Particle acceleration at the knee and beyond

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1 Cosmic Rays

Energy spectrum extends over 12 decades to $> 10^{20} \text{eV} = 10^{11} \text{GeV} = 100 \ \text{EeV} = 16 \ \text{J}$
From the excellent review by Hillas (2006).
2 Acceleration of Cosmic Rays

2.1 Magnetic scattering

Charged particle motion in magnetic field

\[ r_{\text{gyro}} = \frac{p}{qB} \quad \text{(gyro-radius)} \]

where \( p \) is momentum, \( q \) is charge, \( B \) is magnetic field.

- Particle scatters when gyro-radius is comparable to scale of irregularity:
  \[ r_{\text{gyro}} \approx \frac{2\pi}{k} \approx \Delta x \quad \text{(resonant scattering).} \]

- Magnetic energy density per wavenumber \( k \) is
  \[ I(k) = \frac{B(k)^2}{2\mu_0}, \quad B(k) = \int_{-\infty}^{\infty} e^{ikx} B(x) dx. \]
2.2 Turbulence

- Energy density per wavenumber $k$: $I(k) = I_0 (k/k_0)^{-\beta}$ ($k_0 < k < k_{\text{max}}$) where $k_0$ is the smallest wavenumber (corresponding to largest length scale):
  - $\beta = 5/3$ (Kolmogorov turbulence)
  - $\beta = 1$ (saturated turbulence)

2.3 Charged particle scattering and diffusion

- Resonance condition gives scattering mean free path

$$\lambda_{\text{sc}}(p) = r_{\text{gyro}}(p) \frac{B^2/2\mu_0}{I(k)k} \propto p^\alpha$$

with $k = 2\pi/r_{\text{gyro}}$, $r_{\text{gyro}} = p/qB$, charge $q$ (SI units).

- Diffusion coefficient, $\kappa(p) = \lambda_{\text{sc}}(p)v/3 \propto p^\alpha$.

- Minimum or (Bohm diffusion coefficient), $\kappa_B(p) = r_{\text{gyro}}(p)v/3 \propto p^\alpha$.
2.4 Maximum rate of increase in particle momentum

- The maximum electric field induced by motion at speed $v_{\text{bulk}}$ of magnetic fields $B$ is $\sim v_{\text{bulk}} B$.

- For relativistic particles of charge $Ze$ the rate of energy gain (in SI units)
  \[
  \frac{dE}{dt} \bigg|_{\text{acc}} \sim Z e v_{\text{bulk}} B c < Z e c^2 B.
  \]

- Hence, we can write
  \[
  \frac{dE}{dt} \bigg|_{\text{acc}} = \xi Z e c^2 B
  \]
  where $\xi < 1$ is an acceleration rate parameter which depends on the details of the acceleration mechanism.

- Astrophysical shocks have turbulent magnetic fields, bulk motion at speeds approaching $c$, and converging flows.

- Depending on the alignment of $\vec{B}$ with the shock, and the shock speed, the acceleration rate can approach the maximum possible.
3 Important Interactions and Propagation

- pion photoproduction
- photon–photon pair production
- inverse Compton scattering
- Bethe–Heitler pair production
- synchrotron radiation
4 Acceleration and losses in 2.7K CMBR and magnetic field

— Maximum energy as a function of magnetic field of protons for maximum possible acceleration rate parameter $\xi = 1$ (upper), $\xi = 0.04$ (middle), $\xi = 1.5 \times 10^{-4}$ (lower).
— Dot-dot-dot-dash curves are lines of constant Larmor radius as labeled.
4.1 Possible sites of cosmic ray acceleration

— “Hillas plot” showing (chain curves) magnetic field vs. gyroradius.
— Neutron stars (ns), white dwarfs (wd), sunspots (ss), magnetic stars (ms), AGN (ag), ISM (is), SNR (sn), RG lobes (rg), galactic disk (d) and halo (h), clusters of galaxies (cl) IGM (ig), jet-frame synch. proton blazar (bl), jet-frame GRB models (gb).
## 5 Supernovae

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<td>Cas A</td>
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<td>many GRBs</td>
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<td>$&gt; 8 M_\odot$</td>
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<td>SN 1993J</td>
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Candra observations of SN 1006, Tycho’s SNR, and Cas A
5.1 Supernova shocks

- Shown above is an idealization of a type Ia supernova.
- Diffusive shock acceleration can take place at the shocks, most importantly the external shock indicated.
- In other types of supernova the shock propagates in a slow dense (SN type II) or fast low density (SN type Ib) fossil stellar wind.
6 Diffusive Shock Acceleration

- A few charged particles diffuse back and forth and gain energy with each shock crossing:

\[
\frac{\langle \Delta E \rangle}{E} \sim \frac{4(R - 1) V_S}{3 \frac{R}{c}}.
\]
• The region of width \( \left( \frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2} \right) \) is populated by the particles undergoing acceleration.

