki phase

Quenched ⟵

Aoki Gocksch PRD **45**, 3845 (1992)
Aoki Gocksch PLB **231** (1989) 449
Aoki Gocksch PLB **243**, 409 (1990)
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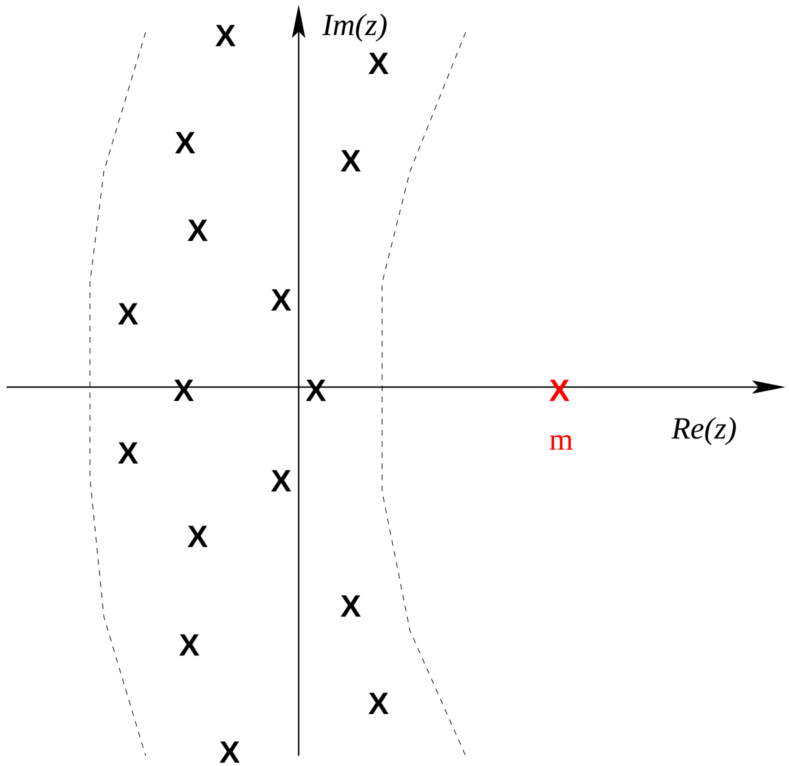
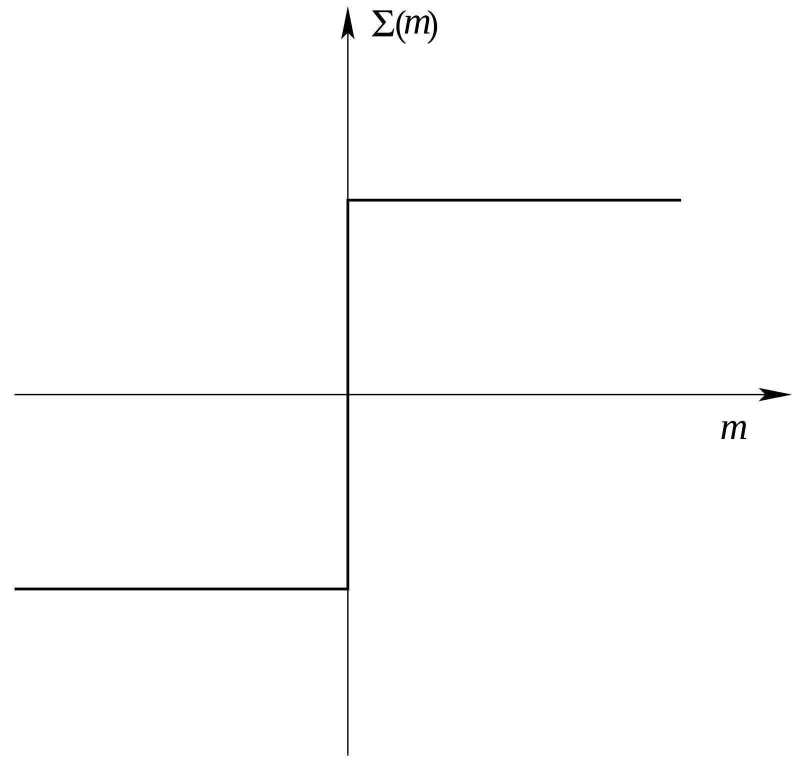
Baron et al. (ETM collab) JHEP08(2010)097

Sharpe-Singleton scenario

Golterman Sharpe Singleton PRD 71 (2005) 094503

Sharpe Singleton (1st order)





Goal: *analytic predictions for the Wilson Dirac spectrum* $a \neq 0$

Method: *Wilson Chiral Perturbation Theory*

Sharpe PRD 74 (2006) 014512

Wilson CPT

The chiral Lagrangian for Wilson fermions has new terms

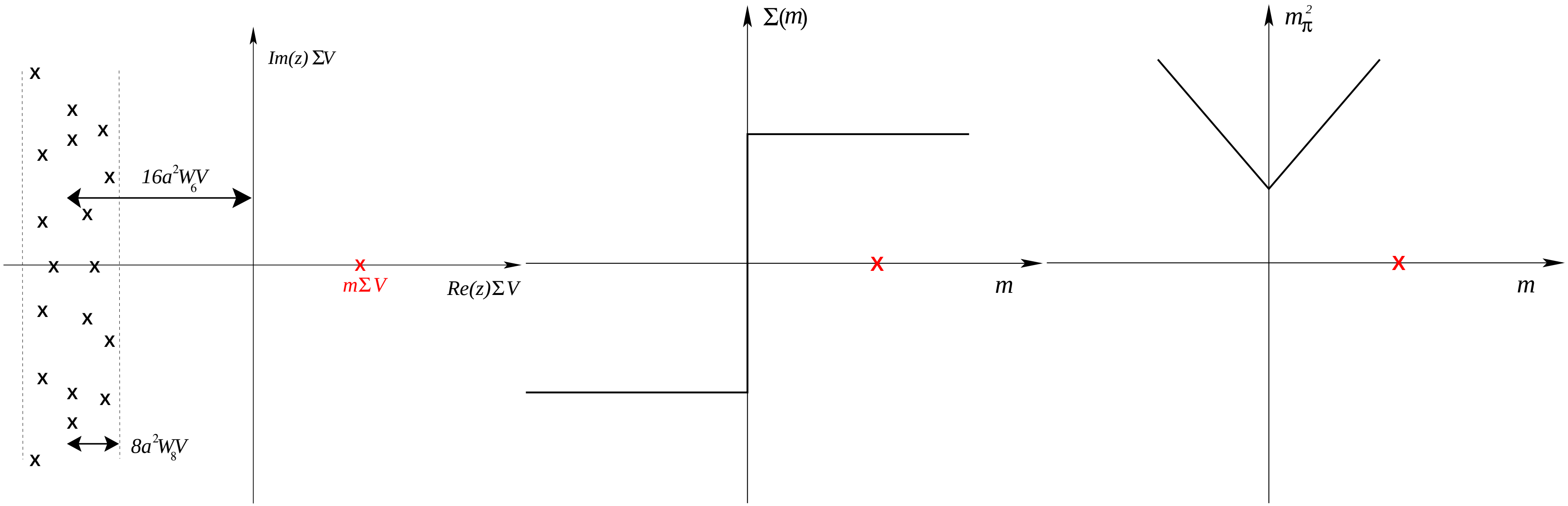
$$\mathcal{L} = \frac{F_\pi^2}{4}\mathrm{Tr}\left(d_\mu U d_\mu U^\dagger\right) + \frac{m}{2}\Sigma\mathrm{Tr}(U + U^\dagger) \\ -a^2 W_6[\mathrm{Tr}\left(U + U^\dagger\right)]^2 - a^2 W_7[\mathrm{Tr}\left(U - U^\dagger\right)]^2 \\ -a^2 W_8 \mathrm{Tr}(U^2 + {U^\dagger}^2)$$

Sharpe Singleton PRD **58**, 074501 (1998)
Rupak Shoresh PRD **66**, 054503 (2002)
Aoki PRD 68:054508,2003
Bar Rupak Shoresh PRD **70**, 034508 (2004)
Sharpe Wu PRD **70**, 094029 (2004)
Aoki Baer PRD 70 (2004) 116011
Golterman Sharpe Singleton PRD **71**, 094503 (2005)
Del Debbio Frandsen Panagopoulos Sannino JHEP0806:007 (2008)
Shindler PLB 672, 82 (2009)
Bar Necco Schaefer JHEP 0903, 006 (2009)
Bar Necco Shindler JHEP 1004:053,2010

with new constants W_6, W_7 and W_8

$a \neq 0$ # Sharpe Singleton scenario (1st order)

$W_8 + 2W_6 < 0$



Gap and pion mass

$$\frac{m_\pi^2 F_\pi^2}{2} = |m|\Sigma - 8(W_8 + 2W_6)a^2$$

Sharpe Singleton PRD 58 (1998) 074501

Kieburg Splittorff Verbaarschot PRD 85 (2012) 094011

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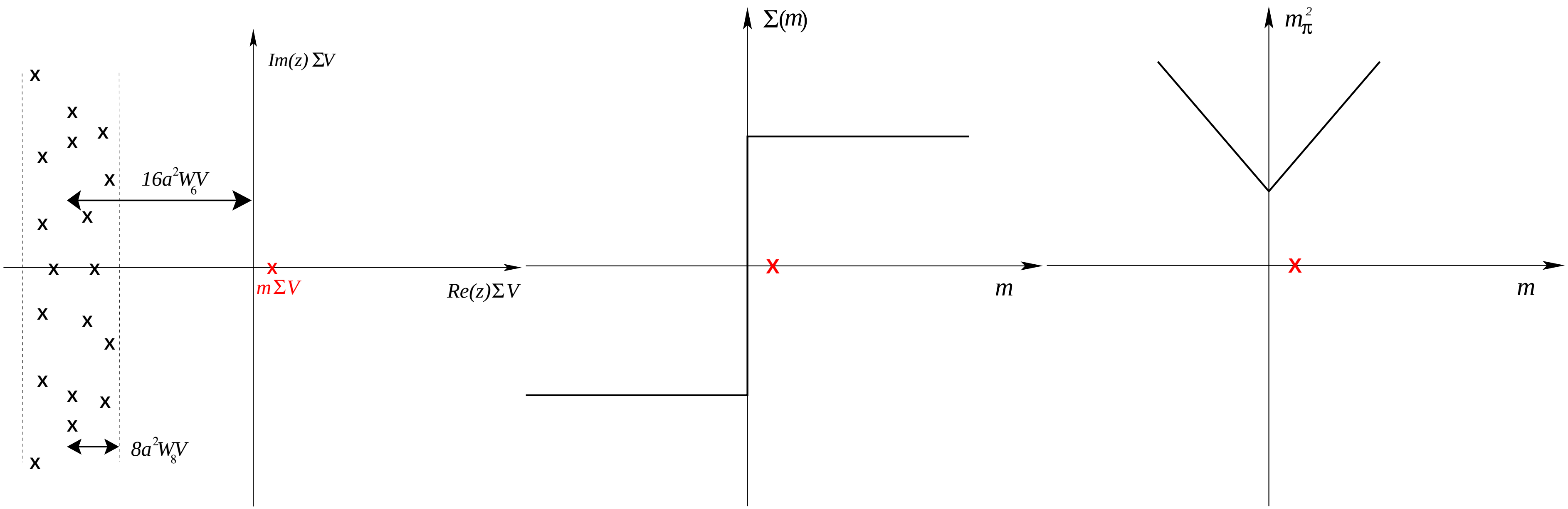

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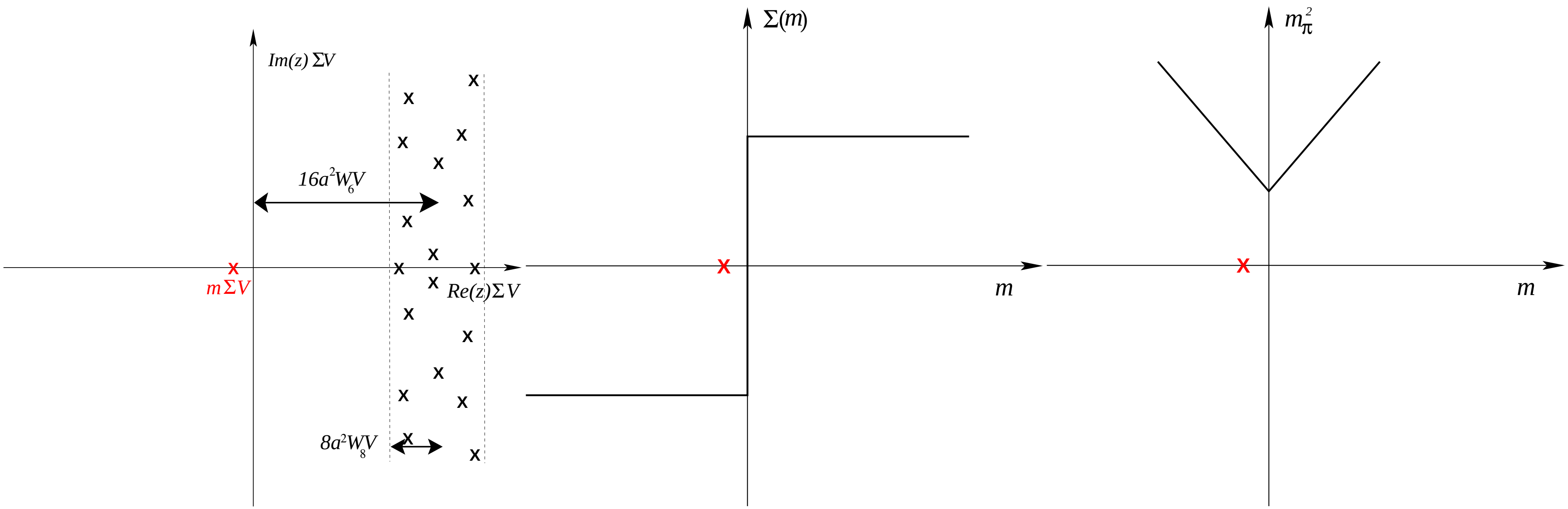

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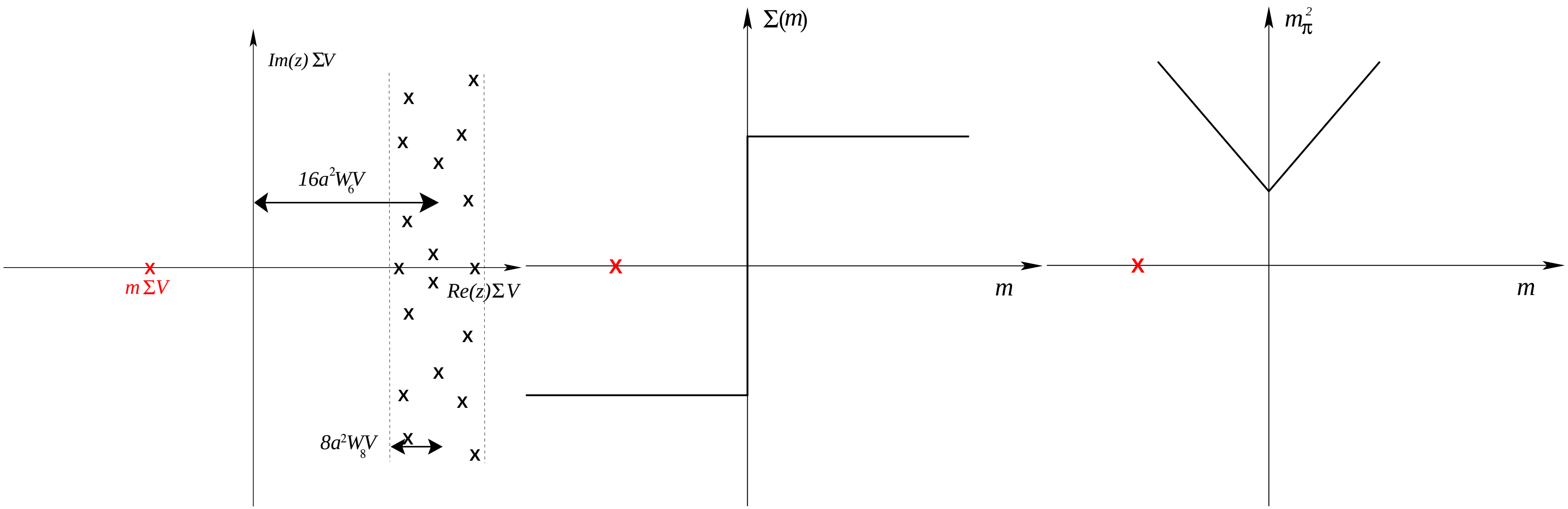

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Sharpe Singleton PRD 58 (1998) 074501

Kieburg Splittorff Verbaarschot PRD 85 (2012) 094011

Farchioni, Gebert, Montvay, Scorzato Eur.Phys.J. C26 (2002) 237

The W_i have fixed signs

Only Wilson CPT with

$$W_6, W_7 < 0 \text{ and } W_8 > 0$$

corresponds to the γ_5-Hermitian D_W

$$D_W = \frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2}\nabla_\mu\nabla^*_\mu$$

QCD ineq: Hansen and Sharpe PRD 85 (2012) 054504, PRD 85 (2012) 014503

Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

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Both Aoki ($W_8 + 2W_6 > 0$) and Sharpe Singleton ($W_8 + 2W_6 < 0$)

- allowed by γ_5 hermiticity !