• The time for one acceleration cycle is

\[
t_{\text{cycle}} \approx \frac{4}{c} \left( \frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2} \right).
\]

(2)

• The acceleration rate is then given by

\[
\frac{dE}{dt}_{\text{acc}} = \frac{\langle \Delta E \rangle}{t_{\text{cycle}}} \approx E \frac{(R - 1)u_1}{3R} \left( \frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2} \right)^{-1}.
\]

(3)

• For rapid acceleration one needs:

  – High compression ratio \( R \)
  – High shock velocity \( V_s \equiv u_1 \)
  – Low diffusion coefficient \( k \) on both sides of the shock.

• A low diffusion coefficient requires high magnetic field and strong turbulence.

How low can the diffusion coefficient be? Is the Bohm diffusion coefficient valid here?
6.1 The diffusion coefficient at the shock from X-ray data

- The cut-off energy in the synchrotron X-rays is directly related to the maximum electron energy and the magnetic field.

\[ \nu^{\text{max}} = 1.2 \times 10^{16} \left( \frac{E_{e}^{\text{max}}}{10 \text{ TeV}} \right)^2 \left( \frac{B}{10 \mu \text{G}} \right) \text{ Hz.} \]

- The maximum electron energy is determined by equating the energy gain rate \( \frac{dE}{dt} \big|_{\text{acc}} = \xi e c^2 B \) to the synchrotron loss rate, giving

\[ E_{e}^{\text{max}} = 1.9 \times 10^3 \xi^{1/2} \left( \frac{B}{10 \mu \text{G}} \right)^{-1/2} \text{ GeV.} \]

- Hence,

\[ \nu_c^{\text{max}} = 4.5 \times 10^{22} \xi \]

- As we have seen,

\[ \xi(E_{e}^{\text{max}}) \propto E_{e}^{\text{max}} \; u_1^2 \kappa(E_{e}^{\text{max}})^{-1} \propto u_1^2 \left( \frac{\kappa_B(E_{e}^{\text{max}})}{\kappa(E_{e}^{\text{max}})} \right). \]

and so we can find \( \kappa_B(E_{e}^{\text{max}})/\kappa(E_{e}^{\text{max}}) \) directly from the X-ray cut-off frequency.
• $\kappa_B(E^\text{max}_e)/\kappa(E^\text{max}_e)$ was found by Stage et al. (2006) to be in the range 2.1–38 for Cas A.

• Parizot et al. found similar values, $\kappa_B(E^\text{max}_e)/\kappa(E^\text{max}_e)$ ranging between 0.2 and 10, for five X-ray SNR.
6.2 Proton acceleration in SN shocks: RX J1713.7-3946

HESS images and profiles
- TeV emission from RX J1713.7-3946 detected by CANGAROO-II (Enomoto et al., 2002, Nature, 416, 823).


- If gamma-rays are from $\pi^0$ decay, spectrum requires acceleration of protons to 200 TeV.

- If gamma-rays are from inverse Compton on CMBR, spectrum requires acceleration of electrons to 100 TeV – unlikely as

$$\left. \frac{dE}{dt} \right|_{acc} = \xi ec^2 B, \quad E_e^{\text{max}} = 1.9 \times 10^3 \xi^{1/2} \left( \frac{B}{10 \, \mu\text{G}} \right)^{-1/2} \text{GeV} \quad \text{and} \quad \xi \ll 1.$$
Galaxy gas distribution with and without CR pressure
(Pfrommer et al., astro-ph/0611084)
6.3 Effect of cosmic ray pressure: shock modification

Simulations show that DSA is quite efficient, whereby up to 50% of the internal energy of shocked downstream plasma can be in cosmic rays.

- As seen in the shock frame, cosmic rays diffusing upstream add their pressure ($P_c$ in diagram) to the thermal pressure upstream.
- This cosmic ray pressure slows the upstream plasma approaching the shock having two effects:
  - increases the overall compression ratio;
  - changes the velocity profile.