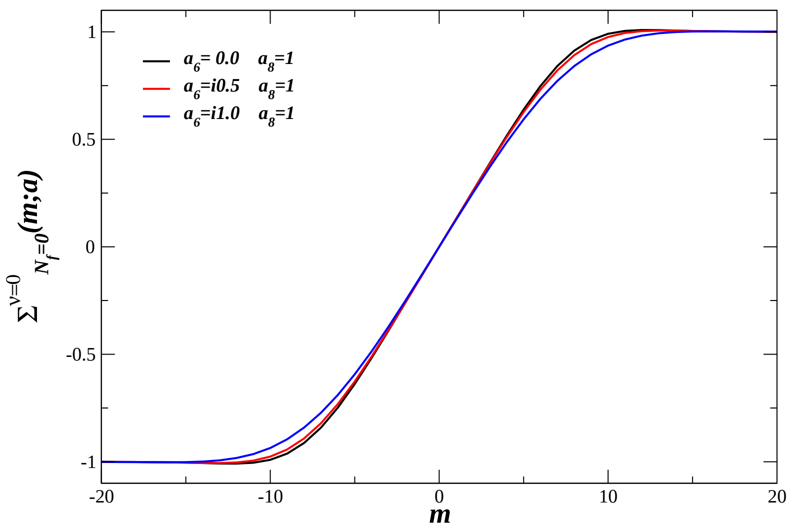
QCD ineq: Hansen and Sharpe PRD 85 (2012) 054504, PRD 85 (2012) 014503

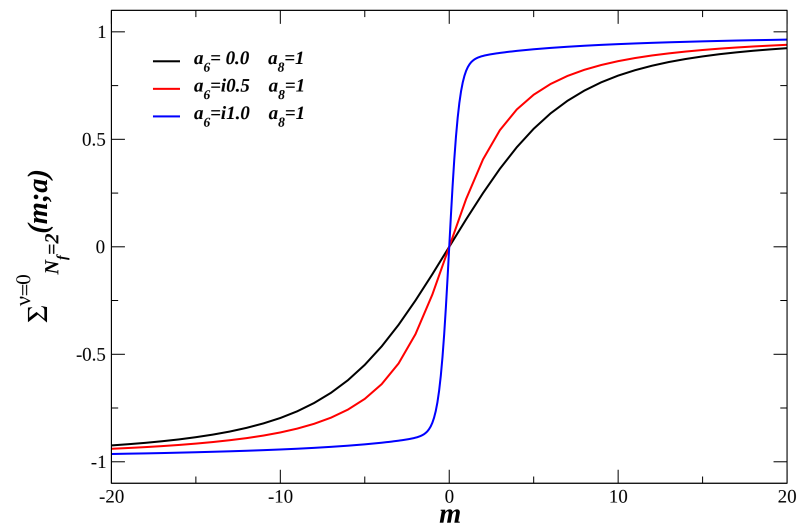
Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

Quenched and unquenched condensate

$W_8 + 2W_6 > 0$ $W_8 + 2W_6 = 0$ $W_8 + 2W_6 < 0$

Quenched $N_f = 2$





Sharpe Singleton only for $N_f > 0$

Golterman Sharpe Singleton PRD 71 (2005) 094503

Kieburg Splittorff Verbaarschot PRD 85 (2012) 094011

Microscopic spectrum:

Wilson CPT in the ϵ-regime $(m\Sigma V \sim a^2 V W_i \sim 1)$

The partition function in a sector ν

$$Z_{N_f}^{\nu} = \int_{U(N_f)} dU \, \det{}^{\nu}(U) \, e^{S}$$

with

$$\begin{aligned} S \quad = \quad & +\frac{m}{2}\Sigma V \,\mathrm{Tr}(U + U^{\dagger}) \\ & -a^2 V W_6 [\mathrm{Tr}\left(U + U^{\dagger}\right)]^2 - a^2 V W_7 [\mathrm{Tr}\left(U - U^{\dagger}\right)]^2 \\ & -a^2 V W_8 \,\mathrm{Tr}(U^2 + U^{\dagger\,2}) \end{aligned}$$

Akemann, Damgaard, Splittorff, Verbaarschot, PRD 83:085014, 2011

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Non trivial fact: In sector ν the Wilson Dirac operator D_W has index ν

index $=\sum_k \mathrm{sign}(\langle k|\gamma_5|k\rangle)$

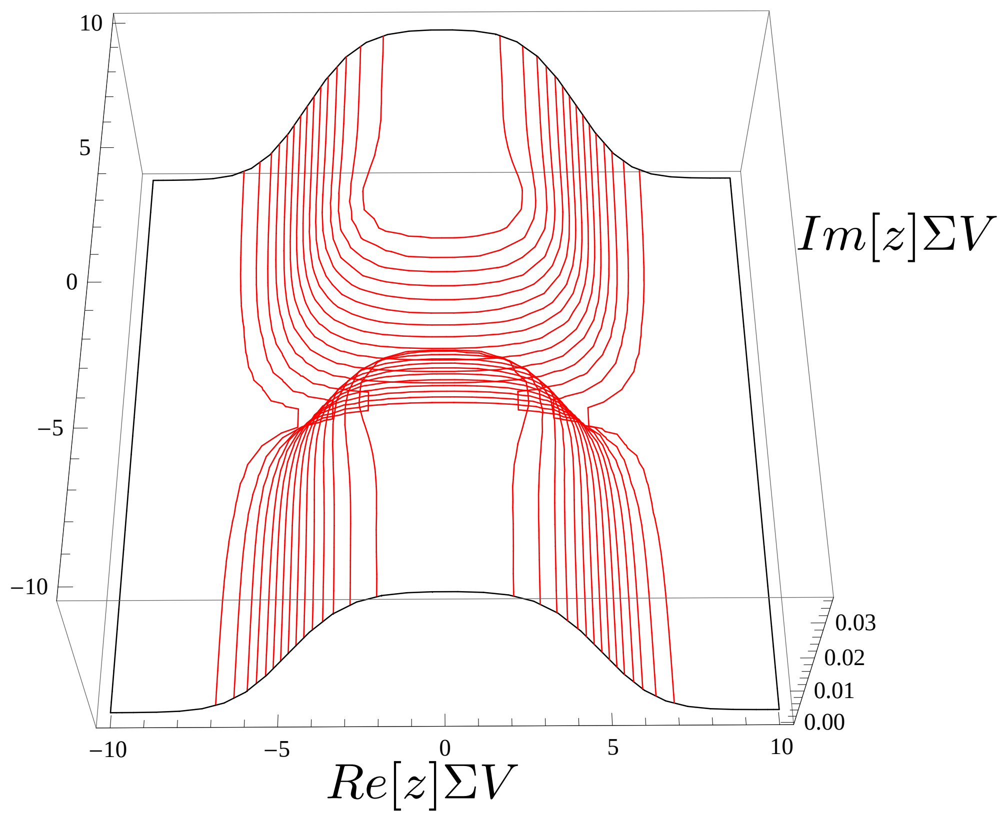
Akemann, Damgaard, Splittorff, Verbaarschot, PRD 83:085014, 2011

The complex eigenvalues of D_W for $\nu = 0$

$N_f = 0$

$a\sqrt{W_8 V} = 0.75$

$W_6 = W_7 = 0$



Kieburg, Verbaarschot, Zafeiropoulos PRL 108, 022001 (2012) ($N_f = 0$)

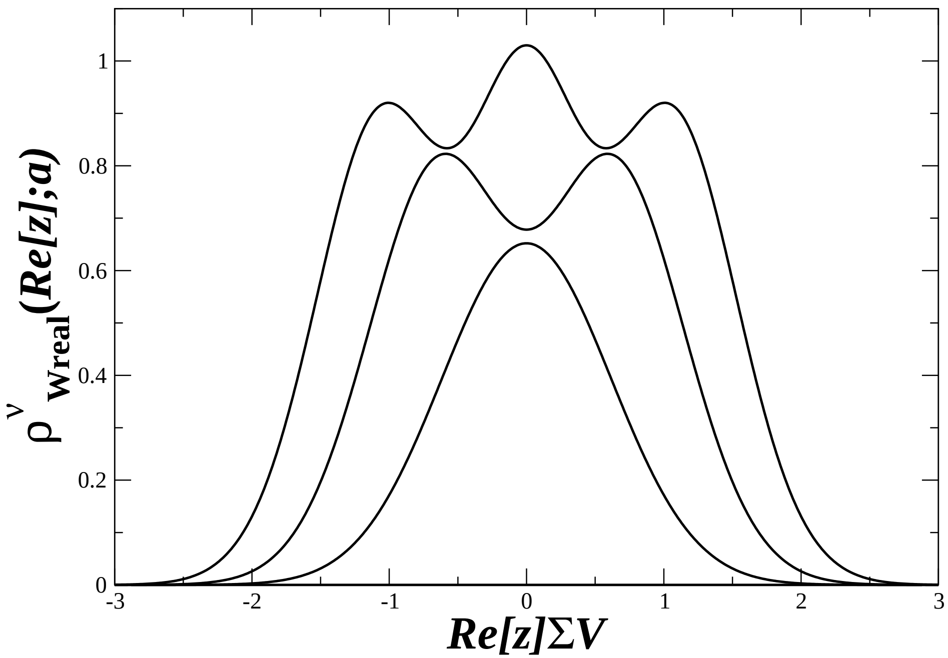
Kieburg Splittorff Verbaarshot PRD 85 (2012) 094011 ($N_f = 2$)

The real eigenvalues of D_W in sector $\nu = \sum_k \mathrm{sign}(\langle k|\gamma_5|k\rangle) = 1, 2, 3$

$N_f = 0$

$a\sqrt{W_8 V} = 0.2$

$W_6 = W_7 = 0$



Gattringer Hip Lang NPB 508 (1997) 329

Hernandez NPB 536 (1998) 345

Damgaard Splittorff Verbaarschot PRL 105:162002,2010

Kieburg, Verbaarschot, Zafeiropoulos PRL 108, 022001 (2012)

The Hermitian Wilson Dirac operator D_5

Introduce

$$D_5 \equiv \gamma_5(D_W + m)$$

The Hermitian Wilson Dirac operator D_5

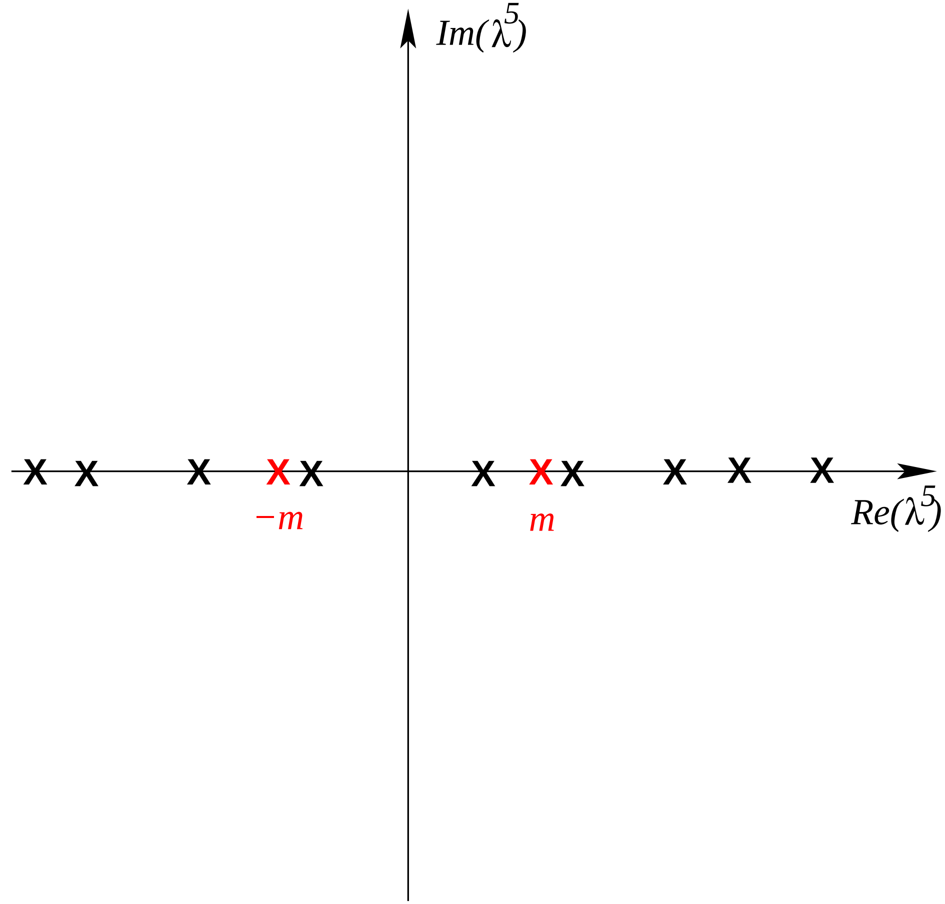
Introduce

$$D_5 \equiv \gamma_5 (D_W + m)$$

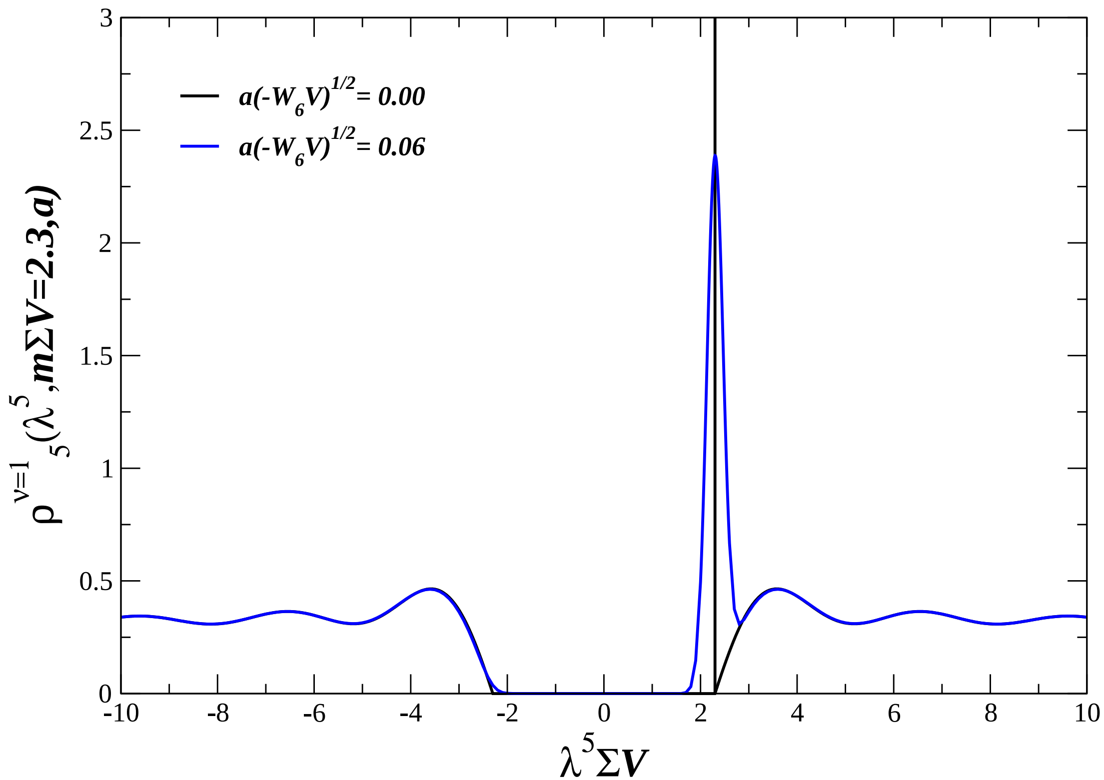
γ_5-Hermiticity of D_W Hermiticity of D_5

$$D_W^\dagger = \gamma_5 D_W \gamma_5 \qquad \Rightarrow \qquad D_5^\dagger = D_5$$