(from Diamond and Malkov 2006)
6.4 **Effect of cosmic ray pressure: field amplification**

Cosmic ray pressure gradient gives rise to cosmic ray streaming upstream.

- CR streaming causes Alfvén wave growth upstream, and field is further enhanced by shock passage.
- Bell & Lucek (2001) showed that non-linear amplification of magnetic field by cosmic rays to many times the pre-shock value.
- Markowith, Lemoine & Pelletier (2006) argue that cosmic rays lose energy in generating this upstream turbulence.
6.5 Overall effect of cosmic ray pressure

Higher, and turbulent, magnetic field implies slower diffusion, and faster acceleration, but...

- Overall compression ratio is increased \((R = 8.3\) in example right) from that of a strong hydrodynamical shock, \(R = 4\).

- The full compression ratio is felt by the highest energy cosmic rays which can diffuse rapidly, thereby giving a spectrum flatter than \(E^{-2}\) near the maximum energy.

- The lowest energy cosmic rays, which diffuse slowly, see a smaller compression ratio \((R = 1.9\) in example), giving a spectrum steeper than \(E^{-2}\) at the lowest energies.
6.6 Expected energy spectrum

We expect a curved spectrum, e.g. as calculated by Berezhko, Pühlhofer & Völk (2003) for Cas A (right).

- Markowith, Lemoine & Pelletier (2006) who argue that cosmic rays lose energy in generating this upstream turbulence claim that this steepened the overall spectrum to $E^{-2.3}$ if the SN expands into the warm ISM.

- The maximum energy is determined by:
  - $dE/dt_{acc}$ for sweep-up time
  - scattering length $\sim$ SNR radius

and could extend significantly above a rigidity of $10^{14}$ V expected from the original theory (before shock modification).
Model propagation of CR protons from SNR of given age and distance.

- Source at 0.1 kpc arriving after: (1) $10^{3}$, ... (9) $10^{7}$ years.
### Table of SNR used in modeling the CR spectrum

<table>
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<th>SNR name</th>
<th>distance $r$ (kpc)</th>
<th>Age $t$ (yrs)</th>
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<td>G65.3+5.7</td>
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</tr>
<tr>
<td>G73.9+0.9</td>
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<td>Cygnus Loop</td>
<td>0.4</td>
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<td>HB21</td>
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<td>G114.3+0.3</td>
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<td>CTA1</td>
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<td>HB9</td>
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<td>Vela</td>
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<td>G299.2-2.9</td>
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<td>Monogem$^1$</td>
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<tr>
<td>Geminga$^2$</td>
<td>0.4</td>
<td>340000</td>
</tr>
</tbody>
</table>

$^1$ Plucinsky et al. 1996;  
$^2$ caraveo et al. 1996;
• Concludes there is a missing $10^5$ yr old SNR at 0.1 kpc to give low E part of spectrum.
Zatsepin and Sokolskaya astro-ph/0601475

Also need a special sources To fit low E data.

Model spectra with 3 classes of source:
I. SN into ISM
II. SN into local superbubble
III. Novae.
Nova Cyni 1992 (Hubble images taken about 6 months apart)

- Nova stars produce expanding shells similar to SNR with energy $10^{46}$-$10^{47}$ erg.
- With about 100 per year, the power is $3 \times 10^{40}$-$3 \times 10^{41}$ erg/s.
- This is comparable with that of SN.
- In the model of Zatsepin and Sokolskaya the spectra extend to 200 GV, and are steeper $E^{-2.6}$. 
7 Conclusions

- Cosmic rays at energies up to $\sim 10^{17}$ eV are probably galactic.
- Galactic cosmic rays are probably accelerated SNR shocks
  - up to $\sim 10^{15}$ eV by “normal” SNR
  - up to $\sim 10^{17}$ eV by SNR of massive stars in magnetized winds (e.g. Biermann models)
- There is strong evidence for electron acceleration to $> 10$ TeV, diffusion near the Bohm limit at this energy, and magnetic field amplification.
- Predicted TeV gamma ray fluxes for SNR (e.g. Drury, Aharonian and Völk 1994) depend on matter density in SNR emission region, and distance.
  - Only one detection RXJ1713.7-3946 (CANGAROO and HESS) gives very strong evidence for hadronic acceleration.
  - Distance to RXJ1713.7-3946 is not well known, but observed flux is not in conflict with Drury, Aharonian and Völk.
• Acceleration in pulsar outer gaps and PWN (discussed by Gaenssler) may also be important for some objects, but probably not significant for CR origin.