D_5 is Hermitian but spectrum *not* symmetric: *not* $(\lambda^5, -\lambda^5)$



Eigenvalue density of D_5 $\nu = \sum_k \text{sign}(\langle k|\gamma_5|k\rangle) = 1$ $N_f = 0$



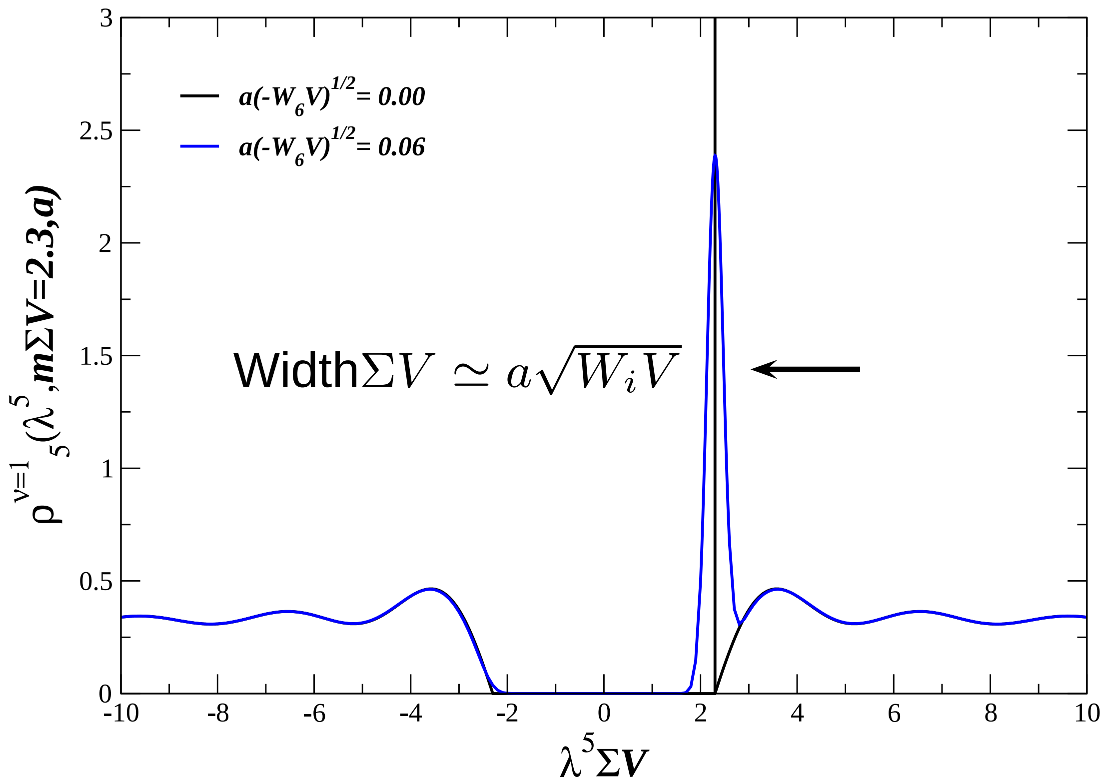
- Aoki phase if gap closes

Bitar Heller Narayanan PLB **418** 167 (1998)

Verbaarschot Zahed Phys.Rev.Lett. 70 (1993) 3852

Damgaard Splittorff Verbaarschot Phys.Rev.Lett.105:162002,2010

Eigenvalue density of D_5 $\qquad \nu = \sum_k \text{sign}(\langle k|\gamma_5|k\rangle) = 1 \qquad N_f = 0$



- Aoki phase if gap closes

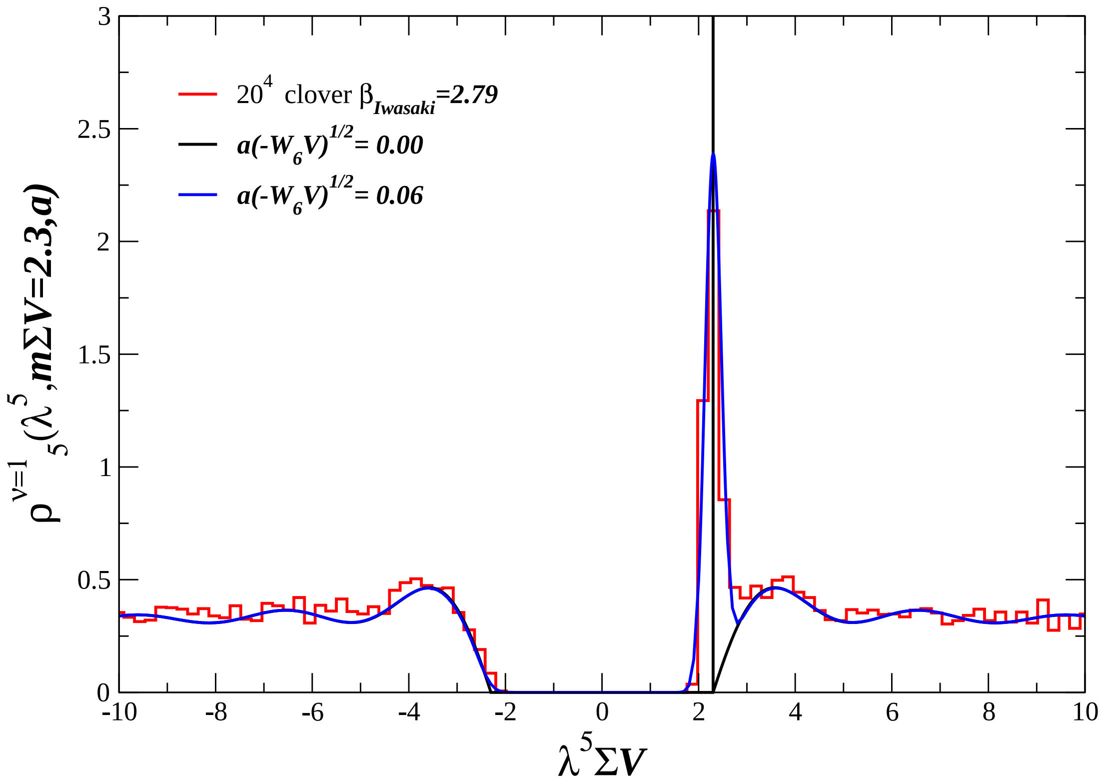
$a\sqrt{W_i V} \ll 1$ only affects the index peak

Bitar Heller Narayanan PLB **418** 167 (1998)

Verbaarschot Zahed Phys.Rev.Lett. 70 (1993) 3852

Damgaard Splittorff Verbaarschot Phys.Rev.Lett.105:162002,2010

Eigenvalue density of D_5 VS lattice $N_f = 0$



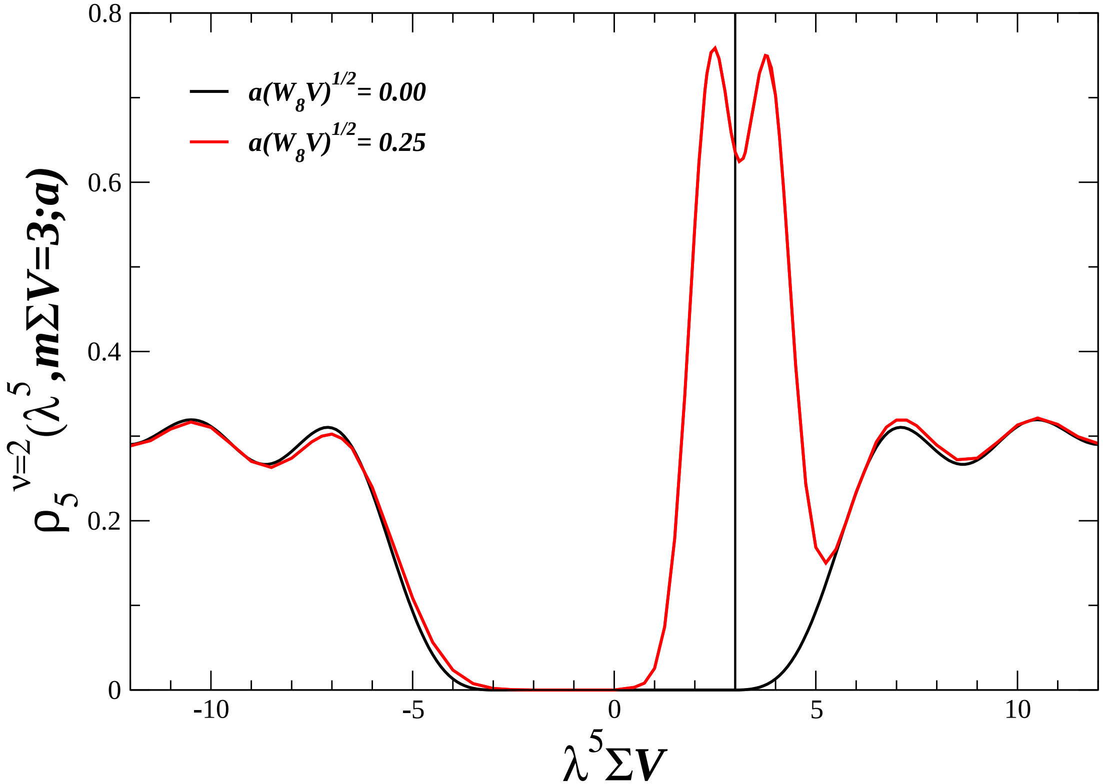
measure Σ, m, W_6, W_7 and W_8

Necco Shindler JHEP 1104 (2011) 031

Deuzeman Wenger Wuilloud JHEP 12 (2011) 109

Damgaard Heller Splittorff Phys.Rev. D85 (2012) 014505, arXiv:1206.4786

Eigenvalue density of D_5 $\nu = \sum_k \mathrm{sign}(\langle k|\gamma_5|k\rangle) = 2$ $N_f = 2$



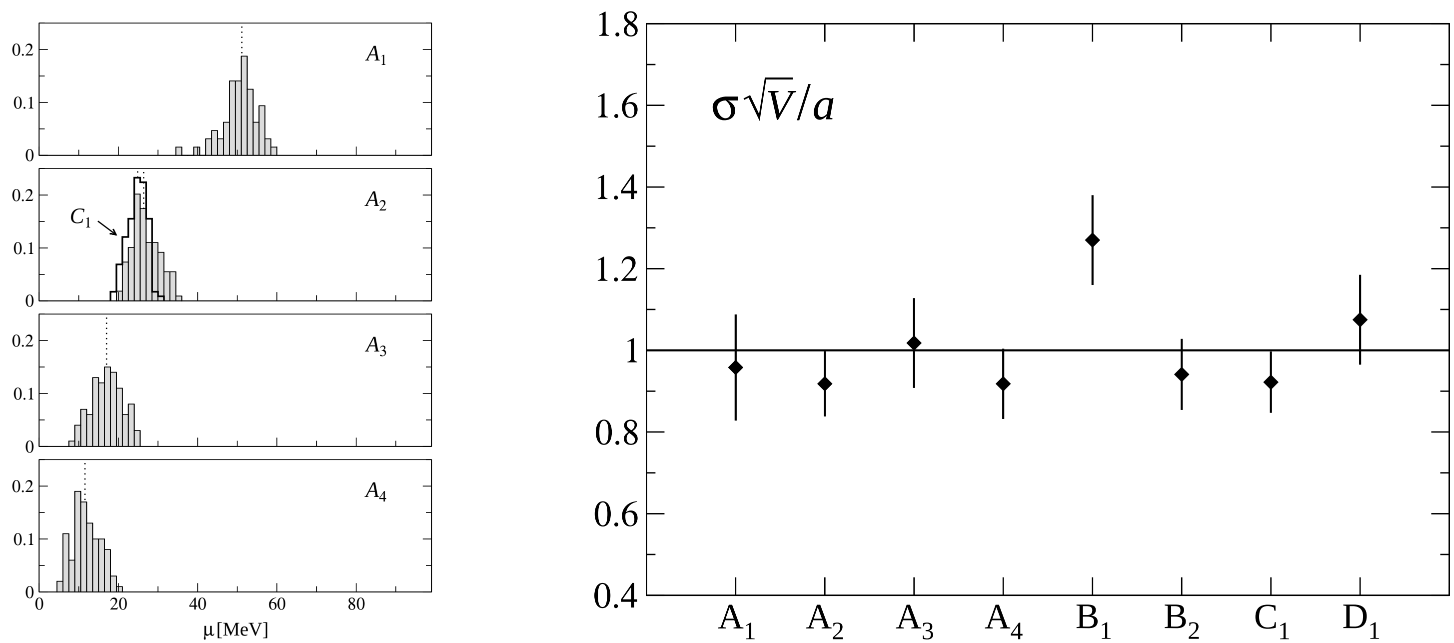
Unquenched:

$\rho_5(\lambda^5 = 0) = 0$ since $\det^2(D_W + m) = \det^2 D_5(m) = \prod_j \lambda_j^5(m)^2$

Lüscher JHEP0707:081,2007

Splittorff Verbaarschot PRD 84 (2011) 065031

Smallest eigenvalue of D_5 lattice $N_f = 2$



$\sigma \simeq \frac{a}{\sqrt{V}}$ exactly as for $a\sqrt{W_i V} << 1$

Del Debbio Giusti Lüscher Petronzio Tantalo JHEP 0602 (2006) 011, JHEP 0702 (2007) 082
Damgaard Splittorff Verbaarschot Phys.Rev.Lett.105:162002,2010

Akemann Ipsen JHEP 4 (2012), 102

Conclusions

Dirac eigenvalues as a tool to understand and test chiral dynamics with Wilson fermions

The Sharpe Singleton scenario *- only occurs unquenched*

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Constraints on the parameters of WCPT from γ_5-Hermiticity

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Microscopic eigenvalue density of D_W and D_5 - $a/\sqrt{V}$ *scaling*

Conclusions

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The Sharpe Singleton scenario *- only occurs unquenched*

Constraints on the parameters of WCPT from γ_5-Hermiticity

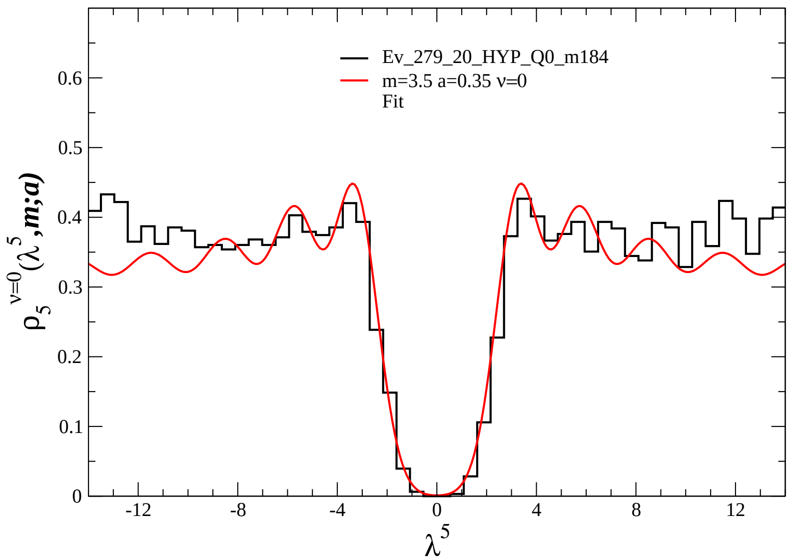
Microscopic eigenvalue density of D_W and D_5 *-* $a/\sqrt{V}$ *scaling*

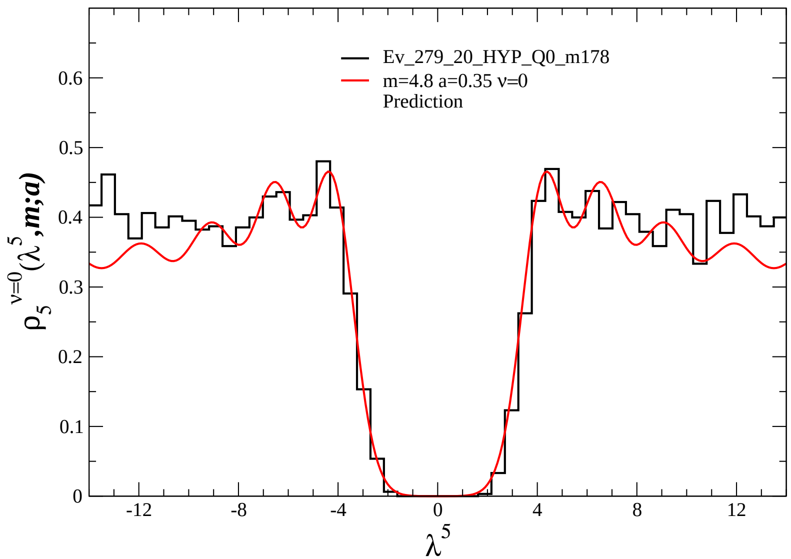
More: Spinodal line, Twisted mass, individual eigenvalues, Random matrix approach, $\beta_D = 1, 4$*, (Savvas Zafeiropoulos talk 3:50), Staggered (James Osborn PoS(Lattice 2011)110).*

Additional slides

Spectrum of D_5 on 20^4 lattice smaller coupling

Histograms: lattice Curves: WCPT





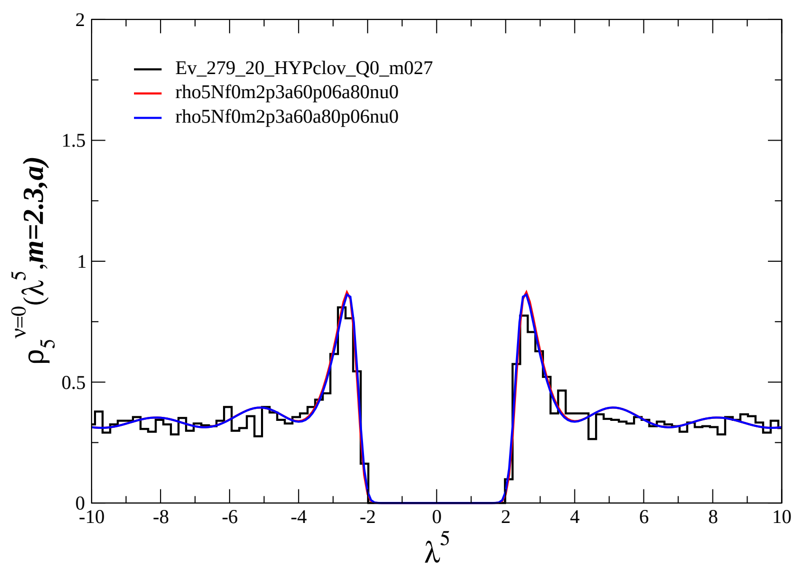
LHS fit (ΣV, $m\Sigma V$ and a_8) RHS prediction: mass scaling

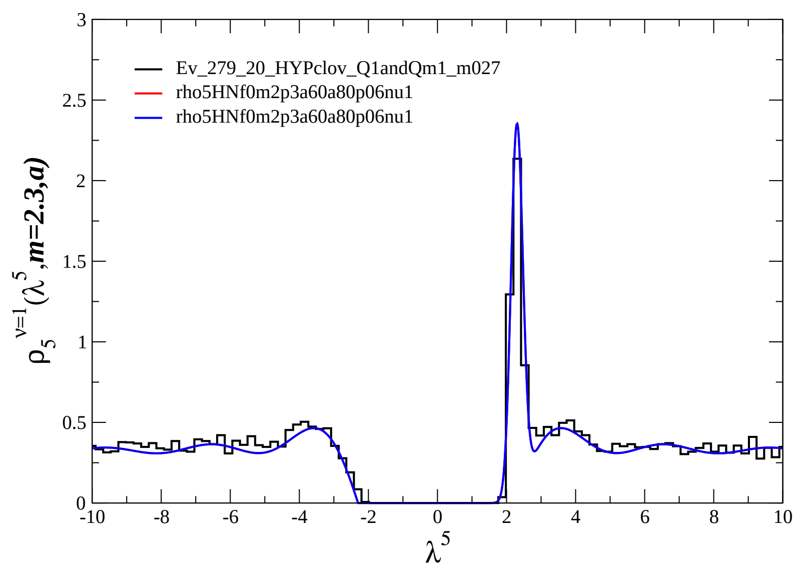
$$N_f = 0$$

Damgaard Heller Splittorff Phys.Rev. D85 (2012) 014505

Spectrum of $D_5 \equiv \gamma_5(D_W + m)$ on 20^4 lattice **clover improved**

Histograms: lattice Curves: WCPT





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Damgaard Heller Splittorff arXiv:1206.4786

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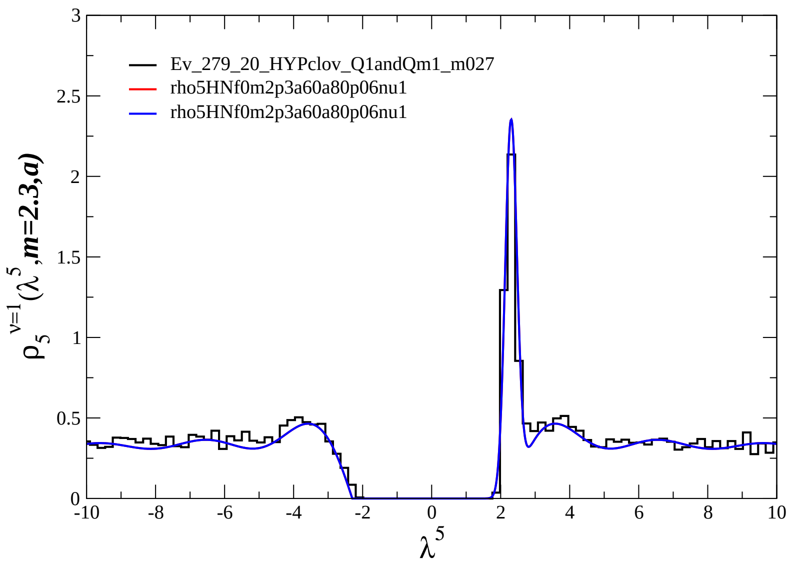
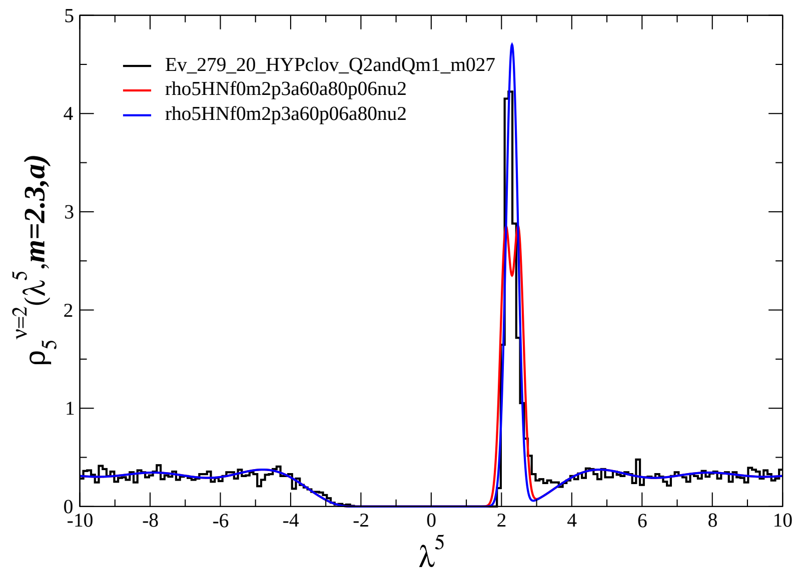




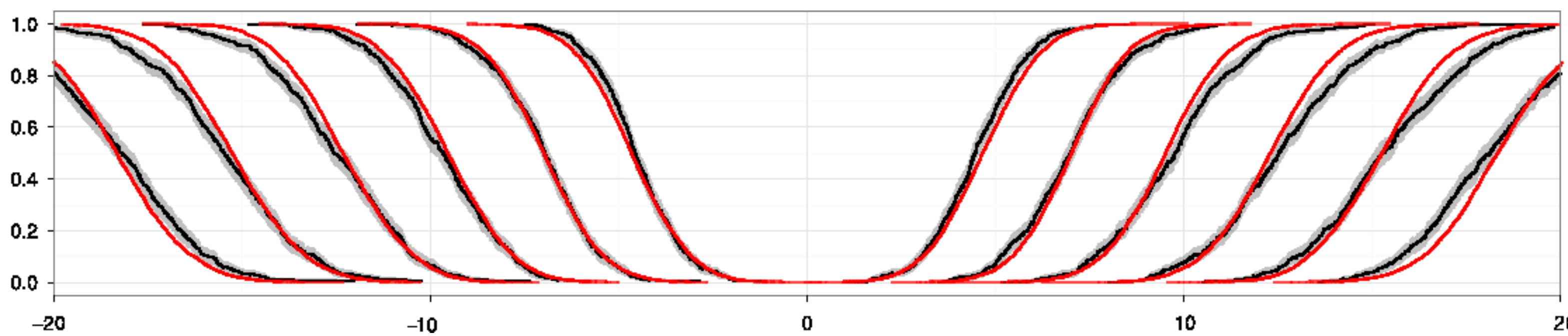
$$N_f = 0$$

Damgaard Heller Splittorff arXiv:1206.4786

Spectrum of D_5 on 24^4 lattice $\nu = 0$

Cumulative eigenvalue distributions

Black: lattice Red: WCPT



Fit to $\nu = 1$ data

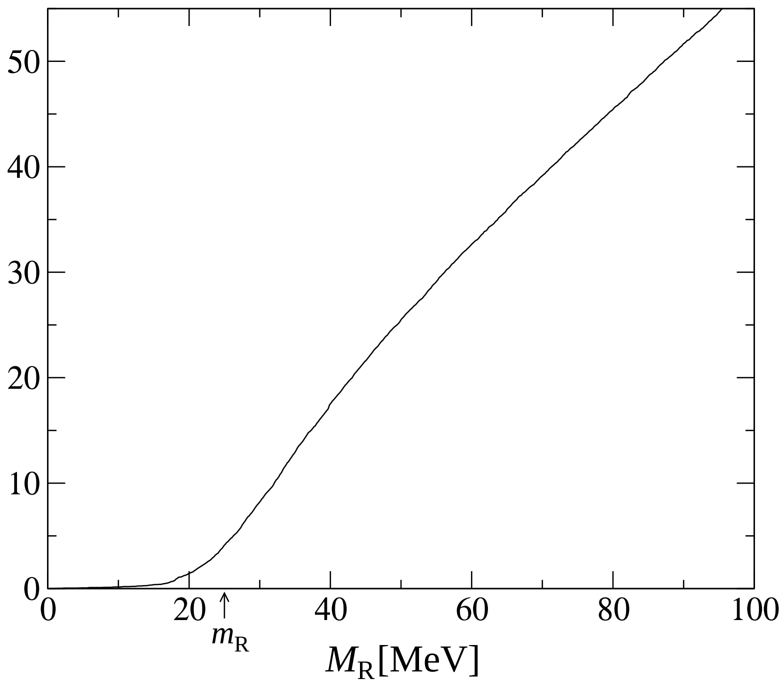
$N_f = 0$

Deuzeman Wenger Wuilloud JHEP 1112 (2011) 109

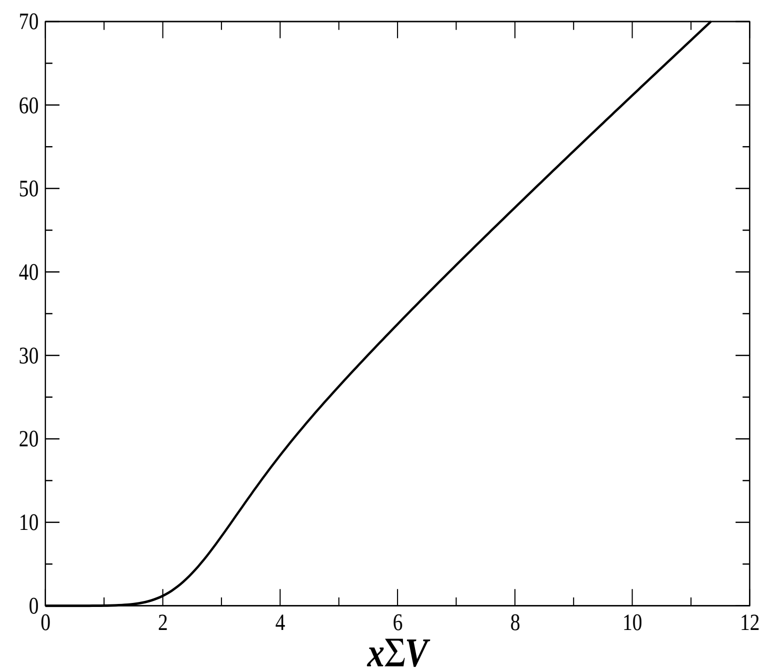
Spectrum of D_5 for $N_f = 0$

- *integrated up from zero & summed over the index*

Lattice $64 \times 32^3 \; a \simeq 0.07 \, fm$



WCPT ($m\Sigma V = 3$, $a_8 = 0.2$)

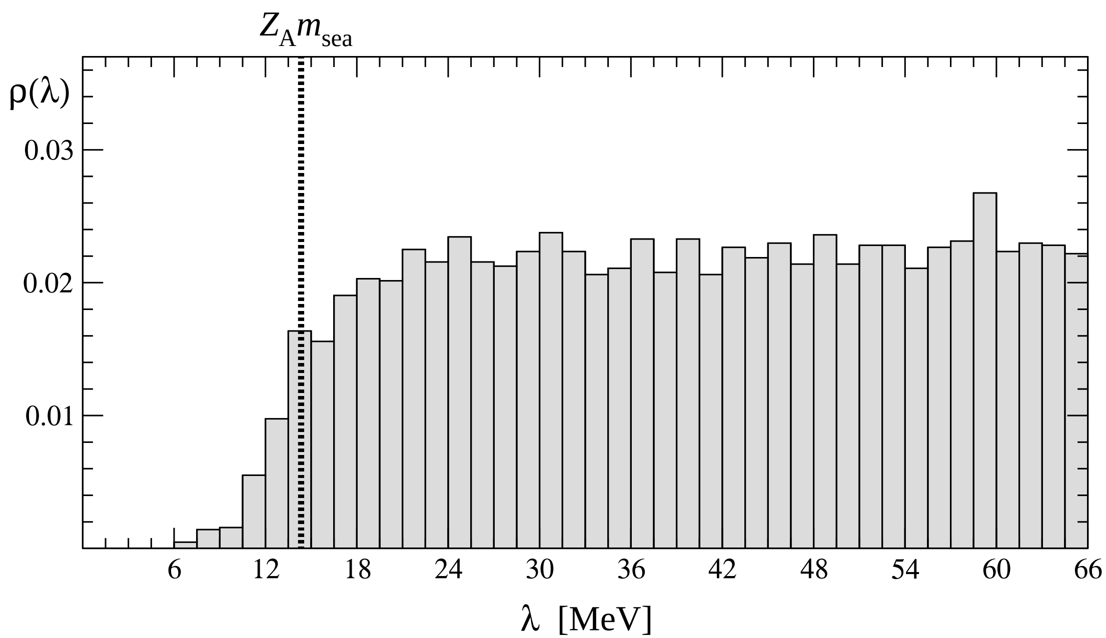


Lüscher Palombi JHEP09(2010)110 Akemann Damgaard Splittorff Verbaarschot PRD 83 (2011) 085014

Necco Shindler JHEP 1104 (2011) 031

Lattice

Spectrum of D_5 for $N_f = 2$



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